QUADRUPOLE TRANSFER FUNCTION FOR EMITTANCE MEASUREMENT*

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Abstract

Historically the use of the quadrupole moment measurement has been impeded by the requirement for large dynamic range, as well as measurement sensitivity to beam position. We investigate the use of the transfer function technique [1-3] in combination with the sensitivity and 160dB revolution line rejection of the direct diode detection analog front end [4] to open the possibility of an emittance diagnostic that may be implemented without operational complication, quasi-parasitic to the operation of existing tune measurement systems. Such a diagnostic would be particularly useful as an emittance monitor during acceleration ramp development in machines like RHIC and the LHC.

INTRODUCTION

QMMs (Quadrupole mode monitors) have historically been used in two different ways. In linacs and transfer lines, several pick-ups may be used to provide a single shot measurement of emittance and Twiss parameters [5]. Although this kind of measurement is also possible in a ring [6,7], in circular accelerators QMMs have primarily been used to measure injection matching, by detecting a frequency component due to beam envelope oscillation at twice the betatron frequency. Both methods have some caveats. In both cases, the quadrupole signal must be measured in the presence of a much stronger sum signal. This problem may be solved by purpose-built magnetic pick-ups [6,7]. Additionally, the quadrupole signal depends on beam position. For the single-shot measurement, this means that the position must be measured and its contribution subtracted before the emittance can be calculated. The injection matching measurement is not sensitive to a DC offset of the beam, since the mismatch signal is separated by frequency. However, any significant beam position oscillation will generate a signal at twice the betatron frequency that may be mistaken for a beam size oscillation.

The approach we describe here offers the possibility of eliminating the classical problems affecting emittance measurement using QMMs. The key to the approach is the Quadrupole Transfer Function (QTF). The QTF shifts the signal from the revolution frequency to approximately twice the betatron frequency. This removes the DC position dependence, just as in the case of injection mismatch measurement, and opens the possibility of using the sensitivity and dynamic range of the Direct Diode Detection Analog Front End (3D AFE).

PICKUP AND BEAM PARAMETERS

For the purpose of presenting quantitative estimates of signal levels in the LHC, one possible set of pickup and beam parameters has been selected. These parameters are shown in Table 1, and are used in the calculations in the following section. It is assumed that pickups similar to existing pickups can at some point be installed in available space in the region of large beta functions adjacent to the 4 o'clock IP.

parameter	symbol	value
electrode radius [cm]	b	3
electrode length [cm]	L	60
subtended angle [deg]	ϕ_0	70
stripline impedance [ohms]	Z	50
Lorentz factor	γ	7461
bunch charge [q]	N	1×10^{11}
number of bunches	N_b	1
rms bunch length [nsec]	σ	0.25
emittance [mm-mrad]	$\varepsilon_x = \varepsilon_y$	4
Hor beta function [m]	β_x	400
Vert beta function [m]	β_y	200
Hor rms beam size [mm]	σ_x	0.46
Vert rms beam size [mm]	σ_y	0.33

Table 1. Pickup and Beam Parameters

The pickup parameters and beta functions shown here are representative of such an installation. The selected beam parameters represent something close to 'worst case', a single bunch at store, where beam size and quadrupole signal are minimized.

BEAM SIGNALS

Given the pickup and beam parameters in Table 1, one can calculate the response of the various modes of the pickup (sum, difference, quadrupole,…) as the beam position and shape changes [8,9]. Signal powers for the various modes as a function of beam offset are plotted in Figure 1. Also shown for purpose of comparison is the spectral power density (per Hz) of the room temperature thermal noise floor.

The sum mode (ie the sum of the power from striplines on opposite sides of the beam) power is ~+9dBm, or a little less than 10mW. To first order sum mode power is not a function of beam position. In frequency domain this power appears at the revolution line.

Power appears in the difference mode (the difference in power from striplines on opposite sides of the beam) when the beam is not centered. If the there is no coherent dipoles excitation, all of this power appear at the revolution line.

Power appears in the quadrupole mode (the difference in sum mode power from the horizontal and vertical planes) if there is beam offset OR an asymmetric beam ($\sigma_x \neq \sigma_y$). If there is no coherent quadrupole or dipole excitation, all the power in the quadrupole mode will appear at the revolution line.

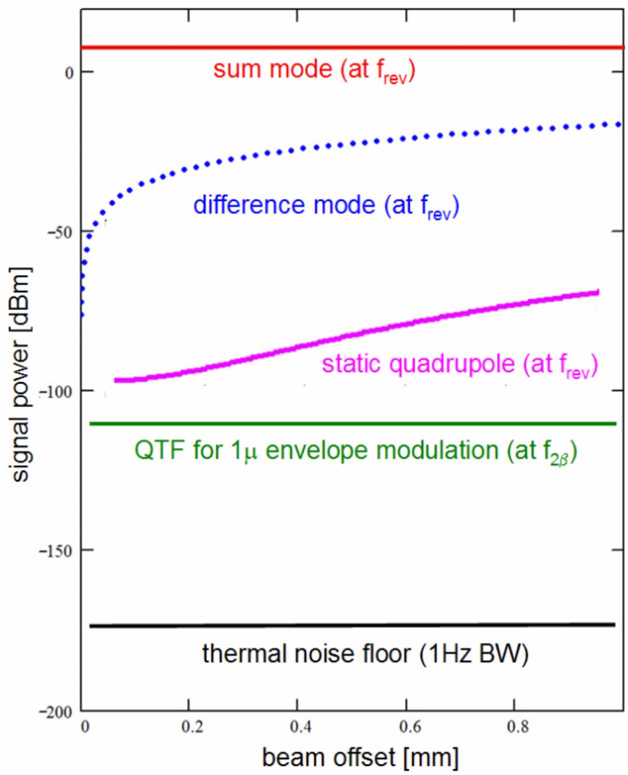

Figure 1. Signal power from the pickup modes as a function of beam position

With the QTF approach, the beam is excited at the quadrupole resonance. Even though a quadrupole kicker may act as a dipole if the beam is offset, the excitation frequency is far for the dipole resonance, so very little coherent dipole oscillation results. Therefore, all signal components except those coming from beam size modulation appear at the revolution line, and can be filtered, giving a measurement of the envelope modulation response which is *separated from interfering sum, difference, and position-dependent quadrupole signals* in the frequency domain.

Kicker power requirements for the 1μ envelope modulation shown in Figure 1 may be estimated by considering the ratio of beam rms radius to kicker radius. This suggests that the kicker field at rms beam radius must be ~x10 that of the BBQ kicker, and the kicker power would then need to be ~x100 that of the BBQ kicker. For the LHC at full energy, this possibly is a formidable requirement. It might be considered to use a resonant kicker in this application.

MEASUREMENT METHOD

When measuring the usual (dipole) transfer function, one excites coherent beam position oscillations using angular kicks from a dipole magnet, and measures the beam position response. The amplitude response

$$F(f) = \frac{\sqrt{\beta_k \cdot \beta_p} \cdot \Delta\theta}{\Delta x}$$

is proportional to the local density of particles with natural oscillation frequencies close to the driving frequency.

In the case of a quadrupole transfer function, one excites fluctuations in the lattice beta function using quadrupole kicks. However, what is measured is not the lattice function directly, but the beam size. The beam size is a function of both the beta function and the beam emittance. This is what opens the possibility to measure emittance with a quadrupole transfer function.

When making a QTF measurement, the observable is actually variations in the square of the beam size. If the beam envelope is being modulated by a quadrupole of integrated strength Δk·l at frequency f, the resulting response can be written

$$\Delta(\sigma^2) = G(f) \cdot \beta_k \cdot \beta_p \cdot \varepsilon \cdot \Delta k \cdot l$$

where G(f) represents the frequency dependence of the quadrupole resonance response, and the beta functions are at the kicker and pickup locations. If one assumes that the resonance response G(f) does not depend on emittance (ie no detuning with amplitude) and that there is no incoherent tune shift (due for instance to space charge), it is not unreasonable to postulate that

F(f) is proportional to G(2f)

which is just another way of saying that the same particles that contribute to the dipole response at f contribute to the quadrupole response at 2f.

Then, by measuring the ratio of the quadrupole transfer function at f and the quadrupole transfer function at 2f, one can obtain a relative measure of emittance.

A POSSIBLE SYSTEM ARCHITECTURE

One possible system block diagram is shown in Figure 2. The outer loop (black) is the conventional Baseband Tune measurement system (BBQ), as employed in RHIC [10] and in development for the LHC [11]. It permits continuous measurement of tune, coupling, and chromaticity. In RHIC, feedback loops have been closed (red) for real-time control of tune and coupling during beam acceleration.

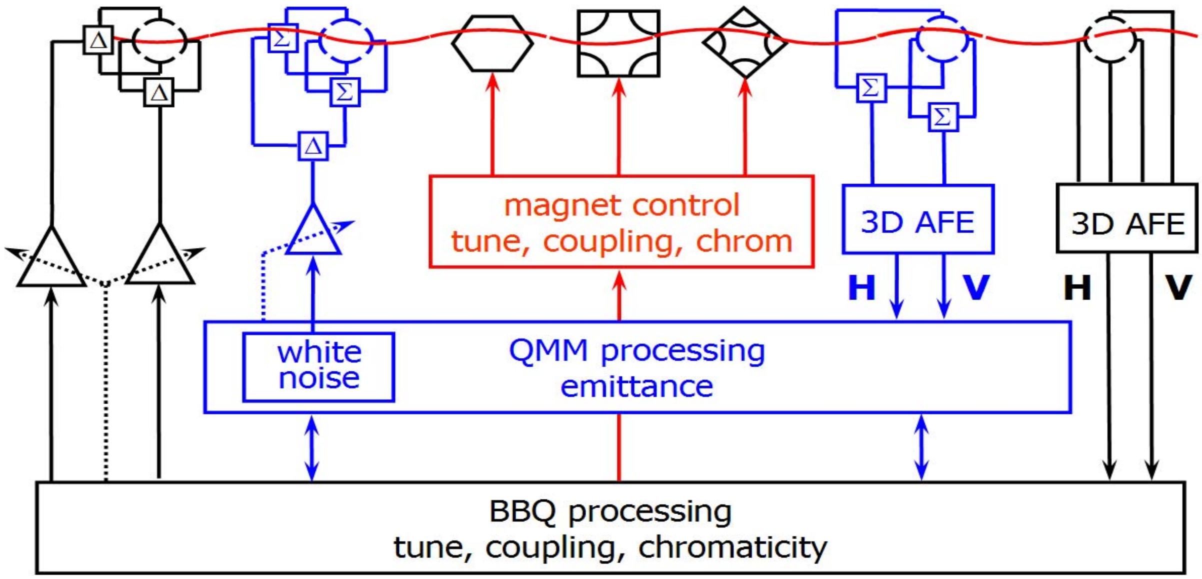

Figure 2. A Possible System Block Diagram

The QMM loop (blue) is shown nestled within the BBQ. In this configuration, it is straightforward to excite the quadrupole resonance with a quadrupole kicker at exactly twice the betatron frequency (the betatron frequency is tracked by the BBQ), to scale the QMM excitation to the BBQ excitation, and to normalize the QMM signal to the BBQ signal, which provides a relative emittance calibration. If the location in frequency domain of the quadrupole resonance relative to the betatron resonance remained constant through the ramp, this would probably be the optimal configuration. However, space charge incoherent tune shift causes this to be untrue, and will be discussed further in a later section.

Space Charge and Noise Excitation

In the LHC, the injection and store incoherent tune shifts due to space charge are estimated to be $\sim 10^{-3}$ and 10^{-5} respectively. It is not yet clear how this shift of the quadrupole mode frequency away from twice the betatron frequency might affect the data quality of the simple approach of exciting the quadrupole resonance at precisely twice the betatron frequency. Machine nonlinearities causing detuning with amplitude may cause similar problems. As an alternative, the quadrupole mode might be excited with white noise, and the 3dB power of the resonance response measured to extract beam size information. Continuous white noise excitation of the betatron resonance during acceleration ramps has been successfully employed in RHIC, for the purpose of observing tunes, coupling, chromaticity and non-linear tune spread. These results suggest that a similar approach may be successful for the quadrupole mode.

CONCLUSION

We have presented a new approach to the application of the quadrupole monitor to emittance measurement. It has the potential to overcome some of the obstacles that have prevented the quadrupole monitor from being more widely used in accelerators. Studies are planned in both RHIC and LHC to validate the method.

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