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A Study of Angular Correlations in 4-jet Final States of Hadronic Z^0 Decays

The OPAL Collaboration

Abstract

Four-jet final states of hadronic Z^0 decays, observed in e^+e^- annihilation around 91 GeV centre of mass energy, are analysed in terms of observables that are sensitive to the non-abelian gauge structure of QCD. After correction for detector resolution and fragmentation effects, the data are compared to QCD and also to predictions of the abelian vector gluon gauge theory. The theoretical expectations are calculated in both second order and leading logarithmic approximation of perturbation theory. The data are compatible with QCD and do not reproduce the predictions of abelian vector gluon models. From the measured topological distributions, upper limits for the relative production rates of $q\bar{q}q\bar{q}$ final states are derived.

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The OPAL Collaboration

M.Z. Akrawy[11], G. Alexander[21], J. Allison[14], P.P. Allport[5], K.J. Anderson[8], J.C. Armitage[6], G.T.J. Arnison[18], P. Ashton[14], G. Azuelos[16,f], J.T.M. Baines[14], A.H. Ball[15], J. Banks[14], G.J. Barker[11], R.J. Barlow[14], J.R. Batley[5], J. Becker[9], T. Behnke[7], K.W. Bell[18], G. Bella[21], S. Bethke[10], O. Biebel[3], U. Binder[9], I.J. Bloodworth[1], P. Bock[10], H. Breuker[7], R.M. Brown[18], R. Brun[7], A. Buijs[7], H.J. Burckhart[7], P. Capiluppi[2], R.K. Carnegie[6], A.A. Carter[11], J.R. Carter[5], C.Y. Chang[15], D.G. Charlton[7], J.T.M. Chrin[14], P.E.L. Clarke[23], I. Cohen[21], W.J. Collins[5], J.E. Conboy[13], M. Couch[1], M. Coupland[12], M. Cuffiani[2], S. Dado[20], G.M. Dallavalle[2], P. Debu[19], M.M. Deninno[2], A. Dieckmann[10], M. Dittmar[4], M.S. Dixit[17], E. Duchovni[24], I.P. Duerdoth[7,d], D.J.P. Dumas[6], H. El Mamouni[16], P.A. Elcombe[5], P.G. Estabrooks[6], E. Etzion[21], F. Fabbri[2], P. Farthouat[19], H.M. Fischer[3], D.G. Fong[15], M.T. French[18], C. Fukunaga[22], A. Gaidot[19], O. Ganel[24], J.W. Gary[10], J. Gascon[16], N.I. Geddes[18], C.N.P. Gee[18], C. Geich-Gimbel[3], S.W. Gensler[8], F.X. Gentit[19], G. Giacomelli[2], V. Gibson[5], W.R. Gibson[11], J.D. Gillies[18], J. Goldberg[20], M.J. Goodrick[5], W. Gorn[4], D. Granite[20], E. Gross[24], J. Grunhaus[21], H. Hagedorn[9], J. Hagemann[7], M. Hansroul[7], C.K. Hargrove[17], I. Harrus[20], J. Hart[5], P.M. Hattersley[1], M. Hauschild[7], C.M. Hawkes[7], E. Heflin[4], R.J. Hemingway[6], R.D. Heuer[7], J.C. Hill[5], S.J. Hillier[1], C. Ho[4], J.D. Hobbs[8], P.R. Hobson[23], D. Hochman[24], B. Holl[7], R.J. Homer[1], S.R. Hou[15], C.P. Howarth[13], R.E. Hughes-Jones[14], R. Humbert[9], P. Igo-Kemenes[10], H. Ihssen[10], D.C. Imrie[23], A. Jawahery[15], P.W. Jeffreys[18], H. Jeremie[16], M. Jimack[7], M. Jobes[1], R.W.L. Jones[11], P. Jovanovic[1], D. Karlen[6], K. Kawagoe[22], T. Kawamoto[22], R.G. Kellogg[15], B.W. Kennedy[13], C. Kleinwort[7], D.E. Klem[17], G. Knop[3], T. Kobayashi[22], T.P. Kokott[3], L. Köpke[7], R. Kowalewski[6], H. Kreutzmann[3], J. Kroll[8], M. Kuwano[22], P. Kyberd[11], G.D. Lafferty[14], F. Lamarche[16], W.J. Larson[4], J.G. Layter[4], P. Le Du[19], P. Leblanc[16], A.M. Lee[15], M.H. Lehto[13], D. Lellouch[7], P. Lennert[10], L. Lessard[16], L. Levinson[24], S.L. Lloyd[11], F.K. Loebinger[14], J.M. Lorah[15], B. Lorazo[16], M.J. Losty[17], J. Ludwig[9], J. Ma[4,b], A.A. Macbeth[14], M. Mannelli[7], S. Marcellini[2], G. Maringer[3], A.J. Martin[11], J.P. Martin[16], T. Mashimo[22], P. Mättig[7], U. Maur[3], T.J. McMahon[1], J.R. McNutt[23], A.C. McPherson[6,c], F. Meijers[7], D. Menszner[10], F.S. Merritt[8], H. Mes[17], A. Michelini[7], R.P. Middleton[18], G. Mikenberg[24], D.J. Miller[13], C. Milstene[21], M. Minowa[22], W. Mohr[9], A. Montanari[2], T. Mori[22], M.W. Moss[14], P.G. Murphy[14], W.J. Murray[5], B. Nellen[3], H.H. Nguyen[8], M. Nozaki[22], A.J.P. O'Dowd[14], S.W. O'Neale[7,e], B.P. O'Neill[4], F.G. Oakham[17], F. Odorici[2], M. Ogg[6], H. Oh[4], M.J. Oreglia[8], S. Orito[22], J.P. Pansart[19], G.N. Patrick[18], S.J. Pawley[14], P. Pfister[9], J.E. Pilcher[8], J.L. Pinfold[24], D.E. Plane[7], B. Poli[2], A. Pouladdej[6], T.W. Pritchard[11], G. Quast[7], J. Raab[7], M.W. Redmond[8], D.L. Rees[1], M. Regimbald[16], K. Riles[4], C.M. Roach[5], S.A. Robins[11], A. Rollnik[3], J.M. Roney[8], S. Rossberg[9], A.M. Rossi[2,a], P. Routenburg[6], K. Runge[9], O. Runolfsson[7], S. Sanghera[6], R.A. Sansum[18], M. Sasaki[22], B.J. Saunders[18], A.D. Schaile[9], O. Schaile[9], W. Schappert[6], P. Scharff-Hansen[7], S. Schreiber[3], J. Schwarz[9], A. Shapira[24], B.C. Shen[4], P. Sherwood[13], A. Simon[3], P. Singh[11], G.P. Siroli[2], A. Skuja[15], A.M. Smith[7], T.J. Smith[1], G.A. Snow[15], R.W. Springer[15], M. Sproston[18], K. Stephens[14], H.E. Stier[9], R. Stroehmer[10], D. Strom[8], H. Takeda[22], T. Takeshita[22], T. Tsukamoto[22], M.F. Turner[5], G. Tysarczyk-Niemeyer[10], D. Van den plas[16], G.J. VanDalen[4], G. Vasseur[19], C.J. Virtue[17], H. von der Schmitt[10], J. von Krogh[10], A. Wagner[10], C. Wahl[9], C.P. Ward[5], D.R. Ward[5], J. Waterhouse[6], P.M. Watkins[1], A.T. Watson[1], N.K. Watson[1], M. Weber[10], S. Weisz[7], P.S. Wells[7], N. Wermes[10], M. Weymann[7], G.W. Wilson[19], J.A. Wilson[1], I. Wingerter[7], V-H. Winterer[9], N.C. Wood[13], S. Wotton[7], B. Wuensch[3], T.R. Wyatt[14], R. Yaari[24], Y. Yang[4,b], G. Yekutieli[24], T. Yoshida[22], W. Zeuner[7], G.T. Zorn[15].

[1]School of Physics and Space Research, University of Birmingham, Birmingham, B15 2TT, UK
[2]Dipartimento di Fisica dell' Università di Bologna and INFN, Bologna, 40126, Italy
[3]Physikalisches Institut, Universität Bonn, D-5300 Bonn 1, FRG

[4]Department of Physics, University of California, Riverside, CA 92521 USA
[5]Cavendish Laboratory, Cambridge, CB3 0HE, UK
[6]Carleton University, Dept of Physics, Colonel By Drive, Ottawa, Ontario K1S 5B6, Canada
[7]CERN, European Organisation for Particle Physics, 1211 Geneva 23 Switzerland
[8]Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago Illinois 60637, USA
[9]Fakultät für Physik, Albert Ludwigs Universität, D-7800 Freiburg, FRG
[10]Physikalisches Institut, Universität Heidelberg,Heidelberg, FRG
[11]Queen Mary and Westfield College, University of London, London, E1 4NS, UK
[12]Birkbeck College, London, WC1E 7HV, UK
[13]University College London, London, WC1E 6BT, UK
[14]Department of Physics, Schuster Laboratory, The University, Manchester, M13 9PL, UK
[15]Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742, USA
[16]Laboratoire de Physique Nucléaire, Université de Montréal, Montréal, Quebec, H3C 3J7, Canada
[17]National Research Council of Canada, Herzberg Institute of Astrophysics, Ottawa, Ontario K1A 0R6, Canada
[18]Rutherford Appleton Laboratory, Chilton, Didcot, Oxfordshire, OX11 0QX, UK
[19]DPhPE, CEN Saclay, F-91191 Gif-sur-Yvette, France
[20]Department of Physics, Technion-Israel Institute of Technology, Haifa 32000, Israel
[21]Department of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel
[22]International Centre for Elementary Particle Physics and Dept of Physics, University of Tokyo, Tokyo 113, and Kobe University, Kobe 657, Japan
[23]Brunel University, Uxbridge, Middlesex, UB8 3PH UK
[24]Nuclear Physics Department, Weizmann Institute of Science, Rehovot, 76100, Israel
[a]Present address: Dipartimento di Fisica, Università della Calabria, 87036 Rende, Italy
[b]On leave from Harbin Institute of Technology, Harbin, China
[c]Now at Applied Silicon Inc
[d]On leave from Manchester University
[e]On leave from Birmingham University
[f]and TRIUMF, Vancouver, Canada

1 Introduction.

The compatibility of Quantum Chromodynamics (QCD) with data from many different hard scattering processes leaves little, if any, doubt about the validity of QCD as the theory of the strong interactions between quarks and gluons [1]. Some of the basic ingredients of QCD are the process of gluon self coupling and, as a physical consequence, the phenomenon of "asymptotic freedom". Asymptotic freedom implies that the QCD coupling strength, α_s, decreases with increasing energy, an expectation which has been confirmed in recent studies of multijet production rates in hadronic final states of e^+e^- annihilation (see [2,3,4] and references quoted therein). One of the most promising signatures of the gluon self coupling or, equivalently, of the triple gluon vertex, is the angular correlation of jet directions in 4-jet hadronic final states of e^+e^- annihilation reactions. Several observables have been proposed to test the specific spin structure of the process $g \rightarrow gg$ within such events [5,6,7,8].

In the analysis presented here, 4-jet final states from a large sample of hadronic decays of the Z^0 boson, measured with the OPAL detector [9] at LEP in e^+e^- annihilations around centre of mass energies, E_{cm}, of 91 GeV, are analysed in terms of two of these observables. The main purpose of this analysis is to test the compatibility of QCD predictions with the measured angular correlations. The data are also compared with the predictions of an abelian vector gluon gauge theory in which the gluon self coupling does not exist. This alternative theory is mainly used as a tool to demonstrate the sensitivity of the utilised observables to the characteristic features of QCD; that the abelian theory is not a realistic theory of the strong interactions is already demonstrated in other measurements, like the energy dependence of multijet production rates in e^+e^- annihilations (see e.g. [2,4]).

In analyses where quark and gluon jets cannot be individually tagged, angular correlations within 4-jet events are more sensitive to the specific spin structure of $q\bar{q}q\bar{q}$ final states rather than to the existence of the gluon self coupling [10]. Nevertheless the experimental study of these observables allows discrimination between QCD and the abelian alternative, since the predicted production rates of $q\bar{q}q\bar{q}$ final states are significantly different. Until now one experimental study of angular correlations in 4-jet final states has been published [11], a study which suffered, however, from low statistical significance and which was based on partly incorrect theoretical predictions of the abelian vector theory [10].

2 Four-Jet Angular Correlations as a Test of QCD.

The gauge structure of QCD is visible only in second or higher perturbative order, where the gluon self coupling contributes through the existence of the triple gluon vertex. In e^+e^- annihilation, the triple gluon vertex as illustrated in Fig. 1a is predicted to be the dominant source of 4-jet events. Other processes that lead to 4-parton final states are double-gluon bremsstrahlung (Fig. 1b,c) and gluon splitting into a quark-antiquark pair (Fig. 1d). The interference between diagrams a, b and c preclude discrimination between the different sources of $q\bar{q}gg$ final states on an event-by-event basis. The specific action of the gluon self coupling can however be tested by comparing the jet distributions predicted by QCD to those derived from a fictitious abelian gauge theory where diagram (a) does not exist.

Within the abelian vector gluon theory, 4-quark final states (diagram (d)) are predicted to be produced more often than in the case of QCD. The relative contributions $R_{q\bar{q}q\bar{q}}$ of 4-quark final states, predicted in second order perturbation theory and normalised to the total number of 4-jet events with jet pair-masses M_{ij} of $M_{ij}^2 > 0.01 \cdot E_{cm}^2$, are about 4.7% for QCD and 31.4% for the abelian theory [10]. Observables that are sensitive to the different spin structures of events of the process (a), namely a gluon of spin 1 decaying to two spin 1 particles, and process (d), a gluon decaying to two spin 1/2 particles, have been proposed to test the existence of the triple gluon vertex. They represent angular correlations of jet momenta $\vec{p}_i$ in 4-jet events. In this analysis, the following observables, which do not require the identification of quark- and gluon-jets within individual events, are studied:

- The angle χ_{BZ}, proposed by Bengtsson and Zerwas [7], which is defined as the angle between the two planes spanned by the jet-momenta $\vec{p}_1$ and $\vec{p}_2$ and by $\vec{p}_3$ and $\vec{p}_4$, where the convention is such that the momentum vectors within a 4-jet event are ordered according to the jet energies, $E_1 \geq ... \geq E_4$. The angle χ_{BZ} is illustrated in Fig. 2a, for a typical configuration of a 4-jet event.
- The angle θ^*_{NR}, originally proposed by Nachtmann and Reiter [6] and as modified by Bengtsson [8] is defined by the angle between the two vectors $\vec{p}_1 - \vec{p}_2$ and $\vec{p}_3 - \vec{p}_4$, where the momenta are again ordered as indicated above. θ^*_{NR} is illustrated in Fig. 2b.

The angle ϕ_{KSW}, proposed by Körner, Schierholz and Wilrodt [5], is not very sensitive to the difference between QCD and the abelian model if quark and gluon jets cannot be individually tagged [10]. This variable is therefore not used in this analysis.

3 Experimental Reconstruction of Four-Jet Angular Correlations.

To reconstruct the jet multiplicity and the corresponding jet momenta in each hadronic event, the jet algorithm introduced by JADE [12] is used. Resolvable jets of particles are defined by demanding that in each event the scaled jet pair masses y_{kl} exceed a threshold value y_{cut}:

$$y_{kl} = \frac{2E_k E_l \cdot (1 - cos\theta_{kl})}{E_{vis}^2} > y_{cut}, \quad (1)$$

where θ_{kl} is the angle between jets k and l and E_{vis} is the total visible energy of the event. This definition of resolvable jets is identical to the corresponding definition of resolvable partons in $O(\alpha_s^2)$ perturbative QCD calculations, resulting in a close agreement between parton rates and experimental jet rates as demonstrated in several studies over a wide range of centre of mass energies; see e.g. [2,12]. For different values of the jet resolution parameter y_{cut} each event is classified as an n-jet event, and the respective four-momenta of the jet axes are calculated from the sum of the particle four-momenta associated with each jet. The measured momenta of both charged and neutral particles are used in this analysis, as described in detail in [2]. For identified 4-jet events that fulfil the additional requirements described in the next section, χ_{BZ} and $\cos\theta^*_{NR}$ are calculated from the reconstructed momenta of jets.

In order to compare the data to the theoretical expectations of QCD and of the abelian theory, the measured angular distributions are corrected for the limited resolution and acceptance of the detector and for the effects of hadronisation, the transition of quarks and gluons to observable hadrons, using a bin-by-bin multiplication method. The correction factors are obtained from two samples of model events, generated with the Jetset QCD shower and hadronisation model [13] with parameters optimised to describe the global properties of hadronic Z^0 decays [14]: the first sample includes hadronisation, initial state radiation, a simulation of the OPAL detector and the same event reconstruction and event selection criteria as applied to the real data, the second sample consists of parton final states without further hadronisation. The correction factors are determined by the ratio of the corresponding distributions from the second and the first sample, calculated for each bin. The corrected distributions are then compared to the theoretical expectations calculated for quark and gluon final states.

This technique automatically includes the correction for background coming from events which after hadronisation are misreconstructed as 4-jet events: Studies with the Jetset QCD shower model at e^+e^- centre of mass energies around the Z^0 pole show that between 67% and 75% of the 4-jet events at y_{cut} values between 0.01 and 0.04, respectively, originate from events which exhibit 4-jet structure at the parton level, i.e. before hadronisation, at identical values of y_{cut}. Most of the remaining background comes from partonic 3-jet events. More than half of these background events, however, are classified as 4-jet events at the parton level in a *window* of ± 0.005 around the particular value of y_{cut} at which the 4-jet structure is tested. This window corresponds to the approximate experimental uncertainty in reconstructing jet pair masses in multijet events, as our studies show; the purity of 4-jet events is larger than 85% if we allow for a corresponding difference between the invariant jet masses on the parton level and after hadronisation. Thus, regardless of how one measures the "purity" of the experimental 4-jet event sample, the background from non-4-jet events is sufficiently small to be included in the overall correction procedure.

In order to render the correction procedure meaningful, the bin sizes of the experimental distributions must be chosen according to the experimental resolutions for reconstructing the angular observables. This resolution is again studied with the Jetset parton shower plus hadronisation program. For each generated event, the observables are calculated from the final partons at the end of the shower and from the reconstructed particles after hadronisation, detector simulation and event reconstruction as described above. At both stages, the jet-multiplicities and the corresponding jet axes are reconstructed with the jet finder using identical values of y_{cut} [1]. For events identified as having 4 jets both before and after hadronisation and that satisfy the additional angle and jet energy requirements described in Section 4, the difference $\Delta O = O_{partons} - O_{jets}$ is calculated for each observable O and sampled in a histogram. Note that in this resolution study no attempt is made to isolate events with kinematic properties besides those which can be applied in a realistic analysis of measured hadronic events.

The distributions ΔO are displayed in Fig. 3 for the observables χ_{BZ} and $\cos\theta^*_{NR}$ from 4-jet events with $y_{cut} = 0.01$ and 0.02. The distributions peak around zero, with r.m.s widths of 20.1 degrees for χ_{BZ} and 0.22 for $\cos\theta^*_{NR}$ at $y_{cut} = 0.01$ and of 17.4 degrees and 0.16 at $y_{cut} = 0.02$, respectively. Note that these numbers contain the effects due to the hadronisation process and due to the limited acceptance and resolution of the detector. The clear peak structures around zero and the limited widths of the distributions demonstrate that the experimental reconstruction of angular correlations in 4-jet events is only moderately affected by these effects. It is therefore concluded that the analyzing power of the experimental method is well suited to study angular correlations in hadronic 4-jet final states of Z^0 decays. As a result of these resolution studies we choose bin-sizes of 18 degrees in χ_{BZ} and 0.2 in $\cos\theta^*_{NR}$ for the further analysis and data correction procedure.

4 Experimental Results and Comparison with Theory.

The following analysis is based on a sample of 80,200 hadronic decays of Z^0 bosons, corresponding to a total integrated luminosity of about 3.5 pb^{-1}. The event selection criteria are as described in [2], with the exception that the requirement $|cos\theta_T| < 0.9$ for the angle between the event thrust axis

[1] The reconstruction and recombination of parton-jets is necessary since a typical parton shower event consists of about 10 final partons, of which many correspond to soft radiation processes with virtualities down to $Q_0 = 1$ GeV/c^2.

and the beam line is replaced by the demand that all reconstructed jet axes satisfy $|cos\theta_{jet}| \leq 0.9$, since the thrust axis is not well defined in spherical, 4-jet like events. A total of 62,365 events remain after these global event selection cuts. Four-jet events are reconstructed for different jet resolution criteria (y_{cut} = 0.010, 0.015, 0.020, 0.030 and 0.040), and for each of these samples the distributions of χ_{BZ} and $\cos\theta^*_{NR}$ are calculated from the reconstructed event jet axes. At this point additional event selection criteria are imposed:

- The observable χ_{BZ} is calculated and sampled only for events in which the angles between those jet axes that define a plane (see Fig. 2) are smaller than 160 degrees. This reduces background from events where one or both of these planes are not well defined.

- In the case of $\cos\theta^*_{NR}$, we require that the energy ratio between the second- and third-largest jet, E_2/E_3, is larger than 2. This condition enhances the expected difference between the expectations from QCD and the abelian theory [10].

These additional cuts reduce the available event statistics by factors of 2 to 10, depending on the observable under study and on the value of y_{cut} used to reconstruct jets. For $y_{cut} = 0.01$, 3880 4-jet events remain for the analysis of χ_{BZ} and 2978 events for $\cos\theta^*_{NR}$. The corresponding numbers for $y_{cut} = 0.02$ are 2080 and 756, respectively, and decrease to 589 and 72 events for $y_{cut} = 0.04$.

The distributions of χ_{BZ} and $\cos\theta^*_{NR}$, after corrections for detector acceptance and resolution and for hadronisation effects as described above, are shown in Fig. 4 and 5 for y_{cut} = 0.010 and 0.020. The evolution of the mean values of these distributions for values of y_{cut} between 0.01 and 0.04 is demonstrated in Fig. 6. The data are compared to the expectations of QCD, calculated from various parton generators based on both second order and leading logarithmic approximations of perturbation theory:

1. Four-parton final states according to second order QCD perturbation theory are generated with Jetset [13]. Predictions of the χ_{BZ} and $\cos\theta^*_{NR}$ distributions were in the past usually based on such calculations [5,6,7,8] since they are exact to a given (second) order. The shape of these distributions does not depend on the strong coupling constant α_s, which is the only free parameter within these calculations. Models based on second order QCD calculations, however, do not provide a good overall description of the highest energy data available today [14,15,16,17], which is probably because of missing contributions from higher order terms.

2. As another option of the Jetset program, parton showers with parton invariant masses Q_g as low as 1 GeV are generated according to probabilities calculated in the leading-logarithmic approximation to all orders. These calculations contain some, but not all, higher order contributions and can therefore be used to estimate the magnitude of higher order effects. Models based on parton showers provide in general a more accurate and complete description of the experimental data than do models based only on second order calculations [14,15,16,17]. Jetset version 7.2 includes simulation of the azimuthal distribution of parton splittings according to gluon spin polarisation and approximations of closest-neighbour colour coherence effects. The QCD shower parameters Λ_{LLA} = 290 MeV and Q_g = 1 GeV are taken from our previous publication [14]. The other options and parameters, such as the choice of the energy scale for the running coupling constant α_s, are kept at their default values.

3. As a further systematic check of the QCD predictions, we also study the QCD shower model HERWIG (version 4.2) [18], which is based on a different approach to the leading-log parton shower cascade. This model includes a correct and more complete treatment of azimuthal correlations in parton splitting processes than does Jetset. The parameters of the HERWIG QCD

parton generator are all kept at their default values, which were determined largely from an optimisation to the global hadronic event shapes observed in our experiment [14].

In addition to these different QCD models, we also compare the data to the predictions of various non-QCD calculations, namely to two different models of the abelian vector theory and to a simple phase space generator of 4-jet final states. With the comparison to such models we verify the sensitivity of the analysed angular distributions to the specific gauge structure of QCD and study the ability to discriminate against simple kinematic effects, rather than aim to exclude these alternative models which are already known not to provide a realistic description of the data in many other respects.

4. The second order generator, as implemented in the Jetset program, is used to generate 4-parton final states according to the abelian vector theory by simply replacing the group constants of QCD by those appropriate for the abelian case (see e.g. [6]): $N_c = 3 \rightarrow 0$, $C_f = 4/3 \rightarrow 1$ and $T_r = N_f/2 \rightarrow 3N_f$; N_c being the numbers of colours and N_f the number of quark flavours. As in the case of QCD, this model contains no free parameter which influences the shape of the χ_{BZ} and $\cos\theta^*_{NR}$ distributions.

5. The Jetset model can also generate parton showers according to the probabilities of an abelian vector theory. The multitude of the available features in this model requires specifying the parameters and options that we have chosen to use: The probabilities for parton branching are calculated according to the abelian vector gluon theory as described above. The abelian coupling strength in the shower, α_a, is set to be constant. Since α_a is expected to *decrease* with decreasing momentum transfer Q^2 (rather than *increase* as α_s in the case of QCD), this is considered to be a conservative approach. We chose $\alpha_a = 0.25$ such that the mean parton multiplicity at an intermediate state of the shower ($Q_g = 6$ GeV) is similar to that expected from QCD. We checked that the results presented below do not depend on the specific value of α_a. We disabled the angular ordering in the parton cascade, which simulates the effect of soft gluon destructive interference in the case of QCD, but we preserved the decreasing order of the parton virtual masses in the shower [19]. We also removed closest-neighbour interference effects in the azimuthal parton distributions, but retained the effects due to the gluon spin and gluon polarisation. The choice of parameters for the abelian shower model is not unique; the predictions of this model, however, are used only to verify the sensitivity of the observables to the physical process under study.

6. We also compare the data to the angular distributions predicted by a simple model where four momentum-balanced jets are generated according to a phase space distribution.

It can be seen in Figs. 4, 5 and 6 that the different QCD calculations qualitatively agree in the overall shapes and slopes of the χ_{BZ} and $\cos\theta^*_{NR}$ distributions, but partly differ in their actual details. For better visibility the areas which are included by the different QCD curves are hatched and labelled "*QCD*". In most cases the $O(\alpha_s^2)$ and the Jetset QCD shower curves define the outer boundaries of the QCD areas, while the Herwig QCD shower curves are usually closer to the $O(\alpha_s^2)$ prediction. We take the differences between the predictions of the various QCD models as systematic uncertainty and point out that the data are always in good agreement with QCD within the given errors.

The distribution predicted by each abelian vector model is significantly different from that derived according to the corresponding QCD model. The $O(\alpha_a^2)$ and the shower model approach for the abelian theory, however, again differ in the detailed shapes of the distributions; the area enclosed by these calculations is therefore hatched and labelled "*Abelian*" to indicate the respective theoretical uncertainty. With the exception of the $\cos\theta^*_{NR}$ distribution at $y_{cut} = 0.02$ and the mean values of the $\cos\theta^*_{NR}$ distribution for $y_{cut} \geq 0.02$, the areas enclosed by the abelian models are clearly separated from those of QCD. Within the regions of clear separation, the observables χ_{BZ} and $\cos\theta^*_{NR}$ are therefore well suited to significantly test the non-abelian gauge structure of QCD. The fact that the data are in good agreement with the QCD predictions but disfavour the expectations of the abelian models, constitutes a significant test of the validity of QCD.

For the sake of better visibility, the predictions obtained from the phase space 4-jet generator are not individually displayed in all the graphs of Figs. 4, 5 and 6 but are discussed in the following. The phase space distributions are basically flat in $\cos\theta^*_{NR}$, as expected from direct analytical calculations [6]. This leads to an almost constant mean value of $\cos\theta^*_{NR}$ independent of the value of y_{cut}, as demonstrated in Fig. 6b. In the case of χ_{BZ}, the phase space model lies always in the shaded area of the abelian models, thus also predicting significant differences from QCD. These studies therefore confirm the previous observation that the agreement of the data with the predictions of QCD constitutes a non-trivial test of QCD.

The most prominent difference between the theoretical curves is observed for the expectation of a hypothetical, pure sample of $q\bar{q}q\bar{q}$ events as shown in Figs. 4 a and 5 a, calculated from the second order QCD generator. Further study reveals that it is mainly the contribution from the process $g \rightarrow q\bar{q}$ that causes the specific differences between QCD and the abelian theory in these observables, rather than the triple gluon coupling $g \rightarrow gg$. Without the individual identification of quark- and gluon-jets, $q\bar{q}gg$ events from QCD (diagrams a-c in Fig. 1) and the abelian theory (including only the diagrams from double bremsstrahlung, b and c) do not exhibit a detectable difference [10]. We therefore point out that the techniques used to analyse angular correlations in 4-jet final states do not allow "direct" observation of the triple gluon vertex. Instead these methods, without individual quark- and gluon-jet identification, provide the possibility to detect and analyse the specific spin structure of $q\bar{q}q\bar{q}$ events. Since QCD and the abelian theory predict significantly different admixtures of $q\bar{q}q\bar{q}$ events in 4-jet final states, this analysis still provides the means of a significant test of the nonabelian nature of the strong interactions.

5 Upper Limits on the Production of $q\bar{q}q\bar{q}$ Final States.

Since the observables studied in this analysis are mainly sensitive to the specific spin structure of 4-quark final states, experimental limits can be obtained on the relative production rate $R_{q\bar{q}q\bar{q}}$ of $q\bar{q}q\bar{q}$ events, normalised by the total rate of 4-jet events observed. While the measurement of this number allows a clear distinction between the predictions of QCD and of the abelian theory ($R_{q\bar{q}q\bar{q}} = 4.7\%$ and 31.4% at $y_{cut} = 0.01$, respectively), it also provides the possibility to place limits on (or find evidence for) other, non-standard processes which lead to additional 4-jet final states by the decay of a gluon to two fermions. An example of such a process is the splitting of a gluon into two light and long-lived gluinos $\tilde{g}$, the supersymmetric spin-1/2 partners of the gluons as postulated by the extended theory of supersymmetry. Because the resulting events would most likely resemble normal 4-jet events, this process would not have been detected by techniques based on missing-energy signatures [20].

We estimate an upper limit on $R_{q\bar{q}q\bar{q}}$ from the measured mean values of the χ_{BZ} and $\cos\theta^*_{NR}$ distributions at $y_{cut} = 0.01$ using the following equation:

$$< O >^{(exp)} = R_{q\bar{q}q\bar{q}} \cdot < O >^{(theor)}_{q\bar{q}q\bar{q}} + (1 - R_{q\bar{q}q\bar{q}}) \cdot < O >^{(theor)}_{q\bar{q}gg}, \qquad (2)$$

where $< O >$ denotes the mean value of an observable, *exp* stands for the experimental measurement and *theor* for the respective theoretical predictions. The expectations for the two different event classes $Z^0 \rightarrow q\bar{q}q\bar{q}$ and $\rightarrow q\bar{q}gg$ are calculated with the second order parton generator of the Jetset model, which provides definite predictions for these different event classes without dependence on model parameters. We do not attempt to determine limits on $R_{q\bar{q}q\bar{q}}$ from a comparison to QCD parton shower models since the definition of resolvable quark- or gluon jets is not unambiguous in the environment of a parton cascade. Different definitions of quark-jets and different kinds of shower models lead to various experimental limits on $R_{q\bar{q}q\bar{q}}$ which can be interpreted only within this particular model.

The experimental mean values are corrected, like the differential distributions discussed above, for the effects of detector acceptance and of the hadronisation process as determined by the Jetset parton shower model. Such a procedure corrects the data to a parton stage which corresponds to minimal parton invariant masses of $Q_g = 1$ GeV, containing much more (soft) gluon radiation than the second order parton generator. Like the hadronisation process, however, the soft part of the parton shower alters the distribution slightly, such that the expected difference between QCD and the abelian theory is increased. This effect is partly responsible for the differences observed in Figs. 4 and 5 between the second order and the parton shower predictions. We did not correct for these differences in the general analysis of the differential distributions presented above, but rather included them in the overall theoretical uncertainty. For estimating an upper limit of $R_{q\bar{q}q\bar{q}}$ from a comparison with $O(\alpha_s^2)$ calculations, however, a more conservative approach is to correct the data also for soft parton shower effects. We have chosen to correct the data to a parton stage which corresponds to minimal parton invariant masses of $Q_g = 6$ GeV, for which the mean multiplicity of parton shower events is similar to the case of second order parton final states at $y_{cut} = 0.01$.

The measured mean values of χ_{BZ} and $\cos\theta^*_{NR}$ for 4-jet events at $y_{cut} = 0.01$ are listed in Table 1. Also listed are the correction factors, the corrected data results and the expectations from the second order parton generator for both $q\bar{q}q\bar{q}$ and $q\bar{q}gg$ events. From these numbers we determine the following limits on $R_{q\bar{q}q\bar{q}}$: $R_{q\bar{q}q\bar{q}} \leq 5.7\%$ or $R_{q\bar{q}q\bar{q}} < 11.8\%$ with 68% or 95% confidence level, respectively, from $< \chi_{BZ} >$ and $R_{q\bar{q}q\bar{q}} < 4.7\%$ or $R_{q\bar{q}q\bar{q}} < 9.1\%$ with 68% and 95% confidence level from $< \cos\theta^*_{NR} >$ (the 68% and 95% confidence levels correspond to the one-tailed gaussian integral of the probability function, restricted to positive values of $R_{q\bar{q}q\bar{q}}$). We checked that similar limits are derived from fits to the *differential* distributions.

The results are compatible with the second order QCD expectation of $R^{(QCD)}_{q\bar{q}q\bar{q}} = 4.7\%$, and clearly rule out the prediction of the abelian model ($R^{(abel.)}_{q\bar{q}q\bar{q}} = 31.4\%$). The possibility of the additional production of $q\bar{q}\tilde{g}\tilde{g}$ events as described above, which would raise the QCD plus Supersymmetry prediction [2] to $R^{(susy)}_{q\bar{q}q\bar{q}} = 4.7\% \cdot 8/5 = 7.5\%$ can currently be ruled out with about 89% confidence level (from $< \cos\theta^*_{NR} >$ at $y_{cut} = 0.01$). Note, however, that this limit is (QCD-) model dependent and is strictly valid only within the second order calculations. More data statistics and a reduction of systematic uncertainties in the correction procedure are necessary to derive more significant limits on the production of non-standard 4-fermion final states.

6 Summary and Discussion.

The OPAL event sample of hadronic Z^0 decays was analysed in terms of 4-jet event observables that are sensitive to the specific nonabelian gauge structure of QCD. Such a study is an important test

[2]Since gluinos are expected to exist in a colour octet, their relative contribution to this process would be equivalent to the production of three new quark flavours, hence the expected rate of four-fermion final states is increased by a factor of 8/5.

of the validity of QCD as the theory of the strong interactions, and is complementary to testing the characteristic running of the coupling strength α_s. For an event sample of almost 8000 identified and reconstructed 4-jet events and after some additional kinematic cuts, the observables are calculated from the reconstructed jet axes. No attempt is made to identify individual quark and gluon jets within the events; instead the observables are calculated after ordering the jets according to their energy. The analysis therefore relies on the statistical assumption that, according to the characteristic bremsstrahlung spectrum of gluons radiated from quarks, the two least energetic jets of a 4-jet event are mainly due to gluons from double bremsstrahlung or from the processes $g \rightarrow gg$ (the triple gluon vertex) and $g \rightarrow q\bar{q}$. The differential distributions are corrected for detector resolution and acceptance, as well as for hadronisation effects determined from the Jetset shower model.

The same model was used to study the influence of the hadronisation process on the experimental reconstruction of the observables. As a result, the r.m.s. resolutions are about or better than 20 degrees for χ_{BZ} and 0.2 for $\cos\theta^*_{NR}$. These resolutions and the fact that hadronisation and soft parton showers affect the shapes and mean values of the analysed distributions by only a few per cent, demonstrate that experimental studies of χ_{BZ} and $\cos\theta^*_{NR}$ in hadronic Z^0 decays are well suited for significant tests of QCD.

The corrected data are compared to theoretical predictions calculated both in second order and in the leading-log approximation of perturbation theory. The differences between the expectations of these different approaches are taken as a systematic uncertainty of the theoretical expectations. In order to demonstrate the sensitivity of the observables to the specific spin structure of 4-parton final states, the data and the QCD predictions are also compared to calculations of the abelian vector gluon gauge theory where the process of gluon self coupling does not exist. The expectations of the abelian theory are again calculated both in second order and in the leading-log approximation of perturbation theory. Although the two perturbative approaches result in different expectations for the detailed shapes of the observables, there is a consistent and clear distinction predicted between QCD and the abelian theory for both observables χ_{BZ} and $\cos\theta^*_{NR}$. The data are always compatible with QCD and clearly disfavour the predictions of the abelian models.

In an analysis where quark and gluon jets are not identified, the observables χ_{BZ} and $\cos\theta^*_{NR}$ are mainly sensitive to the specific spin structure of events where a radiated gluon decays into two fermions, while signatures of the triple gluon vertex cannot be discriminated from double gluon bremsstrahlung. Thus in this analysis we see a clear difference between the predictions of QCD and the abelian theory, but we cannot isolate and identify the process of gluon self coupling. Nevertheless the good agreement between the measured distributions and the QCD expectations is an important consistency test of QCD, since the predicted relative rates of $q\bar{q}q\bar{q}$ final states within the 4-jet sample are significantly different in QCD and in the abelian theory.

From the measured mean values of χ_{BZ} and $\cos\theta^*_{NR}$ and the second order perturbative QCD predictions, we derive upper limits on the production rate of four-fermion final states, relative to all 4-jet events, of $R_{q\bar{q}q\bar{q}} < 9.1\%$ with 95% confidence level. The expectations of QCD and the abelian theory, in second order perturbation theory, are $R_{q\bar{q}q\bar{q}} = 4.7\%$ and 31.4%, respectively. The measured limit therefore rules out the abelian theory but is consistent with QCD. It can also be used to set limits on non-standard processes that lead to additional 4-fermion 4-jet final states.

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	$< \chi_{BZ} >$	$< \cos\theta^*_{NR} >$
measured mean	40.42 ± 0.39	0.634 ± 0.005
corr. factor	1.075 ± 0.014	$0.913 \pm .011$
corrected mean	43.47 ± 0.70	0.579 ± 0.008
$q\bar{q}q\bar{q}$ mean	54.98 ± 0.20	0.388 ± 0.002
$q\bar{q}gg$ mean	43.71 ± 0.26	0.580 ± 0.003
$R_{q\bar{q}q\bar{q}}$ (68% c.l.)	$< 5.7\%$	$< 4.7\%$
$R_{q\bar{q}q\bar{q}}$ (95% c.l.)	$< 11.8\%$	$< 9.1\%$

Table 1: Mean values of χ_{BZ} and $\cos\theta^*_{NR}$ of the data before and after the correction for hadronisation and soft parton shower effects, as well as the predictions for $q\bar{q}q\bar{q}$ and $q\bar{q}gg$ events in second order perturbation theory. Also given are upper limits on the relative production of $q\bar{q}q\bar{q}$ 4-jet final states, calculated from the preceding numbers.

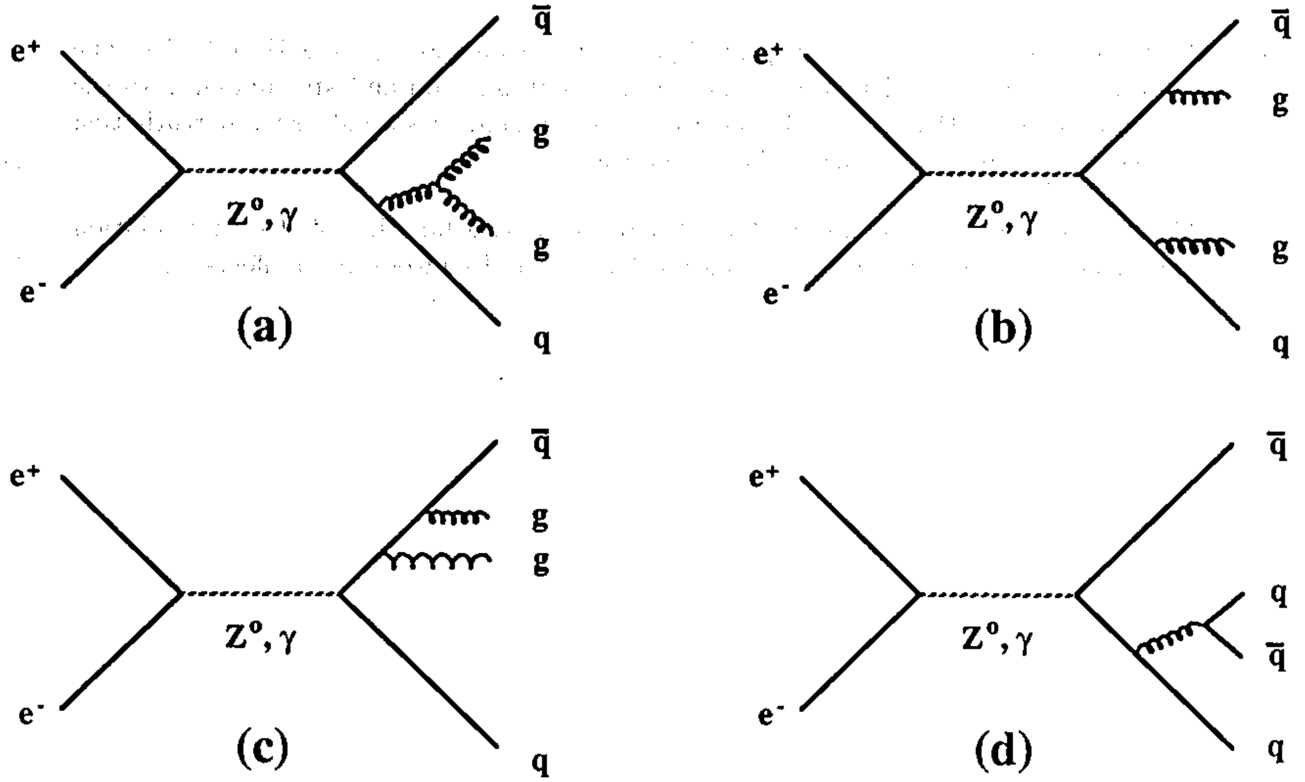

Figure 1: Basic Feynman diagrams for the process $e^+e^- \rightarrow 4$ jets.

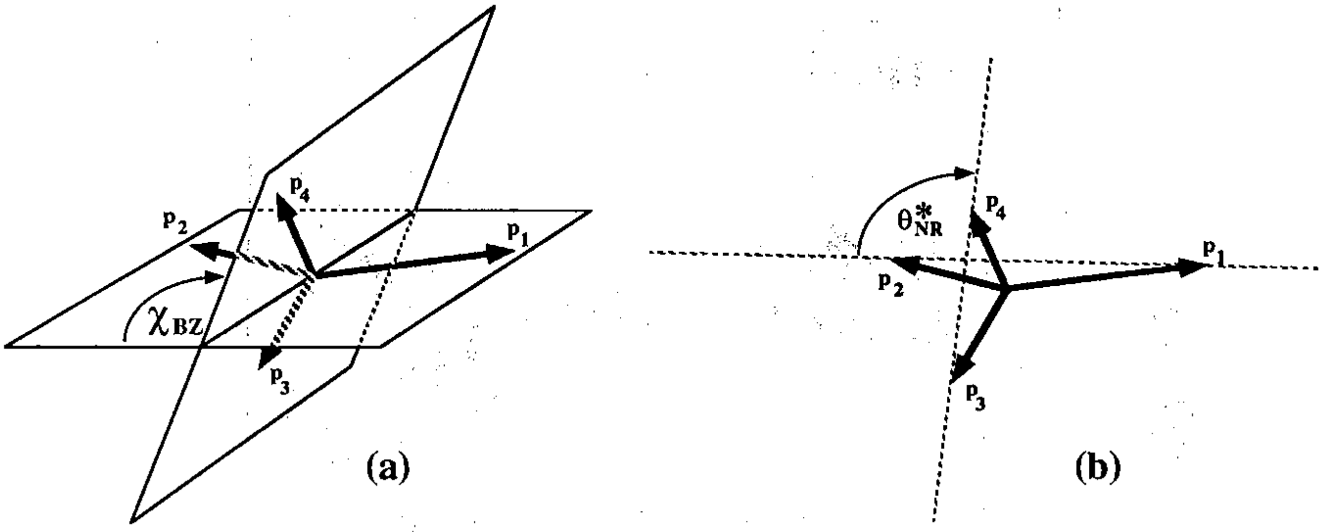

Figure 2: Definitions of χ_{BZ} (a) and of θ^*_{NR} (b) for a typical 4-parton event configuration.

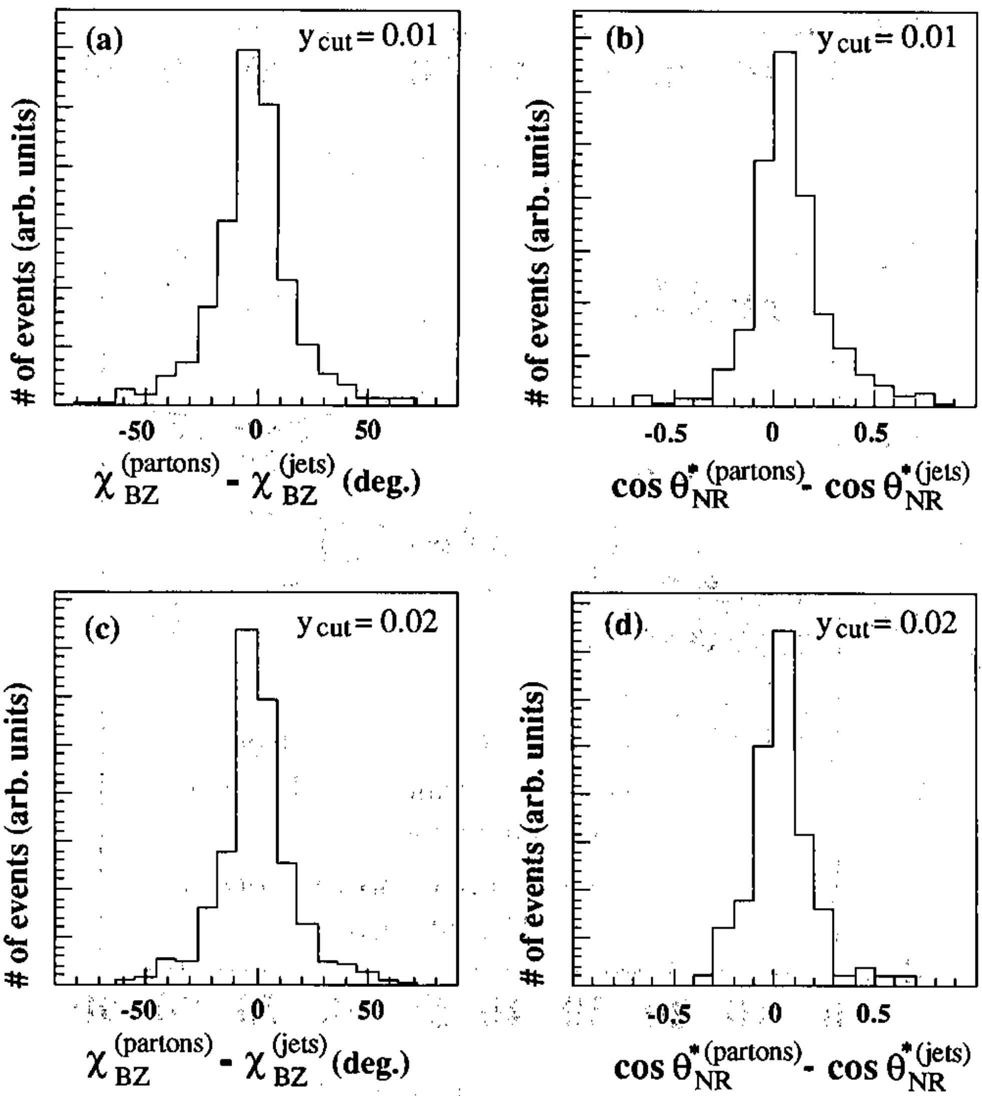

Figure 3: Distributions $\Delta O = O_{partons} - O_{jets}$ for the observables χ_{BZ} and $\cos\theta^*_{NR}$, calculated for Monte Carlo events before the hadronisation process and after hadronisation, detector simulation and event reconstruction.

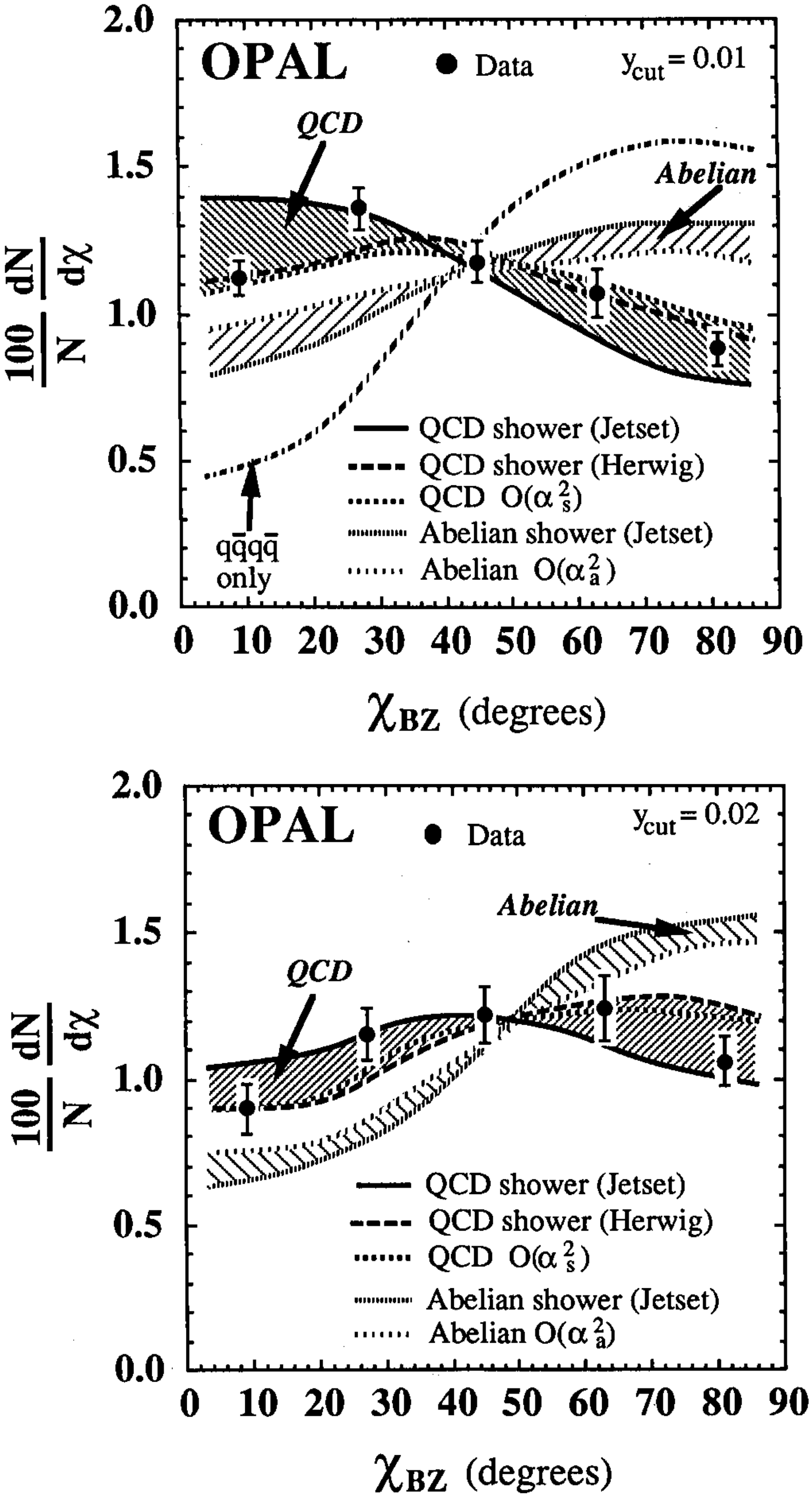

Figure 4: Measured distributions of χ_{BZ} for 4-jet events defined at $y_{cut} = 0.01$ and 0.02. The data are corrected for detector resolution and hadronisation and are compared to the predictions of QCD and the abelian theory, calculated in second order perturbation theory and predicted by parton shower models.

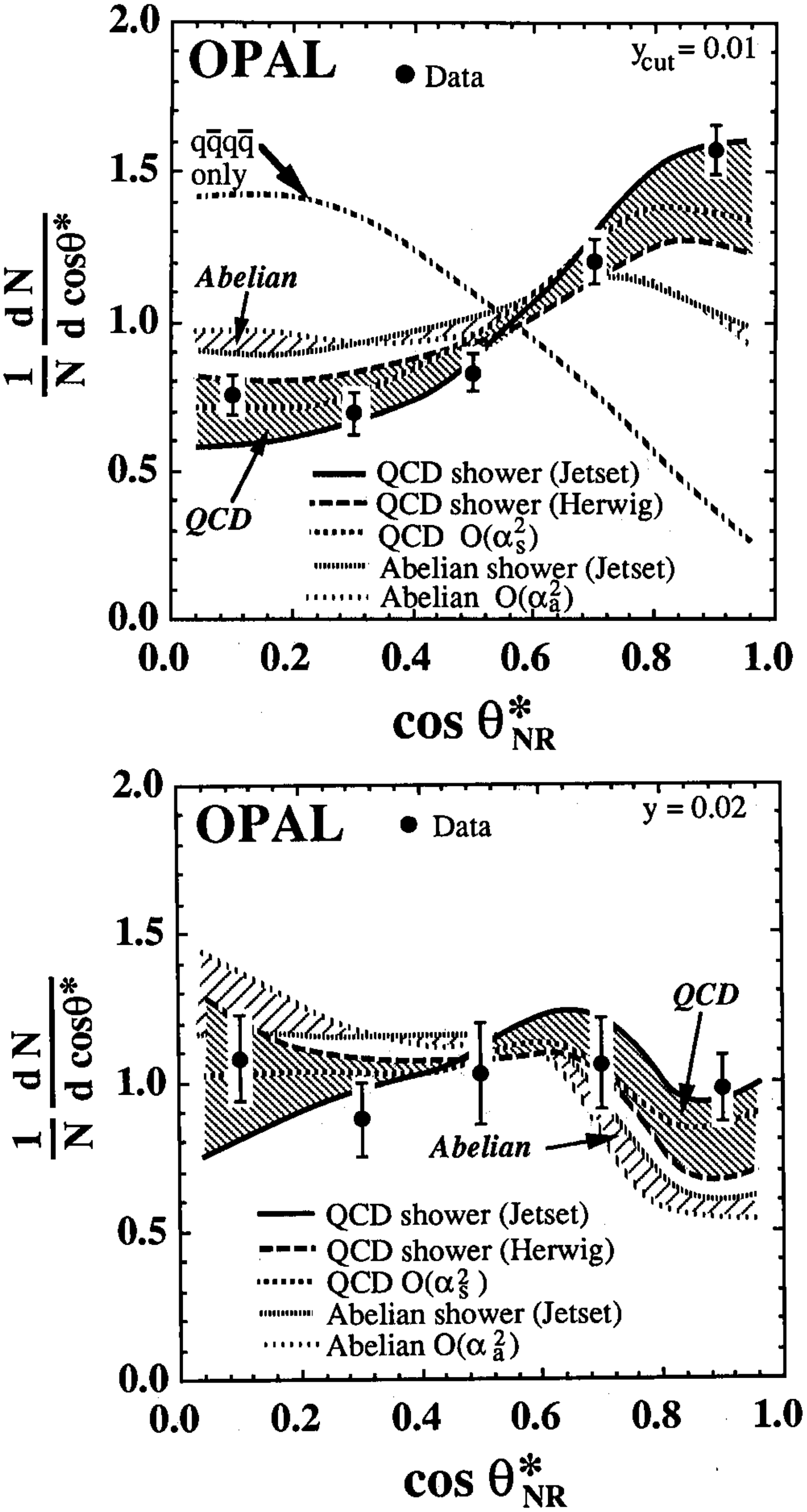

Figure 5: Measured distributions of $\cos\theta^*_{NR}$ for 4-jet events defined at y_{cut} = 0.01 and 0.02. The data are corrected for detector resolution and hadronisation and are compared to the predictions of QCD and the abelian theory, calculated in second order perturbation theory and predicted by parton shower models.

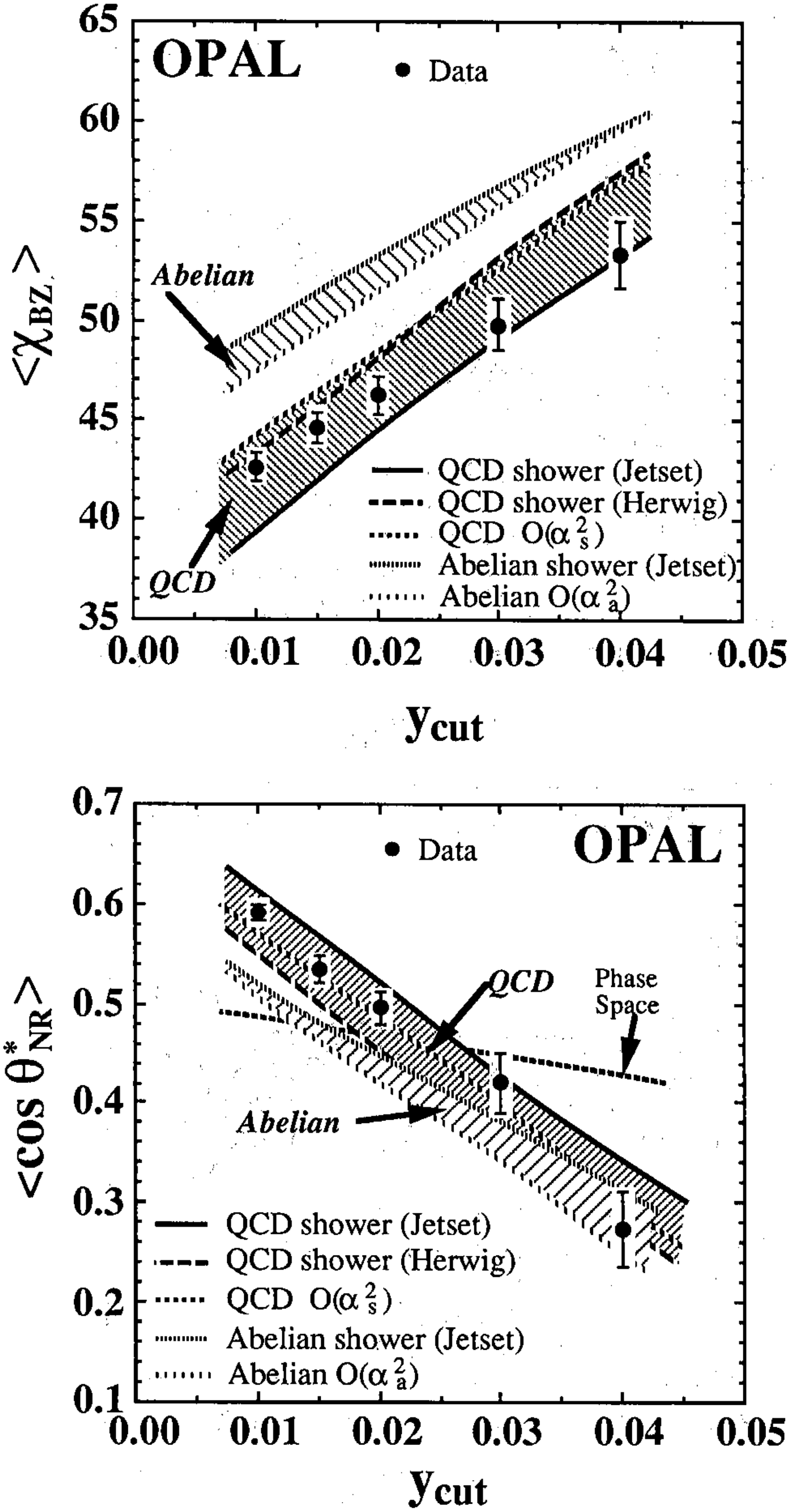

Figure 6: Mean values of the χ_{BZ} and $\cos\theta^*_{NR}$ distributions as a function of the jet resolution parameter y_{cut}; the data are corrected for detector and hadronisation effects.

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