

Effective Planck mass and the scale of inflation

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Abstract. A recent paper argued that it is not possible to infer the energy scale of inflation from the amplitude of tensor fluctuations in the Cosmic Microwave Background, because the usual connection is substantially altered if there are a large number of universally coupled fields present during inflation, with mass less than the inflationary Hubble scale. We give a simple argument demonstrating that this is incorrect.

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Contents

1	Introduction	1
2	Counterarguments	2

1 Introduction

The energy scale of inflation remains very poorly constrained by observation. The upper limit comes from the fact that the amplitude of tensor modes in the Cosmic Microwave Background (CMB) is directly related to the scale of inflation. In particular

$$V^{1/4} \approx \left(\frac{r}{0.01}\right)^{1/4} \times 10^{16} \text{ GeV},$$

where V is the energy density at the time the modes exited the horizon and r is the ratio of tensor to scalar power. Hence the current constraint on the amplitude of tensor fluctuations, $r \lesssim 0.1$ [1], puts an upper limit on the scale of inflation.

The constraint on r is forecast to improve by several orders of magnitude over the next few years, possibly down to a standard deviation $\sigma_r \sim 10^{-4}$ with next generation satellite experiments [2]. If the simplest high-scale inflation models are correct, tensor modes will be discovered with high significance. A lack of discovery would put a powerful constraint on the inflationary energy scale, ruling out many models.

Clearly, the relationship between the scale of inflation and the amplitude of tensor modes is of first importance. There have been several attempts to demonstrate that the amplitude of tensor modes can be enhanced through various mechanisms involving extra degrees of freedom that are produced during inflation [3–5]. This appears to be possible, although it requires a relatively complex mechanism and some tuning, and is often associated with large non-Gaussianity [6, 7]. There is another source of degeneracy if the tensor modes do not propagate on the light-cone during inflation ($c_t \neq 1$). Again one would generically expect large non-Gaussianity since as shown in [8] the cosmological predictions of any such model coincide with another model with $\tilde{c}_t = 1$ and $\tilde{c}_s = c_s/c_t$, where c_s is the propagation speed of scalar fluctuations and its deviation from unity is correlated with non-Gaussianity [9].

Very recently, a simple idea emerged: that the presence of a large number of universally coupled spin 2 fields of mass less or of the order of the expansion rate H_* could change the effective strength of gravitational interactions during inflation, thereby substantially affecting the connection between the amplitude of tensor modes and the scale of inflation [10]. More precisely, in the presence of N such universally coupled fields (including the graviton) the authors of [10] conclude that tensor perturbations are given by

$$\left\langle \gamma_{\mathbf{k}}^s \gamma_{\mathbf{k}'}^{s'} \right\rangle_{\text{vac}} = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') \delta^{ss'} \frac{H_*^2}{M_*^2 k^3}, \quad M_*^2 \equiv \frac{M_{\text{Pl}}^2}{N}. \quad (1.1)$$

If the mass of these fields satisfies $\text{TeV} \ll m \lesssim H_*$, they would have no detectable effect on collider or gravitational experiments, but for large N would have a strong influence during inflation.

2 Counterarguments

In this note we give two counterarguments and conclude that the connection between the tensor modes and H_* cannot be changed in the presence of such fields, regardless of N .

The first counterargument is extremely simple, and invalidates the assumption of having any light (universally coupled) spin-2 particles during inflation, let alone a large number. Light tensor modes are forbidden in de Sitter space by the Higuchi bound [11], which states that massive spin-2 particles of mass $m^2 < 2H_*^2$ have a ghost-like longitudinal excitation. The Higuchi bound is a consequence of group theory: there is no unitary massive spin-2 representation of the 4d de Sitter isometry group with $m^2 < 2H_*^2$ [12–15]. The bound holds in any background with 4d SO(1,4) isometries, regardless of its origin. In particular, it puts a powerful, *universal* constraint on de Sitter compactifications of higher dimensional gravitational theories. This is true regardless of any “warping” or other details (see [16] for one explicit example). If one defines the “size” of the extra dimensions by the inverse mass of the lightest Kaluza-Klein resonance of the graviton, the Higuchi bound forbids any compactification in which the extra dimensions are larger than and $\mathcal{O}(1)$ factor times the de Sitter radius.¹

The generalization to inflation, which deviates from exact de Sitter by the slow-roll parameter $\epsilon = -\dot{H}_*/H_*^2 \ll 1$, is by the assumption of continuity of the spectrum — a reasonable assumption given that ghost instabilities are effectively instantaneous. Moreover, if the spectrum of spin 2 fields was gapped at $\epsilon = 0$ and ungapped at any $\epsilon \neq 0$, the whole scenario of slow-roll inflation would be put into question, since it is based on the well-verified assumption that physics in a background with $\dot{H}_*/H_*^2 \ll 1$ is well approximated by physics on a de Sitter background.

Heavier spin-2 particles are dynamically less relevant at energy scales of order H_* , and for these the assumption that the effective gravitational strength at distances of order H_*^{-1} increases proportionally to their multiplicity is no longer valid.

One can still consider the case of having several universally coupled scalar fields which are much lighter than H_* , or assume that many massive spin-2 fields are accumulated just above $\sqrt{2}H_*$. It is then conceivable to have a stronger force between (non-relativistic) sources and an effectively larger gravitational strength M_*^{-2} . However, the following argument rules out any direct connection between M_* and the cosmologically observable tensor fluctuations.

In order to discuss the scale of inflation one first needs to fix the system of units, and in particular the conformal frame for the metric. What we mean by the scale of inflation is its value in the Einstein frame, in which the Einstein-Hilbert action is normalized by the Planck mass. In this frame, an increase in the strength of attraction between two sources can arise from the exchange of new degrees of freedom (whose fluctuations about their VEV is collectively called χ_a) in addition to the massless graviton $h_{\mu\nu}$. The dynamics of the latter is still governed to lowest order in derivatives by the Einstein-Hilbert action:

$$S_{\text{EH}} = \frac{1}{2}M_{\text{Pl}}^2 \int d^4x \sqrt{-g} R. \quad (2.1)$$

This action fully fixes the vacuum fluctuations of $h_{\mu\nu}$ in a quasi-de Sitter background; the transverse-traceless fluctuations are given by (1.1) but with $M_* \rightarrow M_{\text{Pl}}$. Higher order curvature corrections are suppressed by factors of H_*^2/M_*^2 , and hence they are negligible as long as gravity is weakly coupled. Specifically, a term $\propto R^2$ can be removed by a well-known

¹M.K. thanks N. Arkani-Hamed for discussions on this point.

field redefinition [17] while a term $cR_{\mu\nu}R^{\mu\nu}$ sets the cutoff of the theory at M_{Pl}/\sqrt{c} . Corrections δP_T to the tree-level power spectrum of tensor modes, P_T , are at best $O(cH^2/M_{Pl}^2)P_T$. Clearly $\delta P_T \ll P_T$ for the curvature expansion to make sense.

Next we discuss the fluctuations of other light degrees of freedom χ_a during inflation and argue that they have no observable effect on the tensor spectrum. For massive spin-2 fields satisfying the Higuchi bound the super-horizon fluctuations decay during inflation according to

$$\chi(t) \propto \begin{cases} a^{-3/2+\sqrt{9/4-m_\chi^2/H_*^2}} & \text{if } \sqrt{2}H_* < m_\chi < 3H_*/2; \\ a^{-3/2} & \text{if } m_\chi \geq 3H_*/2, \end{cases} \quad (2.2)$$

where a is the scale factor. This rapid decay means that the fluctuations in the massive regions do not produce observable features in the CMB.

On the other hand for minimally coupled scalar fields the fluctuations do not decay significantly during inflation if $m_\chi \ll H_*$. However, if these fields have universal coupling to matter with gravitational strength, astrophysical observations and laboratory experiments constrain them to be much heavier than the Hubble scale at recombination. The super-horizon fluctuations of such fields start to decay in a fashion similar to (2.2) once the expansion rate falls below their mass during the cosmic evolution. This behavior falls in the category of multifield inflationary models, where isocurvature fluctuations of a scalar field χ can convert into adiabatic fluctuations and enhance the scalar power, but have no effect on the tensor fluctuations. In any case this can only reduce the tensor to scalar ratio r , and does not alter the connection between H_* and the tensor power.

Vectors or higher spin fields also have no effect, because vectors decay outside the horizon too rapidly to be of any relevance, and fields with spin higher than 2 and mass less than $\sim H_*$ lead to ghost instabilities similar to those of spin 2 [15].

In conclusion, contrary to the claims of [10], $N \gg 1$ universally coupled fields cannot alter the connection between the amplitude of tensor fluctuations and the scale of inflation.

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