HYPERON-NUCLEON INTERACTIONS † (*)

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For an examination of hyperfragment binding energies a potential was constructed largely on phenomenological grounds. If one assumes that pion exchange is to generate Λ-N forces and such forces are to conserve isotopic spin, then the simplest effective pion-Λ interaction would be $\boldsymbol{\varphi}_\pi(x) \cdot \boldsymbol{\varphi}_\pi(x)\, \bar{\psi}_\Lambda(x)\, \psi_\Lambda(x)$. The main characteristics of such a potential are apparent from the form of the coupling; first, there are three-body forces coming from the emission of two pions at each vertex; secondly, the Λ spin is not present in the potential, and thirdly, the range of the direct Λ-N force should be $^1/_2\, m_\pi$, as it comes from double pion exchange. The form of the potential and the approximate ranges were obtained from the lowest order non-vanishing static potential produced by such an interaction, and the standard N-π interaction.

The interaction potential for two nucleons and a Λ-particle actually used in the variational calculation of the binding energies was

$$V(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_\Lambda) = -V_1\left[e^{-\lambda(\mathbf{r}_1-\mathbf{r}_\Lambda)^2} + e^{-\lambda(\mathbf{r}_2-\mathbf{r}_\Lambda)^2}\right] - V_2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\, e^{-\lambda[(\mathbf{r}_1-\mathbf{r}_\Lambda)^2+(\mathbf{r}_2-\mathbf{r}_\Lambda)^2]}\ .$$

The constant λ was chosen to be $1.05\, f^{-2}$, so that the range of the two-body force corresponds to $^1/_2\, m_\pi$. A trial function of the form $\exp\{-\alpha(\mathbf{r}_1-\mathbf{r}_2)^2 - \beta(\mathbf{r}_1+\mathbf{r}_2-2\mathbf{r}_\Lambda)^2\}$ was used, and α and β were varied independently. With $V_1 = 107$ MeV, $V_2 = 37.5$ MeV, the non-existence of $_\Lambda H^2$ and $_\Lambda He^3$, and the binding energies of $_\Lambda H^3$, $_\Lambda H^4$ and $_\Lambda He^4$, were successfully reproduced. For $_\Lambda H^3$, states other than a Λ attached to a deuteron were considered but found not to bind.

From the symmetric pion-baryon couplings proposed by many authors, one can also form a potential by a perturbation expansion with static baryon sources for the pion field. Qualitatively the major difference between this potential and the former is that V_1 here becomes Λ spin dependent, with the Λ-N system more attractive in a spin singlet than triplet state. With the assumption that the strength of the two-body force of this potential for its more attractive state is also 107 MeV, one may estimate that $g_{\Sigma\Lambda\pi}/g_{N\pi} \sim 1$. Since $V_1 = 107$ MeV corresponds to the Λ-N system just unbound, one may make another estimate of this ratio by determining the potential necessary so that $K \cot \delta$ at zero energy for Λ-N scattering just becomes negative. Such a calculation will be described below: it gives the same result as the one already outlined.

Some estimates of the binding energy of a Λ in an " infinite " nucleus were also made. The above potential and the Fermi gas model of the nucleus were used. Because the Pauli principle does not effect the Λ, the binding energies were very large (~ 50 MeV), although much of that energy might well come from potentials which were made too attractive in order to fit the light hyperfragments within the approximate variational treatment. With the spin dependent potential the binding energy could drop to about 10 MeV.

A particularly simple form of potential was considered for some estimates of the s-wave phase-shifts in the scattering of the Σ-Λ-N system. One way to look at the proposed baryon symmetry is to suppose that the two-dimensional isotopic spin matrices of the pion-nucleon interaction are replaced by four-dimensional ones, corresponding either to a $(N\Xi)$ or a $(\Sigma\Lambda)$ multiplet. The $\boldsymbol{\tau}$ matrix for the $(\Sigma\Lambda)$ system decomposes into $\mathbf{t} + \mathbf{s}$, where $\mathbf{t}$ represents the ordinary isotopic spin matrix for a Σ ($T = 1$) and $\mathbf{s}$ is a matrix which mixes Λ and Σ, i.e.

$$\mathbf{s}_x = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \text{etc.}$$

In a most naïve way, if in the second order static N-N potential

$$\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{g_N g_N}{4\pi\, m_N{}^2} (\boldsymbol{\sigma}_1 \cdot \nabla)(\boldsymbol{\sigma}_2 \cdot \nabla) \frac{e^{-m_\pi \varrho}}{\varrho}$$

$\boldsymbol{\tau}_2$ is replaced by $\mathbf{t} + \mathbf{s}$, one g_N by g_Σ, and one m_N by m_Σ, then one has a Σ-Λ-N potential with a direct Σ-N interaction and with an interaction exchanging Σ and Λ, but no direct Λ-N interaction. Such a potential is the one to be used here.

Of course, while there would be a direct Λ-N force coming from higher order terms, the usefulness of the concept of a potential seems sufficiently questionable that

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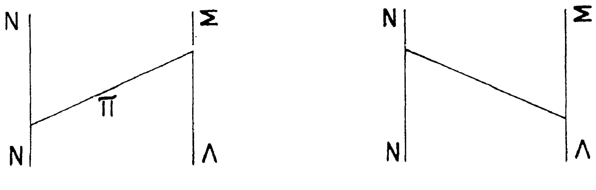
at present a simple model without direct Λ-N terms should suffice. For consider the " potential " :

$$V_{\Lambda \to \Sigma}(\rho) = \frac{g_N g_{\Sigma\Lambda}}{4 m_N m_\Lambda} \tau_1 \cdot \mathbf{s} \, \frac{(\sigma_1 \cdot \nabla)(\sigma_2 \cdot \nabla)}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{e^{i\mathbf{k}\rho}}{E_\pi} \times \left(\frac{1}{E_\pi + \Delta m} + \frac{1}{E_\pi} \right)$$

formed in second order perturbation theory corresponding to the diagrams :



where Δm is the actual Σ-Λ mass difference. The "potential" $V_{\Sigma \to \Lambda}$ would have the sign of Δm reversed. Thus the two potentials are not the same, and if one were to write coupled equations for the Λ and Σ wave functions with $V_{\Lambda \to \Sigma}$ in one and $V_{\Sigma \to \Lambda}$ in the other equation, the system of equations would not be self-adjoint, and therefore there would not be particle conservation. An evaluation of the integrals suggests that the difference between $V_{\Sigma \to \Lambda}$ and $V_{\Lambda \to \Sigma}$ is not too great, but the general problem remains. In the simple way the potential was constructed, such difficulties do not appear. The form of the potential will be taken as $V(\mathbf{r}) = V_1(\mathbf{r})\, \mathbf{t} \cdot \tau + V_2(\mathbf{r})\, \tau_1 \cdot \mathbf{s}$, where V_1 and V_2 may be spin dependent and tensor forces have been suppressed.

The standard variational principles for phase-shifts as well as those which may be used to generate effective range expansions may be readily applied to systems with mixtures of Σ's and Λ's. The scattering naturally divides into two regions : (i) below Σ-production threshold :

$$E_\Lambda = (K^2/2\overline{m}_\Lambda) > 0; \; E_\Sigma = (k^2/2\overline{m}_\Sigma) < 0$$

(the bar refers to reduced mass); (ii) Σ-N scattering and Λ-N scattering where the two hyperons may interchange, or $k^2 > 0$. For $K^2 \to 0$ one has the standard effective range expansion of $K \cot \delta$, but for $k^2 \to 0$ it no longer exists. For $k^2 > 0$ let the two eigenphases $\delta_{1,2}$ for the $(\Sigma$-$\Lambda)$-N system be defined by

$$\lim_{r \to \infty} r\psi_{\Sigma N} \sim \sin(kr + \delta_{1,2})$$

$$\lim_{r \to \infty} r\psi_{\Lambda N} \sim M_{1,2} \sin(Kr + \delta_{1,2}) \, .$$

M_1 and M_2 satisfy $k + M_1 M_2 K = 0$. Let 2 refer to that phase for which $M_2|_{k=0} \neq 0$. Then

$$k \cot \delta_1 = (k \cot \delta_1)\Big|_{k=0} \cdot \left[1 + \frac{k}{(KM_2^2)_{k=0}} \right],$$

$$K \cot \delta_2 = (K \cot \delta_2)\Big|_{k=0} \cdot \left[1 - \frac{k}{(KM_2^2)_{k=0}} \right],$$

and for Λ-N scattering just below Σ-production threshold

$$K \cot \delta = (K \cot \delta_2)\Big|_{k=0} + \frac{|k|}{(M_2^2)_{k=0}} \, .$$

These expansions are exact to first order in k and depend solely on the difference in wave number of the Σ and Λ, and the existence of the part of the potential exchanging Σ and Λ. Thus, even with a direct Λ-N force added, such expansions are valid.

The explicit potentials used in the variational calculations were : (*)

	Σ-N potential		$(\Sigma\Lambda)$-N potential	
	$S = 0$	$S = 1$	$S = 0$	$S = 1$
$T = {}^1/_2$	$44.0\, e^{-ra}$	$-110\, e^{-ra}$	$58.7\, e^{-ra} \tau \cdot \mathbf{s}$	$36.7 e^{-ra}\, \tau . \mathbf{s}$
$T = {}^3/_2$	$-88.0\, e^{-ra}$	0	—	

$a = 0.796 f$ (i.e. about $1/m_\pi$ for the range).

The values do not correspond exactly to the potential of the lowest order static potential — a slight attempt has been made to correct for some higher order effects. If $g_{\Sigma\pi}$ were of opposite sign to $g_{N\pi}$ — which would still preserve the symmetries — the potentials would have opposite sign. The computations were also done in this case.

The s-wave phase-shift expansions are given in Tables I and II along with an estimate of Coulomb corrections in $\Sigma^+ p$ scattering.

If the lowest order static potential had been used with $(g_{\Lambda\Sigma\pi}/g_{N\pi}) \approx 1$, $K \cot \delta/_{K=0}$ would be just negative in the spin singlet state, corresponding to a barely bound Λ-N system. A resonance in Λ-N scattering at ~ 60 MeV (all energies in c.m. system) occurs for most of the potentials, as well as a resonance in eigenphase 2 just above threshold. These seem to be more a property of the existence of a strong interaction mixing Σ-Λ rather than of specific potential parameters.

Cross-sections from the above phase-shift estimates are given below. It has been assumed throughout that the Σ-mass differences are negligible. For the $(\Sigma^+ n)$, $(\Sigma^0 p)$ systems, such seems to be the case, so that the results apply. But as the $(\Sigma^- p)$ and $(\Sigma^0 n)$ mass differences are large, no estimates are given as both the mass differences and Coulomb forces split the isotopic states. The $T = 3/2$, $S = 0$ potential was picked to give, in an exact calculation, a zero energy resonance; this value is of about the right magnitude and thus it is possible to test accuracy of the variational procedure. The huge cross-sections come from this choice. As data becomes available, the strength of the potential may be adjusted accordingly. Unfortunately, the tensor forces and higher angular momentum states — both omitted — probably give important contributions to some of the cross-sections.

(*) Attractive potentials are negative.

TABLE I (*)

The $T = \frac{1}{2}$ system (mixed Σ N and Λ N)

V_1 V_2	44.0 MeV 58.7 MeV	−110.1 MeV 36.7 MeV	−44.0 MeV −58.7 MeV	110.1 MeV −36.7 MeV
low energy Λ-N scattering $K \cot \delta =$	$(2.30 + 3.00\, K^2)$	$4.71 + 3.88\, K^2$	$1.85 + 2.24\, K^2$	$7.53 + 5.8\, K^2$
Λ-N scattering below Σ production threshold $K \cot \delta =$	$-12.2 + 19.3\,\lvert k \rvert$	$-3.77 + 15.0\,\lvert k \rvert$	$-8.04 + 4.75\,\lvert k \rvert$	$-126 + 236\,\lvert k \rvert$
" principal Λ phase " scattering just above Σ threshold $K \cot \delta_2 =$ $M_2 =$	$-12.2\,(1-17.2k)$ -0.228	$-3.77\,(1-13.3k)$ $+0.259$	$-8.04\,(1-4.22k)$ -0.459	$-126\,(1-210k)$ $+0.0652$
" principal Σ phase " scattering just above Σ threshold $K \cot \delta_1 =$ $M_1 =$	$6.14\,(1+17.2k)$ $3.89\,k$	$0.154\,(1+13.3k)$ $-3.43\,k$	$0.539\,(1+4.22\,k)$ $1.93\,k$	$36.1\,(1+210k)$ $-13.60\,k$

TABLE II (*)

The $T = {}^3/_2$ system

V_1		−88.0 MeV	88.0 MeV
$k \cot \delta$ $\Sigma^+ - p$ scattering		$-0.00715 + 1.5\,k^2$	$-0.628 + 0.244\,k^2$
$\dfrac{\pi}{b_0\,(1 - e^{-2\pi\eta})} \cot \delta + \dfrac{1}{b_0}\,[Re\,\psi\,(\eta - \log \eta)]$	$b_0 = \dfrac{\hbar^2}{2\overline{m}_\Sigma e^2}$ $\eta = \dfrac{1}{2kb_0}$	$0.062 \quad + 1.5\,k^2$	$-0.520 + 0.244\,k^2$

TABLE III

Cross-sections

Λ-p scattering	$\dfrac{g_{\Sigma\pi}}{g_{N\pi}} > 0$	$\dfrac{g_{\Sigma\pi}}{g_{N\pi}} < 0$
1) zero energy σ_T	$\sim$ 20 mb	$\sim$ 20 mb
2) resonance $\sim$ 60 MeV σ_T	$\sim$ 80 mb	80 mb
3) Σ-production threshold ($\sim$ 75 MeV) σ_T	$\sim$ 4 mb	0.3 mb
4) at the resonance above Σ-production threshold σ_T	$\sim$ 15 mb	$\sim$ 15 mb
σ_{abs}	$\sim$ 15 mb	$\sim$ 15 mb

Each spin state has its own resonance : for $(g_{\Sigma\pi}/g_{N\pi}) > 0$ the singlet and triplet resonances are at $E_\Sigma \sim {}^1/_2$ MeV; for $(g_{\Sigma\pi}/g_{N\pi}) < 0$ the singlet resonance is at $E_\Sigma \sim 4$ MeV, the triplet at $E_\Sigma \sim 0$. The Λ absorbed appears as ${}^1/_3\,\Sigma^+$ and ${}^2/_3\,\Sigma^0$.

Σ^--n scattering		$\dfrac{g_{\Sigma\pi}}{g_{\pi N}} > 0$	$\dfrac{g_{\Sigma\pi}}{g_{N\pi}} < 0$
1) zero energy	σ_T	$\sim$ 400 b	$\sim$ 55 mb
Σ^+-n scattering			
1) zero energy	σ_T	$\sim$ 46 b	36 mb
	$\sigma_{\Sigma^0\text{-prod}}$	$\sim$ 88 b	100 mb
	$\sigma_{\Lambda\text{-prod}}$	$\sim 15\left(\dfrac{K}{k}\right)$ mb	$4\left(\dfrac{K}{k}\right)$ mb

Σ^+-p scattering

At $E_\Sigma = 1.6$ MeV : in the backward direction :

$\dfrac{d\sigma_T}{d\Omega} \sim 100$ mb/st. for $(g_{\Sigma\pi}/g_{N\pi}) > 0$,

or 5 mb/st for $(g_{\Sigma\pi}/g_{N\pi}) < 0$

(*) The unit of length is 0,8 f, and momenta are measured accordingly.