# Amplitude analysis of charmless $B$ decays at LHCb

Paula Álvarez Cartelle

Imperial College London

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LHCb

# Charmless $B$ decays

- Mediated by $b \to u$ (tree) or $b \to d, s$ (penguin) transitions
- Several comparable amplitudes $\Rightarrow$ large $CP$ asymmetries
- New physics search: New particles adding extra amplitudes could modify observables ($\mathcal{B}$, $CP$-asymmetries...)
- Benchmark to explore flavour symmetries used to deal with QCD contributions
- Rich zoo of charmless final states: Amplitude analysis usually needed to disentangle different decays

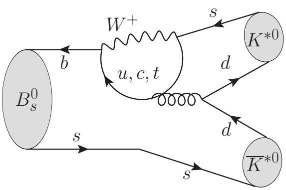
# Amplitude analysis of $B_s^0 \to K^{*0}\overline{K}^{*0}$

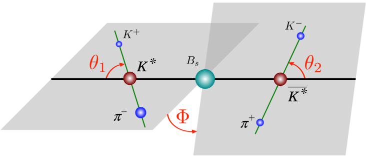
JHEP 07 (2015) 166

$\int \mathcal{L} = 1\,\mathrm{fb}^{-1}$ of 2011 data

# $B_s^0 \to K^{*0}\overline{K}^{*0}$

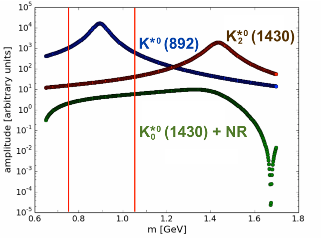
- FCNC, pure penguin transition in the SM
- Final state is a $CP$-eigenstate (3x): Sensitive to $CP$ violation in the interference between $B_s^0$ mixing and decay
- Possibility to use $B^0 \to K^{*0}\overline{K}^{*0}$ to control theoretical uncertainty
- $P \to VV$: Angular analysis needed to disentangle $CP$-even $(L = 0, 2)$ and $CP$-odd $(L = 1)$ final states
- Untagged and time-integrated analysis allows for $CP$-violating Triple Product asymmetries measurement



# $B_s^0 \to K^{*0}\overline{K}^{*0}$

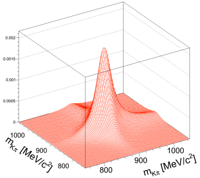
- Reconstructing $B_s^0 \to (K^+\pi^-)_{J_1}(K^-\pi^+)_{J_2}$ in $\pm 150$ MeV $m_{K\pi}$ window around $K^{*0}(892)$
  - P-wave (J=1): $K^{*0}(892)$
  - S-wave (J=0): NR + $K_0^*(1430)$
- D-wave contribution (J=2) is neglected

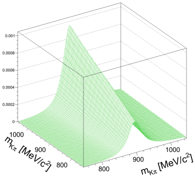


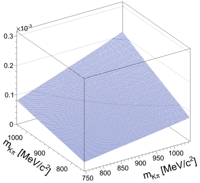

$A_{0,\parallel,\perp}$
$\eta_{0,\parallel} = +1,\ \eta_\perp = -1$

$A_s^+,\ A_s^-$
$\eta_s^+ = -1,\ \eta_s^- = +1$

$A_{ss}$
$\eta_{ss} = +1$



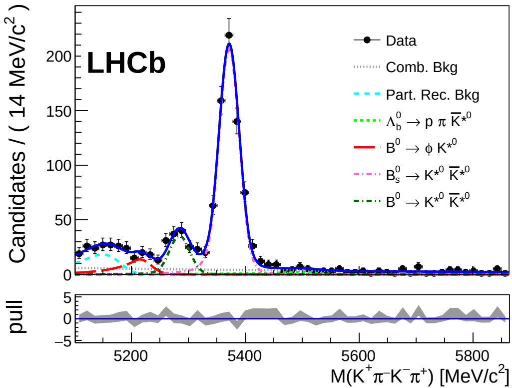




$B \to VV$ $\qquad$ $B \to SV$ $\qquad$ $B \to SS$

# $B_s^0 \to K^{*0}\overline{K}^{*0}$



Analysis performed on $1\,\text{fb}^{-1}$ of 2011 data ($\sqrt{s} = 7$ TeV)




$$N(B_s^0 \to K\pi K\pi) = 697 \pm 31(\text{stat.}) \pm 11(\text{syst.})$$

Time-integrated and untagged analysis: No direct access to the weak phase $\phi_s^{b\to sd\bar{d}}$ but still sensitivity to $CP$-violating observables...

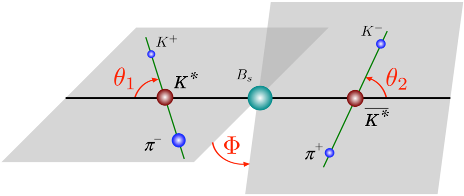
$\mathcal{T}$-odd Triple Product correlations, $q\cdot(\varepsilon_1\times\varepsilon_2)$

$$\mathcal{A}_{T,D}^i = \frac{N(g_{T,D}^i(\Omega)<0) - N(g_{T,D}^i(\Omega)>0)}{N(g_{T,D}^i(\Omega)<0) + N(g_{T,D}^i(\Omega)>0)}$$

$$\mathcal{A}_T^i \propto Im(A_f^* A_\perp - \bar{A}_f^* \bar{A}_\perp)$$

S-wave induced direct $CP$ asymmetries

$$\mathcal{A}_D^i \propto Re(A_f^* A_s^+ - \bar{A}_f^* \bar{A}_s^+)$$




$CP$-even: $f = 0,\ \|,\ s^-,\ ss$
$CP$-odd: $\perp,\ s^+$

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$$\mathcal{A}_T^i \propto Im(A_f^* A_\perp - \bar{A}_f^* \bar{A}_\perp)$$

S-wave induced direct $CP$ asymmetries

$$\mathcal{A}_D^i \propto Re(A_f^* A_s^+ - \bar{A}_f^* \bar{A}_s^+)$$

$CP$-even: $f = 0, \parallel, s^-, ss$
$CP$-odd: $\perp, s^+$

| Asymmetry | Value ± stat. ± syst. |
|---|---|
| $\mathcal{A}_T^1$ | $0.003 \pm 0.041 \pm 0.009$ |
| $\mathcal{A}_T^2$ | $0.009 \pm 0.041 \pm 0.009$ |
| $\mathcal{A}_T^3$ | $0.019 \pm 0.041 \pm 0.008$ |
| $\mathcal{A}_T^4$ | $-0.040 \pm 0.041 \pm 0.008$ |
| $\mathcal{A}_D^1$ | $-0.061 \pm 0.041 \pm 0.012$ |
| $\mathcal{A}_D^2$ | $0.081 \pm 0.041 \pm 0.008$ |
| $\mathcal{A}_D^3$ | $-0.079 \pm 0.041 \pm 0.023$ |
| $\mathcal{A}_D^4$ | $-0.081 \pm 0.041 \pm 0.010$ |

# $B_s^0 \to K^{*0}\overline{K}^{*0}$

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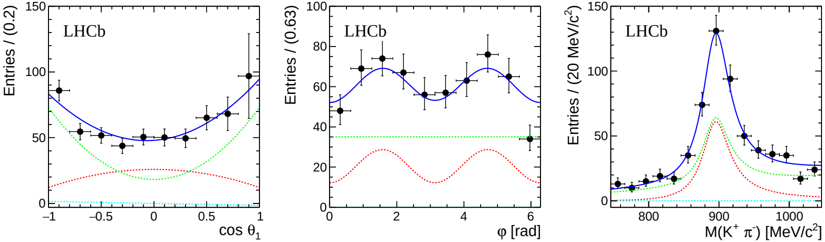
Amplitude analysis assuming no $CP$-violation,

$$\frac{\mathrm{d}^5\Gamma}{\mathrm{d}\Omega\,\mathrm{d}m_{K^+\pi^-}\,\mathrm{d}m_{K^-\pi^+}} = N\sum_{n=1}^{21} K_n(A_i,\,\delta_i;\,m_{K^\pm\pi^\mp})\,F_n(\Omega)$$




$P \to VV$

$P \to SV, VS, SS$

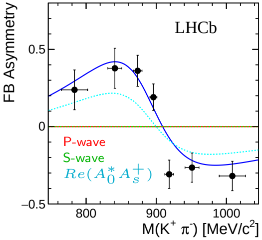
$Re(A_0^* A_s^+)$

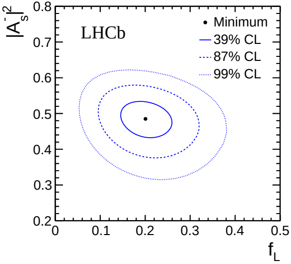
# $B_s^0 \to K^{*0}\overline{K}^{*0}$

JHEP 07 (2015) 166

Systematic dominated by the angular acceptance & the $m_{K\pi}$ description

| Parameter | Value $\pm$ stat. $\pm$ syst. |
|---|---|
| $f_L$ | $0.201 \pm 0.057 \pm 0.040$ |
| $f_\parallel$ | $0.215 \pm 0.046 \pm 0.015$ |
| $\lvert A_s^+\rvert^2$ | $0.114 \pm 0.037 \pm 0.023$ |
| $\lvert A_s^-\rvert^2$ | $0.485 \pm 0.051 \pm 0.019$ |
| $\lvert A_{ss}\rvert^2$ | $0.066 \pm 0.022 \pm 0.007$ |
| $\delta_\parallel$ | $5.31 \pm 0.24 \pm 0.14$ |
| $\delta_\perp - \delta_s^+$ | $1.95 \pm 0.21 \pm 0.04$ |
| $\delta_s^-$ | $1.79 \pm 0.19 \pm 0.19$ |
| $\delta_{ss}$ | $1.06 \pm 0.27 \pm 0.23$ |




Normalising to $B_s^0 \to \phi\overline{K}^{*0}$,

$$\mathcal{B}(B_s^0 \to K^{*0}\overline{K}^{*0}) = (10.8 \pm 2.1(\text{stat.}) \pm 1.4(\text{syst.}) \pm 0.6(f_d/f_s)) \times 10^{-6}$$

# Amplitude analysis of $B^0 \to \rho^0\rho^0$

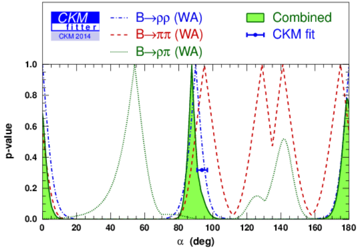
Phys.Lett. B747 (2015) 468

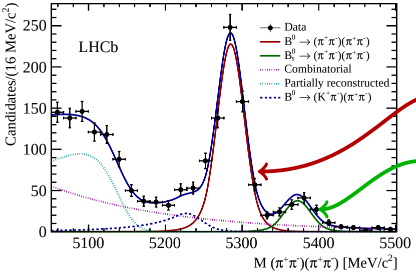
$\int \mathcal{L} = 3\,\mathrm{fb}^{-1}$ of 2011+2012 data

# $B^0 \to \rho^0\rho^0$

Phys.Lett. B747 (2015) 468

- Together with $B^0 \to \rho^+\rho^-$ can be used to constrain the CKM angle $\alpha$
- B-factories found evidence for this decay and reported discrepant $f_L$ measurements $\sim 2\sigma$




$N(B^0) \sim 600$

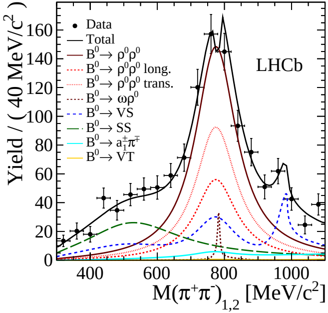
First observation of $B_s^0 \to \pi\pi\pi\pi$

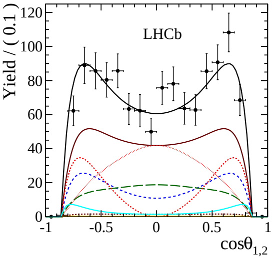
# $B^0 \to \rho^0\rho^0$

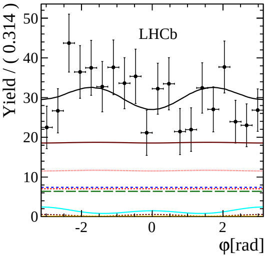
Phys.Lett. B747 (2015) 468

- We look for $B^0 \to (\pi^+\pi^-)(\pi^+\pi^-)$ with $m_{\pi\pi} < 1100$ MeV
  - P-wave: $\rho(980)$, $\omega$
  - S-wave: $f_0(500)$, $f_0(980)$ and NR
  - D-wave: $f_2(1270)$

  11 amplitudes in total!
- Dominant systematics from the mass lineshapes & acceptance correction.



# $B^0 \to \rho^0\rho^0$

Phys.Lett. B747 (2015) 468

- First observation of $B^0 \to \rho^0\rho^0$ (7.1$\sigma$)
- Branching fraction measurement normalised to $B_s^0 \to \phi\overline{K}^{*0}$

$$\mathcal{B}(B^0 \to \rho^0\rho^0) = (0.94 \pm 0.17(\text{stat.}) \pm 0.09(\text{syst.}) \pm 0.06(\text{BF})) \times 10^{-6}$$

- Longitudinal polarisation fraction of $B^0 \to \rho^0\rho^0$

$$f_L = 0.745^{+0.048}_{-0.058}(\text{stat.}) \pm 0.034(\text{syst.})$$

in agreement with BaBar and at $2.3\sigma$ from the Belle measurement

- No evidence for $B^0 \to \rho^0 f_0(980)$ decay mode reported by Belle

$\mathcal{B}(B^0 \to \rho^0 f_0(980)) \times \mathcal{B}(f_0(980) \to \pi^+\pi^-) < 0.81 \times 10^{-6}$ at 90% CL

# Branching ratio of $B_s^0 \to \phi\phi$

LHCb-PAPER-2015-028 in preparation

$\int \mathcal{L} = 3\,\text{fb}^{-1}$ of 2011+2012 data

# $B_s^0 \to \phi\phi$

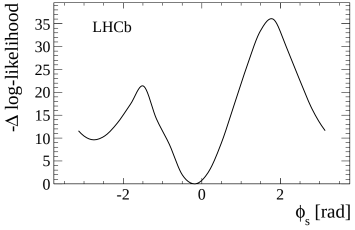
Phys.Rev. D90 (2014) 052011

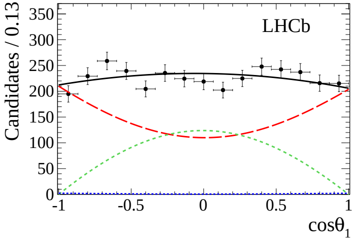
- FCNC $b \to s\bar{s}s$ transition
- $CP$ violation analysis with LHCb Run 1 data ($3\,\text{fb}^{-1}$)
- Tagged, time-dependent amplitude analysis

$$\begin{aligned} \phi_s^{b\to s\bar{s}s} &= 0.17 \pm 0.15(\text{stat}) \pm 0.03(\text{syst})\ \text{rad} \\ |\lambda| &= 1.04 \pm 0.07(\text{stat}) \pm 0.03(\text{syst}) \\ A_U &= 0.003 \pm 0.017(\text{stat}) \pm 0.006(\text{syst}) \\ A_V &= 0.017 \pm 0.017(\text{stat}) \pm 0.006(\text{syst}) \end{aligned}$$

compatible with $CP$ conservation (SM expectation)

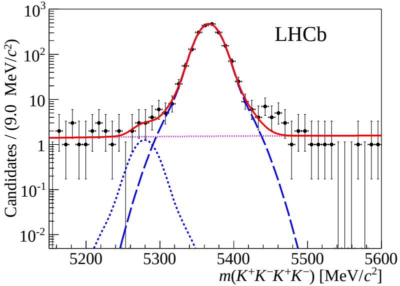
- Polarisation fractions & S-wave contribution also measured

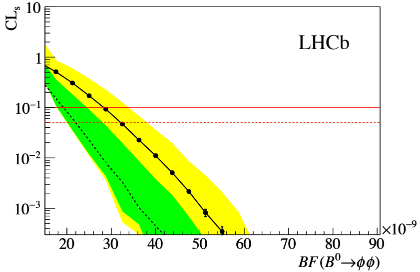


# $B_s^0 \to \phi\phi$

LHCb-PAPER-2015-028
in preparation




- Reoptimisation of the selection
- Using $B^0 \to \phi K^{*0}$ as normalisation and the result of the amplitude analysis

$$\mathcal{B}(B_s^0 \to \phi\phi) = (1.84 \pm 0.05(\text{stat}) \pm 0.07(\text{syst}) \pm 0.11(f_s/f_d) \pm 0.12(\text{norm})) \times 10^{-5}$$

- Signal significance for $B^0 \to \phi\phi < 2\sigma$

$\mathcal{B}(B^0 \to \phi\phi) < 2.8(3.4) \times 10^{-8}$ at 90% (95%) CL

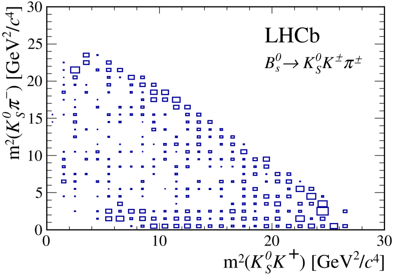
# First observation of $B_s^0 \to K_S \overline{K}^{*0}$

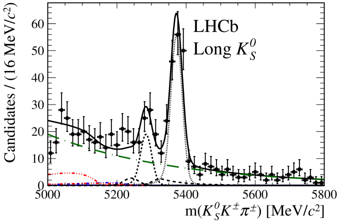
arXiv:1506.08634
(Submitted to JHEP)

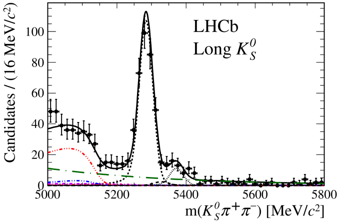
$\int \mathcal{L} = 1\,\text{fb}^{-1}$ of 2011 data

# $B_s^0 \to K_S \overline{K}^{*0}$

- $B^0_{(s)} \to K^0_S hh'$, $h, h' = K, \pi$, interesting for $CP$-violation measurements
- LHCb analysis with $1\,\text{fb}^{-1}$
  - First observation $B_s^0 \to K_S^0 K^+ \pi^-$
  - First observation $B_s^0 \to K_S^0 \pi^+ \pi^-$
  - Confirmation $B^0 \to K_S^0 K^+ \pi^-$
  - Limit for $B^0 \to K_S^0 K^+ K^-$

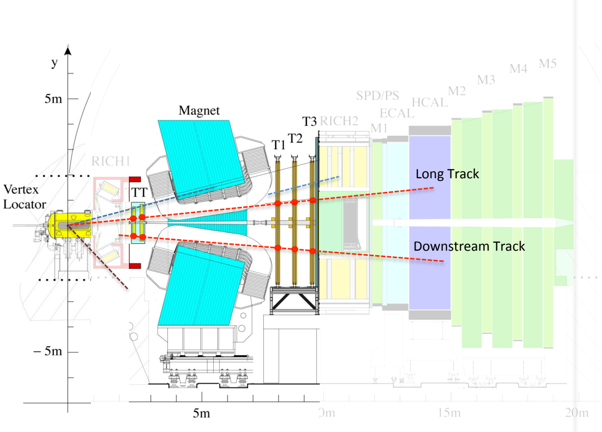


# $B_s^0 \to K_S \overline{K}^{*0}$

- Two $K_S^0(\to \pi^+\pi^-)$ categories, long and downstream
- Simultaneous fit to the $M(K_S^0 K^\mp \pi^\pm) \times M(K^\mp \pi^\pm)$ to extract the $B_{(s)} \to K_S^0 \overline{K}^{*0}$ resonant component



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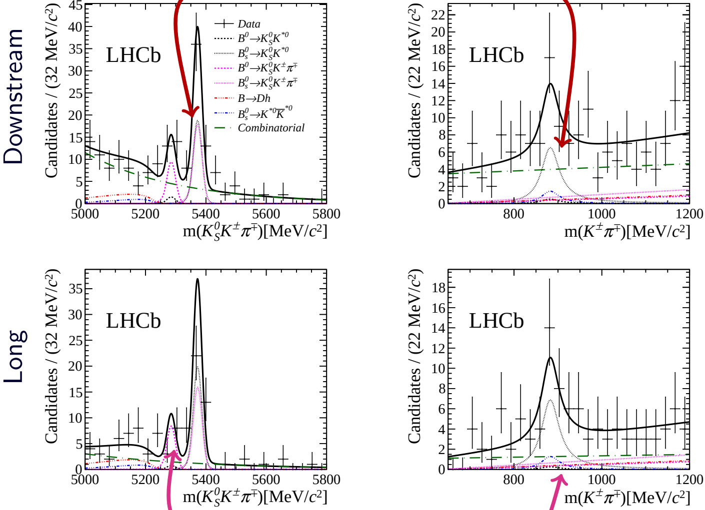
arXiv:1506.08634

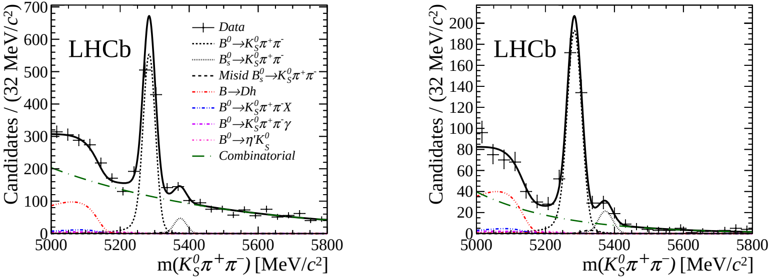
First observation of $B_s^0 \to K_S \overline{K}^{*0}$




No $B^0 \to K^0_S \overline{K}^{*0}$

# $B_s^0 \to K_S \overline{K}^{*0}$

arXiv:1506.08634




- Using $B^0 \to K_S^0\pi^+\pi^-$ as normalisation channel

$$\mathcal{B}(B_s^0 \to K_S \overline{K}^{*0}) = (10.9 \pm 2.5(\text{stat.}) \pm 1.2(\text{syst.})) \times 10^{-6}$$

- No resonant $B^0 \to K_S \overline{K}^{*0}$ component is observed

$\mathcal{B}(B^0 \to K_S \overline{K}^{*0}) < 0.64\,(0.69) \times 10^{-6}$ at 90% (95%) CL

# Summary

- The latest LHCb amplitude analyses in charmless $B$ decays have been presented.
- Discovery of two new decay modes: $B^0 \to \rho^0\rho^0$ and $B_s^0 \to K_S\overline{K}^{*0}$
- The amplitude analysis of $B^0 \to \rho^0\rho^0$ reveals a dominant longitudinal polarisation in agreement with the measurement from BaBar. The $B^0 \to \rho^0 f_0(980)$ signal reported by Belle is not confirmed.
- The analysis of $B_s^0 \to K^{*0}\overline{K}^{*0}$ confirms the low polarisation reported previously by LHCb. The 8 $CP$ asymmetries measured for this mode are found compatibe with the SM expectation of no $CP$ violation
- Word best measurement of the $B_s^0 \to \phi\phi$ branching fraction, and no evidence for $B^0 \to \phi\phi$.

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Thanks for your attention!

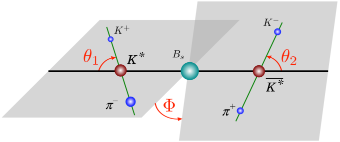
# Backup

# $B_s^0 \to K^{*0}\overline{K}^{*0}$ $CP$ asymmetries

| | $CP$ observable | $g_T^i(\theta_1,\theta_2,\varphi)$ |
|---|---|---|
| $\mathcal{A}_T^{(1)}$ | $\Im(A_\perp A_0^* - \bar{A}_\perp \bar{A}_0^*)$ | $\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\varphi$ |
| $\mathcal{A}_T^{(2)}$ | $\Im(A_\perp A_\parallel^* - \bar{A}_\perp \bar{A}_\parallel^*)$ | $\sin 2\varphi$ |
| $\mathcal{A}_T^{(3)}$ | $\Im(A_\perp^* A_s^- - \bar{A}_\perp^* \bar{A}_s^-)$ | $\mathrm{sign}(\cos\theta_1+\cos\theta_2)\sin\varphi$ |
| $\mathcal{A}_T^{(4)}$ | $\Im(A_\perp^* A_{ss} - \bar{A}_\perp^* \bar{A}_{ss})$ | $\mathrm{sign}(\cos\theta_1+\cos\theta_2)\sin\varphi$ |

| | $CP$ observable | $g_D^i(\theta_1,\theta_2,\varphi)$ |
|---|---|---|
| $\mathcal{A}_D^{(1)}$ | $\Re(A_s^+ A_0^* - \bar{A}_s^+ \bar{A}_0^*)$, $\Re(A_s^+ A_{ss}^* - \bar{A}_s^+ \bar{A}_{ss}^*)$ | $\cos\theta_1\cos\theta_2(\cos\theta_1-\cos\theta_2)$ |
| $\mathcal{A}_D^{(2)}$ | $\Re(A_s^+ A_\parallel^* - \bar{A}_s^+ \bar{A}_\parallel^*)$ | $(\cos\theta_1-\cos\theta_2)\cos\varphi$ |
| $\mathcal{A}_D^{(3)}$ | $\Re(A_s^+ A_0^* - \bar{A}_s^+ \bar{A}_0^*)$, $\Re(A_s^+ A_{ss}^* - \bar{A}_s^+ \bar{A}_{ss}^*)$ | $(\cos\theta_1-\cos\theta_2)$ |
| $\mathcal{A}_D^{(4)}$ | $\Re(A_s^+ (A_s^-)^* - \bar{A}_s^+ (\bar{A}_s^-)^*)$ | $\cos^2\theta_1-\cos^2\theta_2$ |

# $B_s^0 \to (K^+\pi^-)(K^-\pi^+)$ decay rate




$$\frac{d^6\Gamma}{dX} \propto \Big| \Big( A_0(t)\cos\theta_1\cos\theta_2 + \frac{A_\parallel(t)}{\sqrt{2}}\sin\theta_1\sin\theta_2\cos\varphi + i\frac{A_\perp(t)}{\sqrt{2}}\sin\theta_1\sin\theta_2\sin\varphi \Big)\mathcal{M}_1(m_1)\mathcal{M}_1(m_2)$$
$$- \frac{A_s^+(t)}{\sqrt{6}}(\cos\theta_1\mathcal{M}_1(m_1)\mathcal{M}_0(m_2) - \cos\theta_2\mathcal{M}_0(m_1)\mathcal{M}_1(m_2))$$
$$- \frac{A_s^-(t)}{\sqrt{6}}(\cos\theta_1\mathcal{M}_1(m_1)\mathcal{M}_0(m_2) + \cos\theta_2\mathcal{M}_0(m_1)\mathcal{M}_1(m_2))$$
$$- \frac{A_{ss}(t)}{3}\mathcal{M}_0(m_1)\mathcal{M}_0(m_2) \Big|^2$$

$\mathcal{M}_0 \equiv$ LASS parameterisation
$\mathcal{M}_1 \equiv$ Relativistic spin-1 Breit Wigner

# $B_s^0 \to K^{*0}\overline{K}^{*0}$ systematic

| Parameter | $\sigma_{\rm acc}$ | $\sigma_{\rm sim}$ | $\sigma_{\rm rw}$ | $\sigma_{\rm mass}$ | $\sigma_{\rm res}$ | Total |
|---|---|---|---|---|---|---|
| $f_L$ | 0.031 | 0.010 | 0.010 | 0.021 | 0.006 | 0.040 |
| $f_\parallel$ | 0.008 | 0.008 | 0.004 | 0.005 | 0.007 | 0.015 |
| $\lvert A_s^+\rvert^2$ | 0.019 | 0.005 | 0.002 | 0.011 | 0.003 | 0.023 |
| $\lvert A_s^-\rvert^2$ | 0.007 | 0.007 | 0.010 | 0.003 | 0.012 | 0.019 |
| $\lvert A_{ss}\rvert^2$ | 0.003 | 0.001 | 0.000 | 0.005 | 0.003 | 0.007 |
| $\delta_\parallel$ | 0.130 | 0.037 | 0.042 | 0.005 | 0.025 | 0.144 |
| $\delta_\perp - \delta_s^+$ | 0.016 | 0.019 | 0.000 | 0.017 | 0.027 | 0.040 |
| $\delta_s^-$ | 0.160 | 0.036 | 0.075 | 0.033 | 0.030 | 0.186 |
| $\delta_{ss}$ | 0.096 | 0.076 | 0.188 | 0.018 | 0.044 | 0.229 |

# $B^0 \to \rho^0\rho^0$ parameters

| $A_i$ | $\eta_i$ | $f_i$ |
|---|---|---|
| $A^0_{\rho\rho}$ | 1 | $M_\rho(m_1)M_\rho(m_2)\cos\theta_1\cos\theta_2$ |
| $A^\parallel_{\rho\rho}$ | 1 | $M_\rho(m_1)M_\rho(m_2)\frac{1}{\sqrt{2}}\sin\theta_1\sin\theta_2\cos\varphi$ |
| $A^\perp_{\rho\rho}$ | $-1$ | $M_\rho(m_1)M_\rho(m_2)\frac{i}{\sqrt{2}}\sin\theta_1\sin\theta_2\sin\varphi$ |
| $A^0_{\rho\omega}$ | 1 | $\frac{1}{\sqrt{2}}[M_\rho(m_1)M_\omega(m_2)+M_\omega(m_1)M_\rho(m_2)]\cos\theta_1\cos\theta_2$ |
| $A^\parallel_{\rho\omega}$ | 1 | $\frac{1}{\sqrt{2}}[M_\rho(m_1)M_\omega(m_2)+M_\omega(m_1)M_\rho(m_2)]\frac{1}{\sqrt{2}}\sin\theta_1\sin\theta_2\cos\varphi$ |
| $A^\perp_{\rho\omega}$ | $-1$ | $\frac{1}{\sqrt{2}}[M_\rho(m_1)M_\omega(m_2)+M_\omega(m_1)M_\rho(m_2)]\frac{i}{\sqrt{2}}\sin\theta_1\sin\theta_2\sin\varphi$ |
| $A_{\rho(\pi\pi)_0}$ | $-1$ | $\frac{1}{\sqrt{6}}[M_\rho(m_1)M_{(\pi\pi)_0}(m_2)\cos\theta_1+M_{(\pi\pi)_0}(m_1)M_\rho(m_2)\cos\theta_2]$ |
| $A_{\rho f(980)}$ | $-1$ | $\frac{1}{\sqrt{6}}[M_\rho(m_1)M_{f(980)}(m_2)\cos\theta_1+M_{f(980)}(m_1)M_\rho(m_2)\cos\theta_2]$ |
| $A_{(\pi\pi)_0(\pi\pi)_0}$ | 1 | $M_{(\pi\pi)_0}(m_1)M_{(\pi\pi)_0}(m_2)\frac{1}{3}$ |
| $A^0_{\rho f_2}$ | $-1$ | $\sqrt{\frac{5}{24}}\left[M_\rho(m_1)M_{f_2}(m_2)\cos\theta_1(3\cos^2\theta_2-1)+M_{f_2}(m_1)M_\rho(m_2)\cos\theta_2(3\cos^2\theta_1-1)\right]$ |
| $A^{S+}_{a_1\pi}$ | 1 | $\frac{1}{\sqrt{8}}\sum_{\{ijkl\}}\frac{1}{\sqrt{3}}M_{a_1}(m_{ijk})M_\rho(m_{ij})\,[\cos\alpha_{kl}\cos\beta_{ik}+\sin\alpha_{kl}\sin\beta_{ik}\cos\Phi_{kl}]$ |

# $B^0 \to \rho^0\rho^0$ amplitude fit

| Parameter | Definition | Fit result |
|---|---|---|
| $f_L$ | $\lvert A^0_{\rho\rho}\rvert^2/(\lvert A^0_{\rho\rho}\rvert^2+\lvert A^{\parallel}_{\rho\rho}\rvert^2+\lvert A^{\perp}_{\rho\rho}\rvert^2)$ | $0.745^{+0.048}_{-0.058}\pm0.034$ |
| $f'_{\parallel}$ | $\lvert A^{\parallel}_{\rho\rho}\rvert^2/(\lvert A^{\parallel}_{\rho\rho}\rvert^2+\lvert A^{\perp}_{\rho\rho}\rvert^2)$ | $0.50\pm0.09\pm0.05$ |
| $\delta_{\parallel}-\delta_0$ | $\arg(A^{\parallel}_{\rho\rho}A^{0*}_{\rho\rho})$ | $1.84\pm0.20\pm0.14$ |
| $F_{\rho(\pi\pi)_0}$ | $\lvert A_{\rho(\pi\pi)_0}\rvert^2/(\lvert A^0_{\rho\rho}\rvert^2+\lvert A^{\parallel}_{\rho\rho}\rvert^2+\lvert A^{\perp}_{\rho\rho}\rvert^2)$ | $0.30^{+0.11}_{-0.09}\pm0.08$ |
| $F_{\rho f(980)}$ | $\lvert A_{\rho f(980)}\rvert^2/(\lvert A^0_{\rho\rho}\rvert^2+\lvert A^{\parallel}_{\rho\rho}\rvert^2+\lvert A^{\perp}_{\rho\rho}\rvert^2)$ | $0.29^{+0.12}_{-0.09}\pm0.08$ |
| $F_{(\pi\pi)_0(\pi\pi)_0}$ | $\lvert A_{(\pi\pi)_0(\pi\pi)_0}\rvert^2/(\lvert A^0_{\rho\rho}\rvert^2+\lvert A^{\parallel}_{\rho\rho}\rvert^2+\lvert A^{\perp}_{\rho\rho}\rvert^2)$ | $0.21^{+0.06}_{-0.04}\pm0.08$ |
| $\delta_{\perp}-\delta_{\rho(\pi\pi)_0}$ | $\arg(A^{\perp}_{\rho\rho}A^{*}_{\rho(\pi\pi)_0})$ | $-1.13^{+0.33}_{-0.22}\pm0.24$ |
| $\delta_{\perp}-\delta_{\rho f(980)}$ | $\arg(A^{\perp}_{\rho\rho}A^{*}_{\rho f(980)})$ | $1.92\pm0.24\pm0.16$ |
| $\delta_{(\pi\pi)_0(\pi\pi)_0}-\delta_0$ | $\arg(A_{(\pi\pi)_0(\pi\pi)_0}A^{0*}_{\rho\rho})$ | $3.14^{+0.36}_{-0.38}\pm0.39$ |
| $F_{\rho\omega}$ | $(\lvert A^0_{\rho\omega}\rvert^2+\lvert A^{\parallel}_{\rho\omega}\rvert^2+\lvert A^{\perp}_{\rho\omega}\rvert^2)/(\lvert A^0_{\rho\rho}\rvert^2+\lvert A^{\parallel}_{\rho\rho}\rvert^2+\lvert A^{\perp}_{\rho\rho}\rvert^2)$ | $0.025^{+0.048}_{-0.022}\pm0.020$ |
| $f_L^{\rho\omega}$ | $\lvert A^0_{\rho\omega}\rvert^2/(\lvert A^0_{\rho\omega}\rvert^2+\lvert A^{\parallel}_{\rho\omega}\rvert^2+\lvert A^{\perp}_{\rho\omega}\rvert^2)$ | $0.70^{+0.23}_{-0.60}\pm0.13$ |
| $f_{\parallel}^{\rho\omega\prime}$ | $\lvert A^{\parallel}_{\rho\omega}\rvert^2/(\lvert A^{\parallel}_{\rho\omega}\rvert^2+\lvert A^{\perp}_{\rho\omega}\rvert^2)$ | $0.97^{+0.69}_{-0.56}\pm0.15$ |
| $\delta_0^{\omega}-\delta_0$ | $\arg(A^0_{\rho\omega}A^{0*}_{\rho\rho})$ | $-2.56^{+0.76}_{-0.92}\pm0.22$ |
| $\delta_{\parallel}^{\omega}-\delta_0$ | $\arg(A^{\parallel}_{\rho\omega}A^{0*}_{\rho\rho})$ | $-0.71^{+0.71}_{-0.67}\pm0.32$ |
| $\delta_{\perp}^{\omega}-\delta_{\rho(\pi\pi)_0}$ | $\arg(A^{\perp}_{\rho\omega}A^{*}_{\rho(\pi\pi)_0})$ | $-1.72\pm2.62\pm0.80$ |
| $F^0_{\rho f_2}$ | $\lvert A^0_{\rho f_2}\rvert^2/(\lvert A^0_{\rho\rho}\rvert^2+\lvert A^{\parallel}_{\rho\rho}\rvert^2+\lvert A^{\perp}_{\rho\rho}\rvert^2)$ | $0.01^{+0.04}_{-0.02}\pm0.03$ |
| $\delta^0_{\rho f_2}-\delta_{\rho(\pi\pi)_0}$ | $\arg(A^0_{\rho f_2}A^{*}_{\rho(\pi\pi)_0})$ | $-0.56\pm1.48\pm0.80$ |
| $F^{S^+}_{a_1\pi}$ | $\lvert A^{S^+}_{a_1\pi}\rvert^2/(\lvert A^0_{\rho\rho}\rvert^2+\lvert A^{\parallel}_{\rho\rho}\rvert^2+\lvert A^{\perp}_{\rho\rho}\rvert^2)$ | $1.4^{+1.0+1.2}_{-0.7-0.8}$ |
| $\delta^{S^+}_{a_1\pi}-\delta_{\rho(\pi\pi)_0}$ | $\arg(A^{S^+}_{a_1\pi}A^{*}_{\rho(\pi\pi)_0})$ | $-0.09^{+0.30}_{-0.36}\pm0.38$ |

# $B^0 \to \rho^0\rho^0$ systematic

| Systematic effect | Uncertainty on $f_L$ (%) | Uncertainty on $P(B^0 \to \rho^0\rho^0)$ (%) |
|---|---|---|
| Fit bias | 0.1 | 0.8 |
| Model | 3.6 | 6.2 |
| $B^0 \to a_1(1260)^+\pi^-$ | 1.2 | 1.1 |
| S-wave lineshape | 3.4 | 6.1 |
| Lineshapes | $<0.1$ | 0.1 |
| Background subtraction | 0.1 | 0.5 |
| Acceptance integrals | 2.7 | 4.5 |
| Angular/Mass resolution | 0.8 | 1.5 |

# $B_s^0 \to \phi\phi$ systematic

| Source of systematic uncertainty | Relative uncertainty (%) |
|---|---|
| S-wave fraction | 3.1 |
| Relative efficiency between P and S-wave | 1.1 |
| Simulation sample size | 0.8 |
| Fit model | 0.6 |
| Tracking efficiency | 0.5 |
| Hadronic interactions | 0.3 |
| Hardware trigger | 1.1 |
| Particle identification efficiency | 0.3 |
| $\mathcal{B}(\phi \to K^+K^-)$ | 1.0 |
| Quadratic sum of the above | 3.8 |
| Fragmentation fraction ratio $(f_s/f_d)$ | 5.8 |

# $B_s^0 \to K_S \overline{K}^{*0}$ systematic

| Source | $\frac{\mathcal{B}(B_s^0 \to K_S^0 K^{*0})}{\mathcal{B}(B^0 \to K_S^0 \pi^+\pi^-)}$ | | $\frac{\mathcal{B}(B^0 \to K_S^0 K^{*0})}{\mathcal{B}(B^0 \to K_S^0 \pi^+\pi^-)}$ | |
|---|---|---|---|---|
| | Downstream | Long | Downstream | Long |
| Fit | 0.05 | 0.03 | 0.20 | 0.28 |
| Selection | 0.08 | 0.10 | 0.08 | 0.11 |
| PID | 0.01 | 0.01 | 0.01 | 0.01 |
| Trigger | 0.07 | 0.07 | 0.02 | 0.09 |
| Lifetime | 0.05 | 0.05 | - | - |
| Total | 0.13 | 0.14 | 0.22 | 0.31 |
| $f_s/f_d$ | 0.06 | 0.06 | - | - |