$\pi^- + p \to \Lambda^0 + \Theta^0$, $\Lambda^0 \to p + \pi^-$ ANGULAR DISTRIBUTION OF PRODUCTION AND DECAY at 1.12 GeV/c †

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Summary of maximum likelihood fit to the data

At 1.12 GeV/c, 236 Λ^0-decays arising from $\pi^- + p \to \Lambda^0 + \theta^0$ were studied.

The data were fitted with the expression

$$d\bar{N} = \varepsilon(\theta)\,[A_1 + A_2 \cos\theta + A_3 \cos^2\theta + \xi \sin\theta\,(A_4 + A_5 \cos\theta)] \times 2\pi\, d(\cos\theta)\,\frac{d\xi}{2} \qquad (1)$$

$\varepsilon(\theta)$ = probability that a Λ^0 produced at angle θ decays inside the chamber

θ = centre of mass production angle of the Λ^0

ξ = $[\mathbf{p}_{\pi\,\mathrm{inc}} \times \mathbf{p}_{\Lambda} \cdot \mathbf{p}_{\pi\,\mathrm{decay}}]$ / (magnitude of same).

This expression is what one obtains for production involving final s- and p-waves only [1]. (The centre of mass momentum is here 300 MeV/c.)

The likelihood function

$$\mathfrak{L} = \prod_{ij} e^{-\bar{N}_{ij}} (\bar{N})_{ij}^{\;N_{ij}}$$

$i = 1, 2 \,..\, 6$ = histogram interval in $\cos\theta$

$j = 1, 2 \,..\, 4$ = histogram interval in ξ

$\bar{N}_{ij}$ = counts from expression (1)

N_{ij} = observed counts

was formed and maximized by an iterative procedure.

The solution that maximises $\mathfrak{L}$ is:

A_1	A_2	A_3	A_4	A_5
21.892	− 27.008	11.944	14.065	− 32.278

With this solution is associated a 5×5 error matrix $|\delta A_i\, \delta A_j|$

	1	2	3	4	5
1	5.6480	−0.2248	−9.2172	+2.2051	−1.8025
2		8.8182	−7.1432	−2.0162	+1.7779
3			30.2028	−2.0248	−5.4913
4				11.5237	−9.5883
5					50.3981

All five A's are different from zero outside of experimental error.

The best fit to the data are shown in Fig. 1A, and in Fig. 5 of Steinberger's report.

From the plot of

$$\alpha P(\theta) = \frac{\sin\theta\,(A_4 + A_5 \cos\theta)}{A_1 + A_2 \cos\theta + A_3 \cos^2\theta} \qquad (2)$$

(Fig. 5) a lower limit to α can be obtained as follows:

The maximum of αP is: $\alpha P_{\max} = 0.73 \pm 0.14$, where the error is obtained by differentiating (2) and using the error matrix given above.

Then, since the polarization cannot exceed unity, we have a lower limit on α:

$$|\alpha_{\min}| = 0.73 \pm 0.14$$

Our data are consistent with any value of $|\alpha|$ lying between 0.73 and unity.

† Appendix to Session 5. — Experimental.

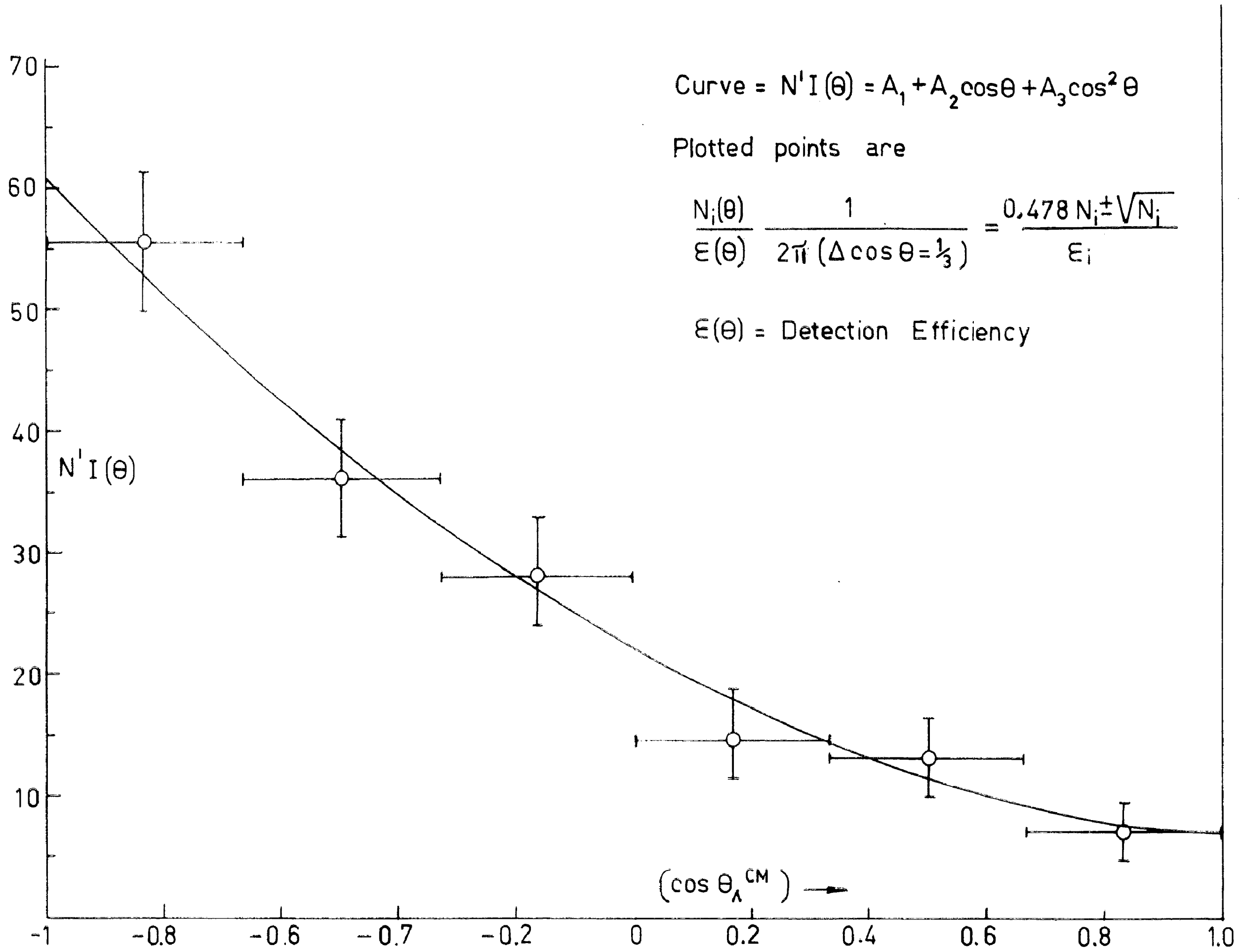


Fig. 1a: $\pi^- + p \rightarrow \Lambda^0 + \theta^0$ *production angular distribution. Curve fitted to the experimental points using s- and p-waves.*

Summary of s- and p-wave analysis of $\pi^- + p \rightarrow \Lambda + K^0$ at 1.1 GeV/c (236 Λ-decays)

The paper of Lee et al.[1] gives the theoretical distribution, $W(\theta, \xi)$, in ξ ("the up-down" direction cosine) and θ of the decay product of the Λ^0. The assumptions of this paper are (*a*) that parity is conserved in the production process, (*b*) but not conserved in the decay of the Λ^0, and (*c*) that only *s*- and *p*-waves are present in the final Λ^0, K^0-state.

$$W(\theta, \xi)\, d\xi d\Omega = \left\{ I(\theta) + \alpha \xi\, I(\theta) P(\theta) \right\} \frac{d\xi}{2} d\Omega$$

$$I(\theta) = |\mathbf{a} + \mathbf{b} \cos \theta|^2 + |\mathbf{c}|^2 \sin^2 \theta$$

$$P(\theta) = \frac{2 \operatorname{Im} \mathbf{c}^* (\mathbf{a} + \mathbf{b} \cos \theta) \sin \theta}{I(\theta)}$$

Now if α were known we could use the 5 least squares constants $A_1, A_2, \ldots A_5$ of

$$dN = \left\{ A_1 + A_2 \cos \theta + A_3 \cos^2 \theta + \xi \sin \theta (A_4 + A_5 \cos \theta) \right\} \times \frac{d\xi}{2} 2\pi d \cos \theta$$

to determine a, b, c, φ and ψ, where

$$\mathbf{a} = a e^{i\varphi}, \quad \mathbf{b} = b e^{i\psi}, \quad \mathbf{c} = c.$$

In order to obtain c, the spin-flip *p*-wave amplitude we must solve a cubic equation. Only two of the three solutions are real. These real solutions are called the $k = 0$ and $k = 2$ solutions. There is, for each of these two solutions, two alternative solutions for the phase angles, φ and ψ. The alternate solution is obtained by reflecting the " normal solution " about the imaginary axis in the complex plane.

a, b, and c have been normalized so that $\int I(\theta) d\Omega = 1$. φ and ψ are given in radians.

$k = 0$ Solution ($\alpha = 0.95$ (*))

Error Matrix

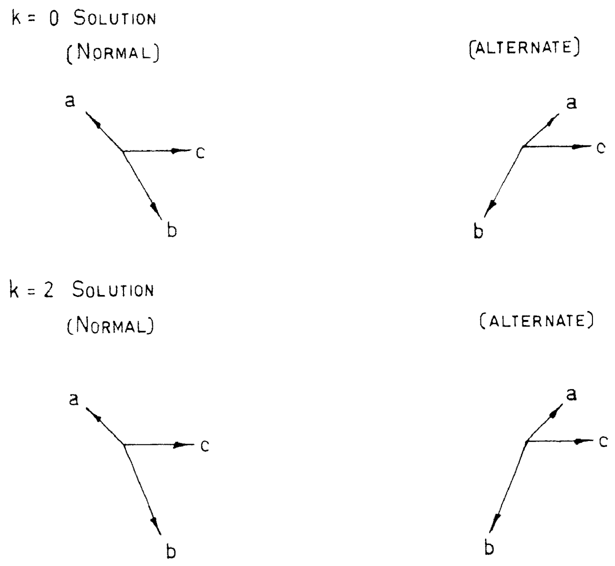
$a = 0.149 \pm 0.044$	$\overline{\delta a^2}$ 0.001902	$\overline{\delta a \delta b}$ − 0.001078	$\overline{\delta a \delta c}$ − 0.001214	$\overline{\delta a \delta \varphi}$ 0.008778	$\overline{\delta a \delta \psi}$ − 0.02174
$b = 0.286 \pm 0.033$		δb^2 0.001098	$\delta b \delta c$ 0.0005559	$\delta b \delta \varphi$ − 0.005680	$\delta b \delta \psi$ 0.01167
$c = 0.212 \pm 0.033$			$\overline{\delta c^2}$ 0.001067	$\delta c \delta \varphi$ − 0.005429	$\overline{\delta c \delta \psi}$ 0.01520
$\varphi = 2.34 \pm 0.31$ (0.70 ± 0.31 alt)				$\overline{\delta \varphi^2}$ 0.09767	$\overline{\delta \varphi \delta \psi}$ − 0.06420
$\psi = 5.24 \pm 0.59$ (4.18 ± 0.59 alt)					$\overline{\delta \psi^2}$ 0.34911

$k = 2$ Solution ($\alpha = 0.95$)

Error Matrix

$a = 0.161 \pm 0.063$	$\overline{\delta a^2}$ 0.003945	$\delta a \delta b$ − 0.002193	$\overline{\delta a \delta c}$ − 0.003125	$\overline{\delta a \delta \varphi}$ − 0.007259	$\overline{\delta a \delta \psi}$ − 0.03127
$b = 0.280 \pm 0.042$		$\overline{\delta b^2}$ 0.001743	$\overline{\delta b \delta c}$ 0.001584	$\overline{\delta b \delta \varphi}$ 0.004829	$\overline{\delta b \delta \psi}$ 0.01608
c $= 0.204 \pm 0.053$			$\overline{\delta c^2}$ 0.002810	$\overline{\delta c \delta \varphi}$ 0.005371	$\overline{\delta c \delta \psi}$ 0.02623
$\varphi = 2.37 \pm 0.25$ (0.77 ± 0.25 alt)				$\overline{\delta \varphi^2}$ 0.06358	$\overline{\delta \varphi \delta \psi}$ 0.01102
$\psi = 5.09 \pm 0.58$ (4.30 ± 0.58 alt)					$\overline{\delta \psi^2}$ 0.3314

(*) From the plot of a, b, etc. vs α we see that these quantities are not rapidly varying and are almost independent of the value of α.



Graphical representation for $\alpha = 0.95$

s-wave cross-section

$$\sigma_{s\text{-wave}} = \sigma_{tot} \frac{\overset{(k=0)\quad(k=2)}{|\mathbf{a}|^2 = 0.0222\ ,\ 0.0259}}{\left(|\mathbf{a}|^2 + \frac{1}{3}|\mathbf{b}|^2 + \frac{2}{3}|\mathbf{c}|^2 = \frac{1}{4\pi}\right)}$$

		$k = 0$	$k = 2$
$\frac{\sigma_{s\text{-wave}}}{\sigma_{tot}}$	=	0.28 ± 0.16	0.33 ± 0.25

Non-spin-flip cross-section

$$\sigma_{\text{non-spin-flip}} = \sigma_{tot} \frac{\overset{(k=0)\quad(k=2)}{\frac{1}{3}|\mathbf{b}|^2 = 0.0273\ ,\ 0.0261}}{(\mathbf{a}^2 + \frac{1}{3}\mathbf{b}^2 + \frac{2}{3}\mathbf{c}^2)}$$

	$k = 0$	$k = 2$
$\frac{\sigma_{\text{non-spin-flip}}}{\sigma_{tot}}$	0.34 ± 0.08	0.33 ± 0.06

Spin-flip cross-section

$$\sigma_{\text{spin-flip}} = \sigma_{tot} \frac{\overset{(k=0)\quad(k=2)}{\frac{2}{3}|\mathbf{c}|^2 = 0.0299\ ,\ 0.0277}}{(\mathbf{a}^2 + \frac{1}{3}\mathbf{b}^2 + \frac{2}{3}\mathbf{c}^2)}$$

	$k = 0$	$k = 2$
$\frac{\sigma_{\text{spin-flip}}}{\sigma_{tot}}$	0.38 ± 0.12	0.35 ± 0.18

Appendix: errors

$k = 0$

$$\delta\left(\frac{\sigma_s}{\sigma_{tot}}\right) = 4\pi\,\delta(a^2) = 2\times 4\pi\, a\,\delta a = 2\cdot 4\pi(0.149)\,(0.044)$$

$$= (0.082)\times 2 = 0.164$$

$$\delta\left(\frac{\sigma_{NSF}}{\sigma_{tot}}\right) = \frac{4\pi}{3}\,\delta(b)^2 = \frac{8\pi}{3}\,b\,\delta b = \frac{8\pi}{3}(0.286)\,(0.033) = 0.078$$

$$\delta\left(\frac{\sigma_{SF}}{\sigma_{tot}}\right) = \frac{8\pi}{3}\,\delta(c)^2 = \frac{16\pi}{3}\,c\,\delta c = \frac{16\pi}{3}(0.212)\,(0.033) = 0.116$$

$k = 2$

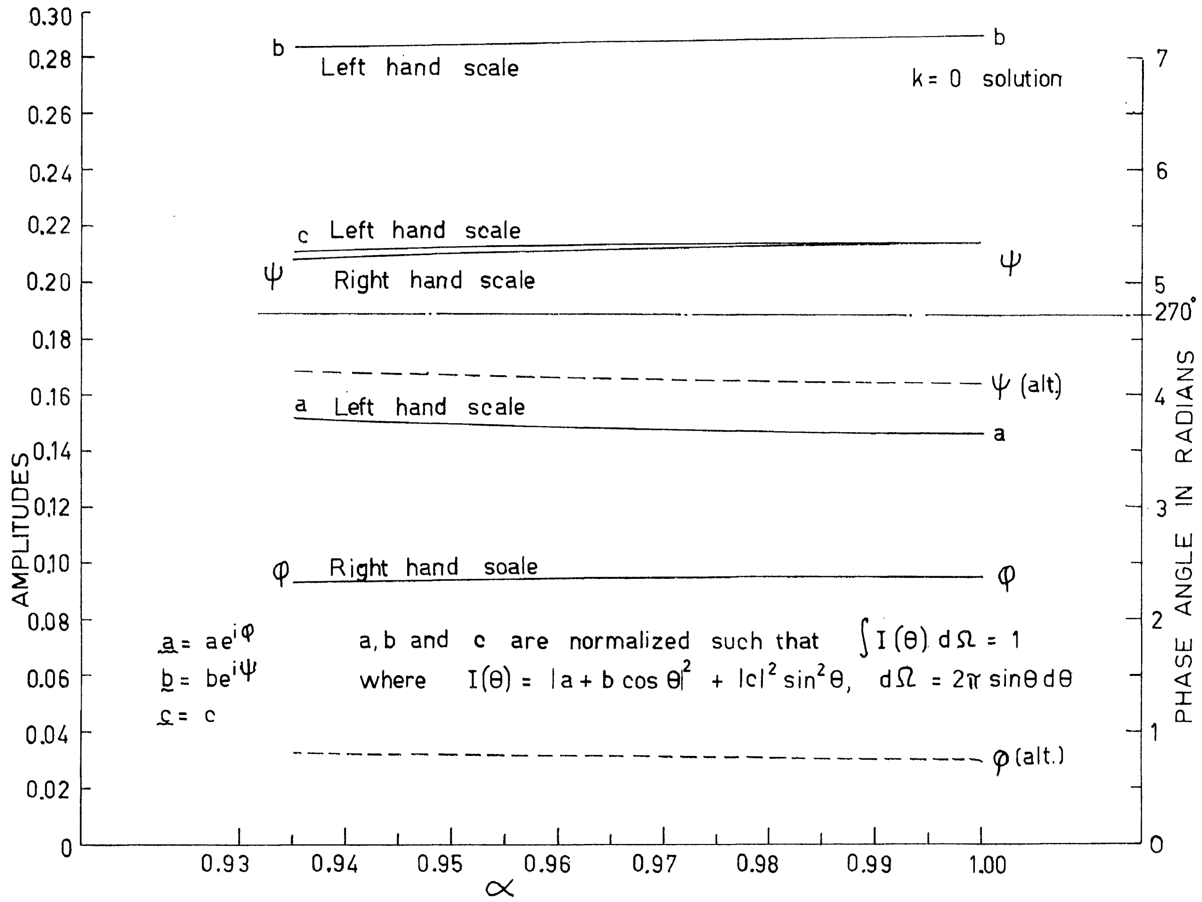
$$\delta\left(\frac{\sigma_s}{\sigma_{tot}}\right) = 8\pi\,(0.161)\,(0.0628) = 0.254$$

$$\delta\left(\frac{\sigma_{NSF}}{\sigma_{tot}}\right) = \frac{8\pi}{3}(0.280)\,(0.0417) = 0.0562$$

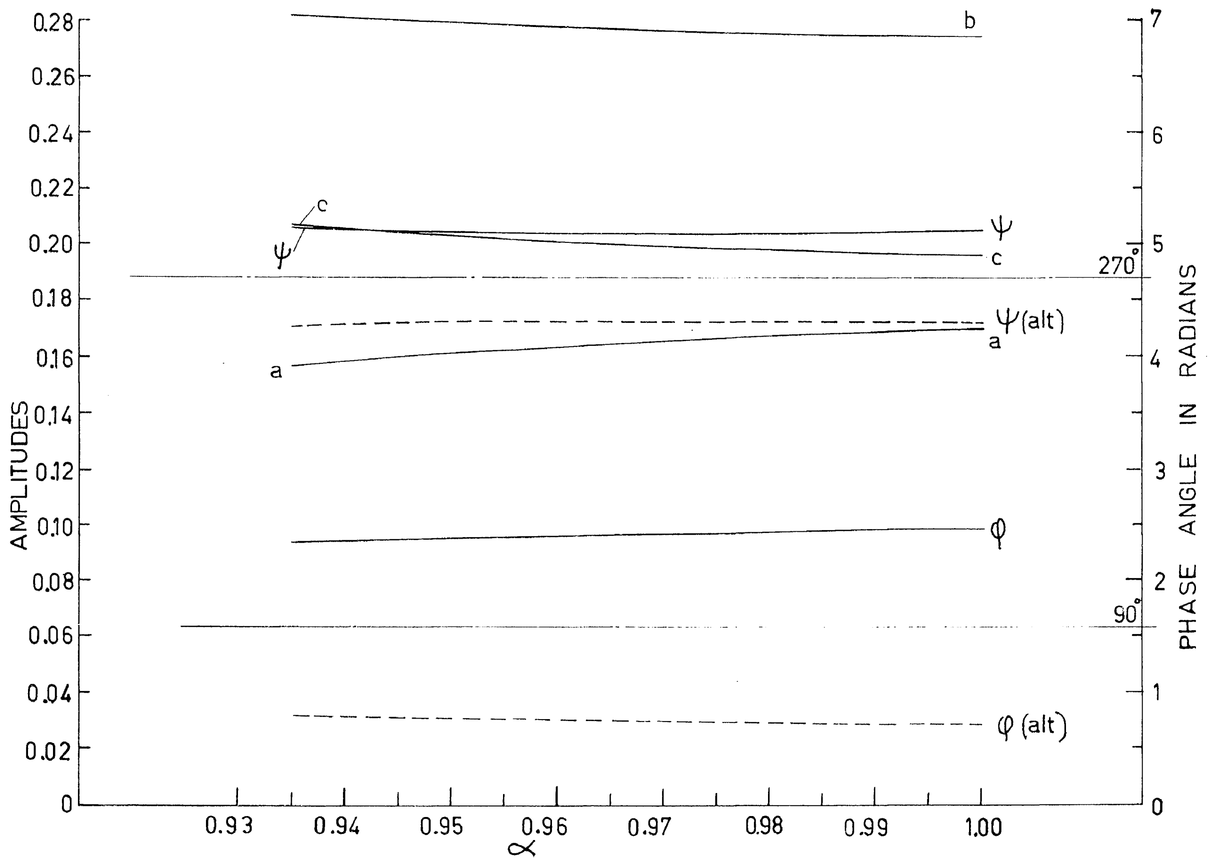
$$\delta\left(\frac{\sigma_{SF}}{\sigma_{tot}}\right) = \frac{16\pi}{3}(0.204)\,(0.0530) = 0.181$$

LIST OF REFERENCES

1. Lee, T. D., Steinberger, J., Feinberg, G., Kabir, P. K. and Yang, C. N., Phys. Rev., *106*, p. 1367, 1957.


k = 0 solution
b Left hand scale b
c Left hand scale
ψ Right hand scale ψ
270°
ψ (alt)
a Left hand scale a
φ Right hand soale φ
a = ae^{iφ}
b = be^{iψ}
c = c
a, b and c are normalized such that ∫ I (θ) dΩ = 1
where I(θ) = |a + b cos θ|^2 + |c|^2 sin^2θ, dΩ = 2π sinθ dθ
φ (alt.)
AMPLITUDES
PHASE ANGLE IN RADIANS
0.30
0.28
0.26
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
0.93 0.94 0.95 0.96 0.97 0.98 0.99 1.00
α
7 6 5 4 3 2 1 0

k=2 solution


b
c
ψ
c
270°
ψ (alt)
a
a
φ
90°
φ (alt)
AMPLITUDES
PHASE ANGLE IN RADIANS
0.28
0.26
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
0.93 0.94 0.95 0.96 0.97 0.98 0.99 1.00
α
7 6 5 4 3 2 1 0