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Super-Kac-Moody algebras and the $N = 2$ superconformal case.

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Abstract

The $N = 2$ superconformal algebra ($SC(N = 2)$) can be seen as the algebra of diffeomorphisms acting as derivations on a $N = 2$ super-Kac-Moody algebra $\widetilde{U(1)}_{N=2}$. We comment on this property. In particular, since $U(1)$ is the only Lie algebra whose $N = 2$ super-Kac-Moody extension can be connected to the $SC(N = 2)$ algebra, one concludes that the construction of a (non-trivial) $N = 2$ super Wess-Zumino-Witten model is impossible.

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The Virasoro algebra $\mathcal{V}$, defined as the central extension of the diffeomorphism algebra $\mathrm{Diff}(S^1)$ of the circle S^1, and any Kac-Moody (or KM) algebra $\tilde{\mathcal{G}}(S^1)$, are intrinsically related, $\mathcal{V}$ acting as the algebra of derivations on $\tilde{\mathcal{G}}(S^1)$. This simple property, i.e. the existence of the semi-direct sum $\tilde{\mathcal{G}}(S^1) \rtimes \mathcal{V}$, is well known to play a fundamental role in two-dimensional conformal field theory [1]. A few years ago, it was realized [2] that a particular central extension performed on the super-loop algebra $\tilde{\mathcal{G}}(\hat{S}^1)$ (where $\hat{S}^1$ is the $(N = 1)$ supercircle[1] $S^1 \otimes G_1$) leads to a super-KM algebra $\tilde{\mathcal{G}}_w(\hat{S}^1)$ such that the set of $\mathrm{Diff}(\hat{S}^1)$ elements "compatible" with $\tilde{\mathcal{G}}_w(\hat{S}^1)$ is exactly the $N = 1$ superconformal algebra $SC(N = 1)$, whose central extension is usually called the Ramond-Neveu-Schwarz (RNS) algebra. Let us make it clear that an element of $\mathrm{Diff}(\hat{S}^1)$ is said to be "compatible" if it acts as a derivation on $\tilde{\mathcal{G}}_w(\hat{S}^1)$. Moreover, the subscript w on $\tilde{\mathcal{G}}_\omega(\hat{S}^1)$ refers to the cocycle w associated to the considered central extension : indeed, if to any usual loop algebra $\mathcal{G}(S^1)$ can be associated only one central extension, up to a multiplicative factor, this is no longer the case for a super loop algebra $\mathcal{G}(\hat{S}^1)$. The structure $\tilde{\mathcal{G}}_w(\hat{S}^1) \rtimes \widetilde{SC}(N = 1)$ is then well adapted for constructing the $N = 1$ super Virasoro minimal models via the Goddard-Kent-Olive (GKO) coset construction [3], as well as for the study of the $N = 1$ supersymmetric Wess-Zumino-Witten (WZW) models [4].

Recently [5], a general study of the central extension relative to the super-KM algebras $\tilde{\mathcal{G}}(S^1) \otimes G_N$ has been performed, as well as the determination, for each central extension, of the "compatible" diffeomorphism algebra. While the RNS algebra, as we already mentioned, shows up directly in this approach, this is not the case for the other (i.e. $N > 1$) usual super-conformal (SC) algebras, also called Ademollo et al. [6] algebras.

In this note, we would like to point out that relaxing the condition of (semi-) simplicity for the $\mathcal{G}$ algebra and, more precisely, taking for $\mathcal{G}$ a one-dimensional $U(1)$ algebra, allows one to construct a super-KM algebra $\tilde{\mathcal{G}}_w(S^1 \otimes G_2)$ such that the subalgebra of Diff $(S^1 \otimes G_2)$ compatible with this central extension is exactly the $SC(N = 2)$ algebra. The realization of this last algebra in terms of bosons and fermions from $\tilde{\mathcal{G}}_w(S^1 \otimes G_2)$ is immediate. Then such a framework appears well adapted to the (re-)interpretation of the $SU(2) \otimes U(1)/U(1)$ coset construction used in [7] for constructing representations of $\widetilde{SC}(N = 2)$ out of representations of $SU(2) \otimes U(1)$ KM algebras. Finally, the twisted and antiperiodic $SC(N = 2)$ algebras are also discussed in this approach, before concluding.

Although our construction may appear as a reformulation of properties more or less known in the superfield approach [8], it has seemed to us of some relevance to emphasize the particular position of the $N = 2$ superconformal algebra with respect to a Kac-Moody current algebra. As discussed in the conclusion, the situation is completely different from the $N = 0$ and $N = 1$

[1] G_N is the Grassman algebra with N Grassman variables $\theta_i (i = 1, ..., N)$ and $\mathcal{G}$ a (semi) simple Lie algebra

cases, and the constraint on $\mathcal{G}$ to be Abelian does not allow the construction of the usual WZW models for $N = 2$. We also note that our result completes the general study performed in [5].

Using the standard notation for a loop algebra [9] $\mathcal{G}(S^1) = C(t, t^{-1}) \otimes \mathcal{G}$ if $t \in \mathbf{C} - \{0\}$, the $N = 2$ superloop algebra relative to $U(1)$ will be written as : $C(t, t^{-1}; \theta_1, \theta_2) \otimes U(1) \equiv U(1)_{N=2}$ with ${\theta_1}^2 = {\theta_2}^2 = 0$ and $\{\theta_1, \theta_2\} = 0$. Denoting by J the $U(1)$ generator, a graded basis is :

$$J_n = t^n \otimes J; \quad \varphi_n = \theta_1 t^n \otimes J; \quad \psi_n = \theta_2 t^n \otimes J; \quad X_n = \theta_1 \theta_2 t^n \otimes J \tag{1}$$

We consider the central extension defined by the bilinear form w:

$$w(J_m, J_n) = m\delta_{m+n} \tag{2}$$

$$w(\psi_m, \psi_n) = w(\varphi_m, \varphi_n) = \delta_{m+n} \tag{3}$$

$$w(X_m, X_n) = -\frac{1}{m}\delta_{m+n} \quad m \neq 0 \tag{4}$$

and w being zero valued everywhere else.

We note that (4) cannot define a central extension in the case of a KM algebra with $\mathcal{G}$ simple, but only when the finite algebra $\mathcal{G}$ is Abelian: this can be seen explicitly by studying the Jacobi identity, or by noting that such a bilinear form cannot be written in terms of a Berezin integral involving a divergenceless vector field, as made clear in [5].

We denote the obtained algebra by $\widetilde{U(1)}_{N=2}$ –omitting an index w for specifying the cocycle, in order not to overload the notations– which satisfies the (non-vanishing) commutation relations :

$$\begin{aligned} [J_m, J_n] &= m\delta_{m+n} \\ \{\psi_m, \psi_n\} &= \{\varphi_m, \varphi_n\} = \delta_{m+n} \\ [X_m, X_n] &= -\frac{1}{m}\delta_{m+n} \quad m \neq 0. \end{aligned} \tag{5}$$

Now, we want to determine among the vector fields on $S^1 \otimes G_2$:

$$\mathcal{L} = (a(t) + \theta_1\theta_2 b(t))\, \partial_t + (\theta_1 c(t) + \theta_2 d(t))\, \partial_1 + (\theta_1 e(t) + \theta_2 f(t))\, \partial_2 \tag{6}$$

and

$$\mathcal{G} = (\theta_1 \alpha(t) + \theta_2 \beta(t))\, \partial_t + (\gamma(t) + \theta_1\theta_2 \epsilon(t))\, \partial_1 + (\mu(t) + \theta_1\theta_2 \zeta(t))\partial_2 \tag{7}$$

(where $\partial_t, \partial_1, \partial_2$ means $\frac{\partial}{\partial_t}, \frac{\partial}{\partial\theta_1}, \frac{\partial}{\partial\theta_2}$ respectively) those which act as derivatives on the above super-KM algebra, that is :

$$\mathcal{L}[X, Y\} = [\mathcal{L}X, Y\} + [X, \mathcal{L}Y\} \tag{8}$$

$$\mathcal{G}[X,Y\} = [\mathcal{G}X,Y\} + (-)^{g(X)}[X,\mathcal{G}Y\} \quad \forall X,Y \in \widetilde{U(1)}_{N=2} \tag{9}$$

where $[,\}$ can be either a commutator or an anti-commutator and $g(X) = 0$ or 1 following X is a boson or a fermion. It follows that:

$$w(\mathcal{L}X,Y) + w(X,\mathcal{L}Y) = 0 \tag{10}$$

$$w(\mathcal{G}X,Y) + (-)^{g(X)}w(X,\mathcal{G}Y) = 0 \tag{11}$$

Expanding $\mathcal{L}$ and $\mathcal{G}$ in Laurent series, the compatible algebra can be seen as generated by:

$$\begin{aligned} L_n &= -t^{n+1}\partial_t + \frac{n+1}{2}t^n(\theta_1\partial_1 + \theta_2\partial_2) \\ T_n &= it^n(\theta_2\partial_1 - \theta_1\partial_2) \\ G_n^1 &= t^n(\theta_1 t\partial_t - \partial_1 + n\theta_1\theta_2\partial_2) \qquad (12) \\ G_n^2 &= t^n(\theta_2 t\partial_t - \partial_2 - n\theta_1\theta_2\partial_1) \end{aligned}$$

(13)

One recognizes the (non-extended) $N = 2$ superconformal algebra $SC(N = 2)$, satisfying the usual commutation relations :

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} \\ [L_m, G_n^i] &= (\frac{m}{2} - n)G_{n+m}^i \\ \{G_m^i, G_n^j\} &= 2\delta^{ij}L_{m+n} + i\epsilon^{ij}(m-n)T_{m+n} \\ [L_m, T_n] &= -nT_{m+n} \\ [T_m, G_n^i] &= i\epsilon^{ij}G_{m+n}^j. \end{aligned} \tag{14}$$

where $i = 1,2$ and $\epsilon^{12} = -\epsilon^{21} = 1$. The action of $SC(N = 2)$ on $\widetilde{U(1)}_{N=2}$ reads :

$$[L_m, \phi_n] = (m(h_\phi - 1) - n)\phi_{m+n} \tag{15}$$

with $\phi \equiv J, \varphi, \psi, X$ and $h_\phi = 1, \frac{1}{2}, \frac{1}{2}, 0$ respectively.

$$\begin{aligned} &[G_r^1, J_n] = n\varphi_{r+n} && [G_r^2, J_n] = n\psi_{r+n} \\ &\{G_r^1, \varphi_s\} = -J_{r+s} && \{G_r^2, \psi_s\} = -J_{r+s} \\ &\{G_r^1, \psi_s\} = -(r+s)X_{r+s} && \{G_r^2, \varphi_s\} = -(r+s)X_{r+s} \\ &[G_r^1, X_n] = -\psi_{r+n} && [G_r^2, X_n] = \varphi_{r+n} \\ &[T_m, J_n] = 0 && [T_m, X_n] = 0 \\ &[T_m, \varphi_r] = \psi_{m+r} && [T_m, \psi_r] = -\varphi_{m+r} \end{aligned} \tag{16}$$

A canonical realization of $\widetilde{SC}(N = 2)$ can be easily obtained. For convenience, let us replace the commutation relations given in (5) by the corresponding O.P.E. associated to the spin 1 boson $J(z)$, the spin $\frac{1}{2}$ fermions $\psi(z)$ and $\varphi(z)$, and the spin 0 boson $X(z)$:

$$J(z)J(w) = \frac{\frac{1}{2}}{(z-w)^2}$$

$$\psi(z)\psi(w) = \varphi(z)\varphi(w) = \frac{\frac{1}{2}}{(z-w)} \tag{17}$$

$$X(z)X(w) = \frac{1}{2}\ln(z-w) \tag{18}$$

Then, setting :

$$\begin{aligned} L(z) &= :\partial X\partial X:(z) + :JJ:(z) + :\partial\psi\psi:(z) + :\partial\varphi\varphi:(z) \\ G^1(z) &= 2:\varphi J:(z) - :\psi\partial X:(z) \\ G^2(z) &= 2:\psi J:(z) - :\varphi\partial X:(z) \\ T(z) &= 2i:\varphi\psi:(z) \end{aligned} \tag{19}$$

(20)

we obtain a realization of the central extended superconformal $\widetilde{SC}(N = 2)$ algebra, with central extension $c = 3$. Indeed :

$$L(z)L(w) = \frac{2L(w)}{(z-w)^2} + \frac{\partial L(w)}{(z-w)} + \frac{1}{(z-w)^4}$$

$$L(z)\phi(w) = h_\phi\frac{\phi(w)}{(z-w)^2} + \frac{\partial\phi(w)}{(z-w)} \tag{21}$$

with $\phi = G^1, G^2, T$ and $h_\phi = \frac{3}{2}, \frac{3}{2}, 1$ respectively

$$G^1(z)G^1(w) = G^2(z)G^2(w) = \frac{2L(w)}{(z-w)} + \frac{2}{(z-w)^3}$$

$$G^1(z)G^2(w) = \frac{2iT(w)}{(z-w)^2} + \frac{i\partial T(w)}{(z-w)}$$

$$T(z)G^1(w) = i\frac{G^2(w)}{(z-w)}$$

$$T(z)G^2(w) = -i\frac{G^1(w)}{(z-w)}$$

while the action of $\widetilde{SC}(N=2)$ on $\widetilde{U(1)}_{N=2}$ reads :

$$L(z)\phi(w) = h_\phi \frac{\phi(w)}{(z-w)^2} + \frac{\partial\phi(w)}{(z-w)}$$

with $\phi = J, \psi, \varphi, X$ and $h_\phi = 1, \frac{1}{2}, \frac{1}{2}, 0$ respectively.

$$\begin{aligned}
T(z)J(w) &= T(z)X(w) = 0 \\
T(z)\varphi(w) &= -\frac{\psi(w)}{(z-w)} \qquad T(z)\psi(w) = \frac{\varphi(w)}{(z-w)} \\
G^i(z)\Theta_h(w) &= \frac{\chi_{h+\frac{1}{2}}(w)}{(z-w)} \\
G^i(z)\chi_{h+\frac{1}{2}}(w) &= 2h\frac{\Theta_h(w)}{(z-w)^2} + \frac{\partial\Theta_h(w)}{(z-w)}
\end{aligned}$$

where h $(h+\frac{1}{2})$ denotes the conformal weight of Θ (χ). Explicitly one can form the two supersymmetric doublets $(\Theta_h, \chi_{h+\frac{1}{2}})$

$$\begin{array}{llll}
(X(z),\psi(z)) & (\varphi(z),J(z)) & \text{under} & G^1(z) \\
(X(z),-\varphi(z)) & (\psi(z),J(z)) & \text{under} & G^2(z).
\end{array} \tag{22}$$

At this point, we must note that our approach is equivalent to that of [8], in which the $\widetilde{U(1)}_{N=2}$ algebra is replaced by a $N=2$ superfield and the above superconformal algebra realization by a supercurrent.

It is natural to wonder how this $\widetilde{U(1)}_{N=2}$ algebra can be used in a coset construction of the unitary representations of $\widetilde{SC}(N=2)$, knowing already the coset construction of [7]. For such a purpose one may try to follow the same spirit as in [2], in which the case of the RNS or $\widetilde{SC}(N=1)$ algebra is considered. There, the starting point is a representation of the direct sum $\tilde{A}_1(S^1) \oplus \tilde{A}_{1,w}(S^1 \otimes G_1)$ constructed by taking a level p representation of the KM algebra relative to $A_1 = SU(2)$ and a level 2 representation of the $N=1$ super-KM algebra generated by :

$$Q^a_n = it^n \otimes J^a \quad ; \quad h^a_n = i\theta T^m \otimes J^a \tag{23}$$

with the commutation relations :

$$\begin{aligned}
[Q^a_m, Q^b_n] &= if^{ab}_c Q^c_{m+n} + m\delta_{m+n}\delta^{ab} \\
[Q^a_m, h^b_n] &= if^{ab}_c h^c_{m+n} \\
\{h^a_m, h^b_n\} &= \delta_{m+n}\delta^{ab}
\end{aligned} \tag{24}$$

before dividing by the bosonic $\tilde{A}_1$-representation of level $(p+2)$. Of course such an approach is completely analogous to the famous GKO construction of [3], where, starting with the level $k=n$ and $k=2$ representations of $\tilde{A}_1$, one also uses, for building the fermionic $G^i(z)$ generators, the spin-$\frac{1}{2}$ fermions which show up explicitly in the realization of the $k=2$ $\tilde{A}_1$-representation. The direct use of the algebra $\tilde{A}_{1,w}(S^1 \otimes G_1)$ with commutation relations given in (24) looks to us more natural, in particular because of the property of the $SC(N=1)$ algebra to be (isomorphic to) the algebra of diffeomorphism compatible with ω.

Thus, let us start with the $\widetilde{U(1)}_{N=2}$ algebra, and consider more precisely the $\tilde{A}_1(S^1) \otimes \widetilde{U(1)}_{N=2}$ algebra. From the representation of level k of $\tilde{A}_1(S^1)$ with generators $T^a_m (a=1,2,3)$, one obtains by the usual Sugawara construction a realization of the Virasoro algebra :

$$L_1(z) = \frac{1}{k+2}\sum_{a=1}^{3} :T^aT^a:(z) \tag{25}$$

with central charge : $c_1 = \frac{3k}{k+2}$, while from the $\widetilde{U(1)}_{N=2}$ generators, one obtains as already introduced in (19) :

$$L_2(z) = :\partial X\partial X:(z) + :JJ:(z) + :\partial\psi\psi:(z) + :\partial\varphi\varphi:(z) \tag{26}$$

with central charge $c_2 = 3$.

Now, let us consider the $\widetilde{U(1)}^3$ algebra with generators $\partial X(z), J(z)$ and $S(z)$, with :

$$S(z) = T^3(z) + 2iI(z) \quad \text{and} \quad I(z) = :\psi\varphi:(z) \tag{27}$$

One can immediately check that

$$\begin{aligned}
\partial X(z)\partial X(w) = J(z)J(w) &= \frac{1}{2}\frac{1}{(z-w)^2} \\
S(z)S(w) &= \frac{\frac{1}{2}(k+2)}{(z-w)^2}
\end{aligned} \tag{28}$$

Then, defining L_{diag} as :

$$L_{diag}(z) \;=\; :\partial X \partial X: + :JJ:(z) + \frac{1}{k+2} :SS:(z)$$

with central charge $c_d = 3$, one remarks that :

$$\bar{L}(z) \;=\; L_1(z) + L_z(z) - L_{diag}(z) \tag{29}$$

acts trivially on $\widetilde{U(1)}^3$, i.e. :

$$\underbrace{\bar{L}(z) X(w)} \;=\; \underbrace{\bar{L}(z) J(w)} \;=\; \underbrace{\bar{L}(z) S(w)} = 0 \tag{30}$$

At this point, we cannot exactly speak about a usual "coset" construction of $\hat{A}_1(S^1) \otimes \widetilde{U(1)}_{N=2}$ by $\widetilde{U(1)}^3$, since $I(z)$ is not an element of $\hat{A}_1(S^1) \otimes \widetilde{U(1)}_{N=2}$ but only constructed from $\psi(z)$ and $\varphi(z)$ in $\widetilde{U(1)}_{N=2}$. However, $\bar{L}(z)$ commutes with $L_{diag}(z)$ and therefore its corresponding central charge satisfies :

$$\bar{c} \;=\; c_1 + c_2 - c_d \;=\; 3\frac{k}{k+2} \tag{31}$$

these values of $\bar{c}$ being those relative to unitary representations of $\widetilde{SC}(N=2)$. Moreover, defining the superconformal generators $\bar{G}^i(z)(i=1,2)$ as :

$$\begin{aligned} \bar{G}^1(z) &= \frac{2}{\sqrt{k+2}}\Big(:\psi T^1:(z) - :\varphi T^2:(z)\Big) \\ \bar{G}^2(z) &= \frac{2}{\sqrt{k+2}}\Big(:\varphi T^2:(z) - :\psi T^1:(z)\Big) \end{aligned} \tag{32}$$

and :

$$\bar{T}(z) = \frac{2}{k+2}\Big(ik : \varphi\psi:(z) + T^3(z)\Big)$$

one easily verifies that these generators act trivially on $\widetilde{U(1)}^3$. Thus, we have obtained a realization of the $\widetilde{SC}(N=2)$ algebra with $c = \frac{3k}{k+2}$ which is actually the same as the one obtained in [7] by the coset construction relative to $SU(2) \otimes U(1)/U(1)$. In the light of the above discussion, one could say that [7] follows the usual GKO construction of [3] while we just use the approach of [2].

Finally, we can remark that the three usual kinds of $SC(N=2)$ algebras, denoted following [10] as the P (periodic) algebra with integer modes for all the generators, the A (antiperiodic) algebra with integer modes for L_m, T_m but half-integer modes ($m \in \mathbb{Z} + \frac{1}{2}$) for G^i_m ($i = 1,2$) and the T (twisted) algebra with integer modes for L_m and G^1_m and half-integer modes for G^2_m and T_m, can all be seen from the super $\widetilde{U(1)}_{N=2}$ algebra.

For such a purpose, let us restrict the $\widetilde{U(1)}_{N=2}$ algebra with generic element $f(t;\theta_1,\theta_2) \otimes I$ to the subalgebra generated by the elements satisfying the condition[2]

$$f(t;\theta_1,-\theta_2) \otimes I \;=\; f(-t;\theta_1,\theta_2) \otimes I \tag{33}$$

One may call the obtained algebra a "twisted" $\widetilde{U(1)}_{N=2}$ algebra, although for a usual KM algebra defined on the circle $\bar{\mathcal{G}}(S^1)$, a real twist involves not only a modulo m periodicity (for a twist of order $m \in \mathbb{N}$) on the continuous part of the generator, but also an (outer) automorphism on the finite dimensional algebra $\mathcal{G}$. A basis for this twisted algebra is :

$$\begin{aligned} J_{2n} &= t^{2n} \otimes I \qquad \varphi_{2n} = \theta_1 t^{2n} \otimes I \\ \psi_{2n+1} &= \theta_2 t^{2n+1} \otimes I \qquad X_{2n+1} = \theta_1\theta_2 t^{2n+1} \otimes I \end{aligned} \tag{34}$$

It is straightforward to deduce a basis for the diffeomorphism algebra compatible with this twisted $\widetilde{U(1)}_{N=2}$ using notations of (12) :

$$L_{2n} \;;\; G^1_{2n} \;;\; G^2_{2n+1} \text{ and } T_{2n+1}. \tag{35}$$

Then, redefining :

$$\begin{aligned} \hat{L}_n &= \frac{1}{2}L_{2n} \;;\; \hat{T}_{n+\frac{1}{2}} = T_{2n+1} \\ \hat{G}^1_n &= \frac{1}{\sqrt{2}}G^1_{2n} \;;\; \hat{G}^2_{n+\frac{1}{2}} = \frac{1}{\sqrt{2}}G^2_{2n+1} \end{aligned} \tag{36}$$

one recognizes the usual basis of the T (twisted) $SC(N=2)$ algebra, the commutation relations being the same as given in (14).

In the same way, imposing on the $\widetilde{U(1)}_{N=2}$ elements the constraint :

$$f(t;-\theta_1,-\theta_2) \otimes I \;=\; f(-t;\theta_1,\theta_2) \otimes I \tag{37}$$

one gives rise to another twisted $\widetilde{U(1)}_{N=2}$ algebra generated by :

$$\begin{aligned} J_{2m} &= t^{2n} \otimes I \;\;;\;\; X_{2n} = \theta_1\theta_2 t^{2n} \otimes I \\ \varphi_{2n+1} &= \theta_1 t^{2n+1} \otimes I \;\;;\;\; \psi_{2n+1} = \theta_2 t^{2n+1} \otimes I \end{aligned} \tag{38}$$

[2] Note that with the notation of [8], where $\theta = \theta_1 + i\theta_2$ and $\bar{\theta} = \theta_1 - i\theta_2$, the transformation $\theta_2 \to -\theta_2$ is the exchange $\theta \leftrightarrow \bar{\theta}$.

The diffeormorphism algebra compatible with this twisted algebra is the A (antiperiodic) $SC(N=2)$ algebra.

As a conclusion, we have seen that the $N=2$ superconformal algebra $SC(N=2)$ appears as the algebra of diffeomorphisms acting on the super-KM algebra $\widetilde{U(1)}_{N=2}$ with a particular cocycle given in (2), (3) and (4). We have also discussed the first implications of this property. We can notice how the remarkable relationship which exists between any KM algebra $\tilde{\mathcal{G}}_\omega(S^1)$ and the Virasoro algebra is restricted when supersymmetry is included. In the case of $N=1$ supersymmetry, the RNS algebra is still the (central extension of) algebra of diffeomorphisms compatible with the super-KM algebra $\tilde{\mathcal{G}}_\omega(S^1 \otimes G_1)$ whatever the simple Lie algebra $\mathcal{G}$, but with a particular cocycle w. Finally, going from $N=1$ to $N=2$, the (semi) simplicity of $\mathcal{G}$ is lost and we are left with a $U(1)$ algebra. One may wonder whether such a construction could be extended to $N>2$. A direct calculation in which the central extension of the $\widetilde{U(1)}_{N=3}$ superalgebra is a direct generalization of the $\widetilde{U(1)}_{N=2}$ case already considered (i.e. this $\widetilde{U(1)}_{N=3}$ algebra contains the $\widetilde{U(1)}_{N=2}$ algebra as a subalgebra), leads to a negative result. In other words, the compatible algebra of diffeomorphisms on $\widetilde{U(1)}_{N=3}$ cannot correspond to the $SC(N=3)$ algebra. Thus, the $N=2$ case looks like a limiting case.

Our result, that is the Abelianity of the obtained KM algebra $\widetilde{U(1)}_{N=2}$, forbids the hope of constructing a $N=2$ WZW model in the framework of the usual $SC(N=2)$ –see [5] for new kinds of "super" conformal algebras. However, the explicit study of this $U(1)$ symmetry could be of some relevance in certain cases. As we have already seen, it shows up directly in the coset construction of $\widetilde{SC}(N=2)$ unitary representations. Finally, let us mention that a similar coset treatment using this $\widetilde{U(1)}_{N=2}$ algebra might work for the determination of unitary representations of the $N=2$ super-W_3 algebra ; indeed the $N=1$ super-W_3 case has recently been solved [11] in a way similar to the $\widetilde{SC}(N=1)$ case, the $\tilde{A}_1(S^1 \otimes G_1)$ algebra being replaced by the $\tilde{A}_2(S^1 \otimes G_1)$ one.

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