DISCUSSION

Symon: I think one consideration with regard to the stochastic method would be the effect of a space charge limit near the injector where you have a much larger current than eventually reaches the target.

Kolomenskij: Symon's thoughts are correct, I think, because actually in the injection region there is, as a rule, a large space charge and because of this there are large particle losses. It is for this reason that we must have a sufficiently intensive $P(\omega)$ in the region of injection in order to keep the particles in this region only a small time. In order to take into account the injection problems we must choose their spectrum very carefully, and because the injection problems are very complicated we have only some evaluations in this direction; I think we may discuss this with Symon and with any other people who want to.

O'Neill: Has Kolomenskij estimated the radio-frequency power required to make a voltage of several kilovolts over a wide frequency spectrum like this?

Kolomenskij: I think that by appropriate choice of the voltage spectrum the power in our case may be reduced to about 10 kilowatts.

Barbier: I would like to know the mean acceleration time to get to 30 MeV and the number of accelerating sections, V_1, V_2, V_3.

Kolomenskij: The average time that corresponds to reaching the maximum energy is approximately

$$t_{\text{accel}} \simeq \left(\frac{E_{\max}}{\Delta E}\right)^2 t_{\text{revol}}$$

and here we must take into account the time of revolution. It is the order of magnitude but in various cases there may be different functions $P(\omega)$.

EXPERIMENTS ON STOCHASTIC ACCELERATION

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I. PRINCIPLES OF STOCHASTIC ACCELERATION

We have constructed a cyclic accelerator producing a continuous extracted beam of protons of 4.4 MeV and an internal beam of the order of 1 μA. This accelerator, which is in some respect similar to a synchro-cyclotron, has a diameter of 50 cm. The magnetic field of 14 000 G decreases by 8% from the centre to the radius of extraction. Its single electrode in a shape of a D is fed by a high frequency generator producing 5 kW and 2 000 V_{rms}. The voltage given by this generator varies at random and its Fourier analysis shows a continuous spectrum in the 21-23 Mc/s band. The idea of stochastic acceleration was put forward for the first time by Burshtein, Veksler and Kolomenskij [1]. We announced the practical application of this idea in an earlier paper [2].

To study the motion of the protons, reference should be made to the phase diagram on Fig. 1 (see a paper by the author [3]).

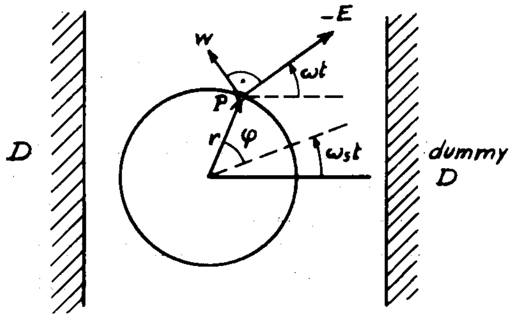

Fig. 1 Phase diagram.

Considering the electric field

$$E = \frac{E_o}{2}(e^{i\omega t} + e^{-i\omega t}) \tag{1}$$

If the position of the proton in the phase diagram is expressed by

$$r = |r|e^{i(\omega_s t + \phi)} \tag{2}$$

the acceleration is produced only by $e^{+i\omega t}$, and the term $e^{-i\omega t}$ represents no more than a perturbation which can be neglected. The accelerating field is

$$E = \frac{E_o}{2}e^{i\omega t}\,. \tag{3}$$

If ω is equal to the synchronous frequency ω_s, point P moves in a straight line with a velocity

$$w = \frac{E_o}{2B} \tag{4}$$

where B represents the magnetic induction.

If ω is not equal to ω_s, point P moves along the arc of a circle, as shown on Fig. 2.

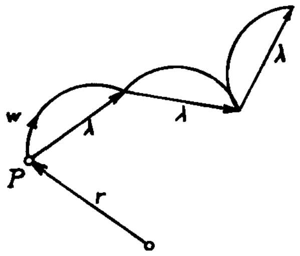

Fig. 2 Random walk.

Let us assume that the high frequency generator produces successive wave trains of duration T, with a random phase jump between each wave train. It can easily be seen that point P will perform Brownian movement. The mean free path is

$$\lambda = \frac{2w}{\omega_s - \omega}\sin\frac{(\omega_s - \omega)T}{2}\,. \tag{5}$$

The motion of the protons is therefore on the whole a sort of diffusion similar to that of a molecule in a gas. Fourier analysis of a wave train of constant amplitude and of duration T actually gives a frequency spectrum of amplitude λ. In the particular case where

$$(\omega_s - \omega)T = 2\pi \tag{6}$$

the arcs of the circles in Fig. 2 become full circles, $\lambda = 0$, and there is no acceleration. The acceleration (or deceleration) is proportional to the amplitude of the frequency spectrum of the generator voltage taken at the synchronous frequency ω_s. In view of the above, the motion of the protons can be expressed as a diffusion equation

$$D\Delta q = \frac{\partial q}{\partial t}\,, \quad \text{or} \quad \operatorname{div}\frac{\lambda}{2}\operatorname{grad}(wq) - \frac{q}{T} = \frac{\partial q}{\partial t} \tag{7}$$

where D is the diffusion constant of the cyclotron $\lambda w/2$ and q is the space charge.

It can be shown that the mean duration of the acceleration is[4)]

$$t = \frac{\Delta\omega B^2 R^4}{5.61\, U^2_{rms}} \tag{8}$$

where $\Delta\omega$ is the width of the spectrum, R the maximum radius of the orbit and U_{rms} the Dee voltage. Calculations yield for the intensity of the beam that can be accelerated

$$i = \text{const}\,\frac{U^2_{rms}}{\Delta\omega R} \tag{9}$$

The beam is in inverse ratio to the radius of the cyclotron.

II. MEASUREMENTS

a. Types of modulation

We have tried four different types of stochastic modulation:

(a) A scintillator excited by an α source illuminates a photomultiplier. The pulses (1 million per sec) are filtered and amplified up to 5 kW. One thus obtains damped wave trains of 22 Mc/s with a band width of ± 1 Mc/s.

(b) A local oscillator with a frequency modulated stochastically by means of a photomultiplier and subsequent amplification.

(c) A saw-tooth frequency modulation sufficiently slow for the protons to be accelerated as in a synchro-cyclotron. A stochastic amplitude modulation is superimposed. The protons are thus accelerated, lost and decelerated at random.

(d) A sinusoidal frequency modulation between 21-23 Mc/s, but too fast (20 000 c/s) for the protons to keep to synchronous conditions. They are accelerated, lost or decelerated according to their instantaneous phase with respect to the accelerating field. Phases are distributed at random so that the acceleration becomes stochastic.

Beams of equal intensities are obtained with the last three methods, while the first method gives a much weaker intensity. We chose the last method, which happens to be the simplest. Fig. 3 shows the beam intensity measured as a function of the modulation frequency.

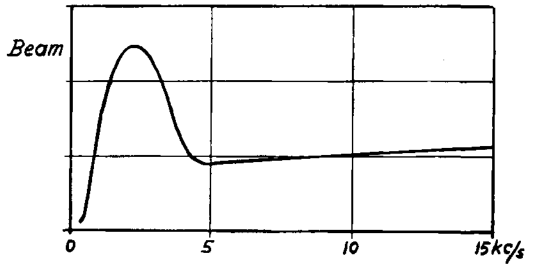

Fig. 3 Beam vs. repetition rate.

b. The acceleration time

The beam is allowed to come out the cyclotron without any special extraction device being used. The protons are counted with a scintillator and a photomultiplier situated a long way away from the cyclotron. From the estimate of solid angles, one can deduce that there are very few losses during extraction. The beam can therefore go through the unstable point $n = 0.2$ during stochastic acceleration.

To measure the duration of acceleration, the ion source is pulsed at the rate of 100 pulses per second. Pulses from the photomultiplier are transmitted to an oscilloscope. Fig. 4 shows the slow decay of the beam after each pulse from the ion source. The average duration is 2 to 3 msec corresponding to formula (8). Fig. 4 also shows the behaviour of the beam under synchronous acceleration. It will be noted that the duration of acceleration is negligible compared to the duration of the pulse from the source.

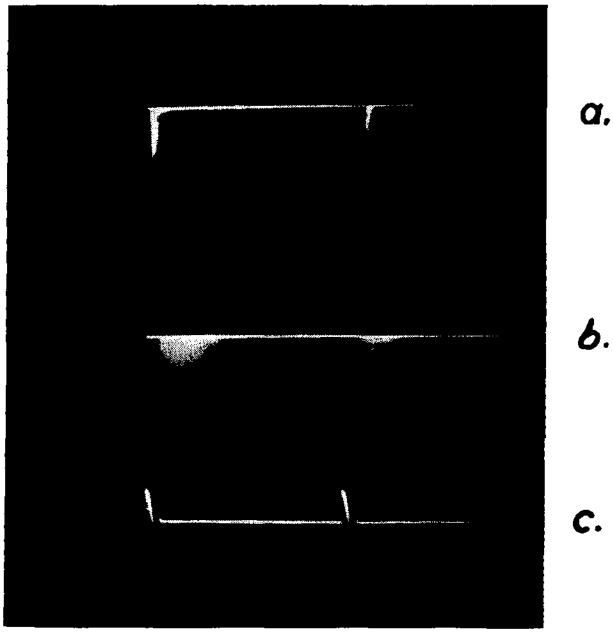

Fig. 4 Oscillogram showing (a) the acceleration time for a synchronous regime, (b) the acceleration time for a stochastic regime and (c) an ion source current with 100 c/s.

c. The beam as a function of radius

The intensity of the beam is measured with a thermocouple moved along the radius. Fig. 5 shows the stochastic beam as a function of radius. In accordance with formula (9), the intensity should vary as $1/R$, but measurements give a somewhat sharper variation. This is probably caused by losses due to gas scattering.

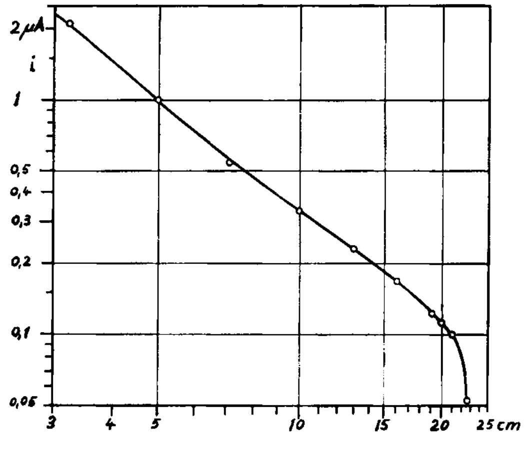

Fig. 5 Beam vs. radius.

d. The beam as a function of the Dee voltage

It will be noted on Fig. 6 that the beam is proportional to the square of the Dee voltage, as one should expect from formula (9).

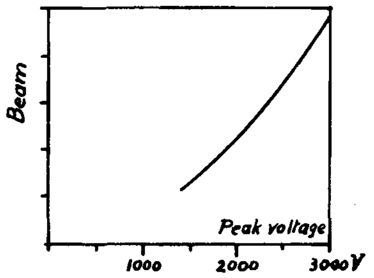

Fig. 6 Beam vs. Dee voltage.

e. Gas scattering loss

Fig. 7 shows the decay of the beam when a gas is introduced in the vacuum chamber. The radiation length which is to be expected from the theory of this phenomenon (which, for the sake of brevity, will not be explained in the present paper) can be deduced from our measurements. For instance, for air we find 52 g/cm^2, whereas the theoretical value should be 36.5 g/cm^2.

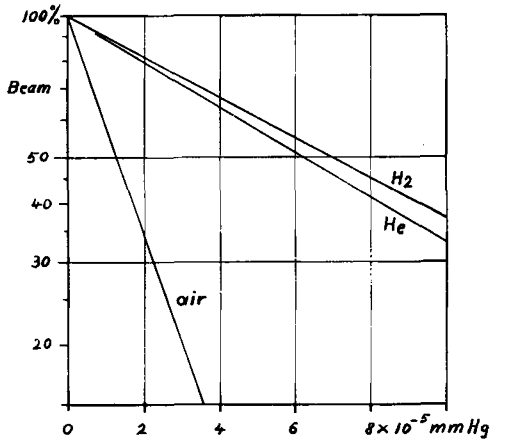

Fig. 7 Beam vs. gas pressure.

f. Energy spread

We made a measurement of the time of flight over 470 cm. The first scintillator has a thickness of 12 mg/cm^2, which slows down protons by 1.4 MeV.

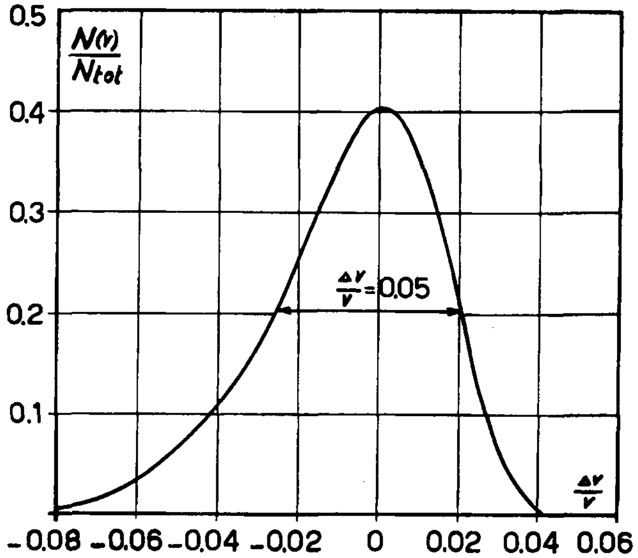

Fig. 8 Velocity spread.

Fig. 8 shows the width of the coincidence curve of the two photomultipliers. The energy spread is $\pm 5\%$. We obtain exactly the same curve in the stochastic as in the synchronous case. Stochastic acceleration therefore does not cause any increase in energy spread.

III. APPLICATIONS OF STOCHASTIC ACCELERATION

From the point of view of intensity, the stochastic cyclotron lies half-way between the fixed frequency cyclotron and the synchro-cyclotron. It is easier to construct than the former and, compared to the latter, it has the advantage of producing a continuous beam.

Above 50 MeV, there are major technical difficulties connected with the radio-frequency. However, other methods are available to improve the performance of a large accelerator[4] :

a) Increase in intensity

In a large synchro-cyclotron, the duration of the ion injection is very short compared to the acceleration cycle and, as a result, the intensity is strictly limited. With stochastic acceleration up to 20 MeV, the ions are continuously accumulated on intermediate orbits. At every accelerating cycle, a large part of these accumulated ions can be recovered and the total intensity can therefore be increased by about a factor 10.

b) Accumulation of polarized protons

In view of the very low intensity of a source of polarized protons, stochastic stacking would be sufficient to increase the duration of the injection and thus avoid the loss of atoms between pulses.

c) Debunching of an extracted beam

A pulsed beam is often inconvenient and difficult to use in the electronic detection circuits set up for experiments. A stochastic extraction would considerably broaden the pulses. All that would be required would be a system of electrodes, near the extreme orbit, fed by a noise generator. The ions would come out almost continuously if conventional accelerating cycles were stopped before normal extraction begins.

d) Passage through transition energy

There are accelerators in which the synchronous frequency is independent of energy for certain energy values. In these regions, the amplitude of phase oscillations becomes very large, which may cause a serious loss of particles. Since in stochastic acceleration the phase of the particles and that of the accelerating field are not related, stochastic acceleration could be used to pass through the transition energy. This method has been proposed by M. Barbier, CERN. The stochastic cyclotron affords one example of this since its transition energy is zero, and particles are accelerated from zero.

LIST OF REFERENCES

1. Burshtein, E. L., Veksler, V. I. and Kolomenskij, A. A. A stochastic method of particle acceleration. AERE (*) Lib/Trans 623. October 1955. (Original Russian appeared *In*: Some problems in the theory of cyclical accelerators. Moscow, USSR Academy of Sciences, 1955. p. 3-6).
2. Keller, R., Dick, L. et Fidecaro, M. Accélération stochastique dans un cyclotron de 5 MeV. C.R. Acad. Sci., Paris, *248*, p. 3154-6, 1959.
3. Keller, R. Le mouvement anharmonique des ions dans un cyclotron et le diagramme de phase. Nuclear Instrum., *4*, p. 181-8, 1959.
4. Keller, R. and Schmitter, K. H. Beam storage with stochastic acceleration and improvement of a synchro-cyclotron beam. CERN (*) 58-13, December 1958.

DISCUSSION

SYMON : I do not remember whether you gave a comparison between the current that you got with a stochastic method compared with a synchro-cyclotron acceleration?

KELLER : In the first slide I showed you the pulse which comes out from the synchro-cyclotron and in the stochastic acceleration the integral of the pulse was about 2 times less than the integral of the pulse in the synchro-cyclotron acceleration. That means we did not lose very much; the beam is limited in the centre by space charge effects but not at a bigger radius.

KHOE KONG TAT : You mentioned that your calculations also can be applied at the centre of the cyclotron but your equation did not take into account the fact of the ion source and I think the motion at the centre of the cyclotron is more complicated than you mention here. In your equation for the electrical field you do not take the ion source into account. The ion source will distort the electrical field. This equation is not applicable for the centre of the cyclotron because due to the ion source the electrical field will be distorted.

KELLER : Of course, the electrical field is disturbed by the ion source, but not very much. The ion source is in such a position with respect to the electrodes that the field is nearly constant in the middle.

SANDS : In line with the question of Symon, you must operate the synchro-cyclotron in a pulsed regime and I assume you can operate the stochastic acceleration in a continuous regime. Do you have a figure for the average intensity operating on a continuous regime?

KELLER : Except for the acceleration time measurement the ion source works continuously. In the case of a d.c. voltage applied on the ion source, we measured the beam intensity versus modulation frequency. Below 5000 c/s the regime is synchronous and above it is stochastic. In the second case the beam is nearly independent of the frequency and has half the value of the maximum in the first case. If we turn the rotating condenser of an SC much faster than is usual we just have a stochastic regime.

(*) See note on reports, p. 696.

Lawson : I should like to ask Keller about the comparison of the currents in the stochastic and synchro-cyclotron regimes. Is the system running continuously in the stochastic regime? On the slide the output in the stochastic regime appeared to be pulsed. I would like also to know what the ratio of the voltage is in the two cases.

Keller : The ion source was in a constant discharge regime in both cases.

Bakker : I still think there is one piece of information which did not come out of Keller's talk and I think it is something in which everybody is interested : that is, the factor 8 in increase in intensity that Keller expects to achieve in our 600 MeV cyclotron by introducing stochastic acceleration, and which is based on his experiments on the stochastic method in the small cyclotron which he constructed; I think this should be mentioned. This is so encouraging that we in CERN have now decided to try shortly the method which Keller described in our 600 MeV cyclotron.

Keller : Adding to your remark, in the small cyclotron which we tried out we did not try to increase the intensity. We tried out the stochastic method. But in the big synchro-cyclotron, because the space-charge density increases when the diffusion constant decreases at a larger radius, we can stack very high beam space charge and inject this space charge in the main frequency programme and in this case we will gain a factor of about 10.

Goodell : I think from a practical point of view this scheme offers quite a bit of interest. In particular, it appears that you can construct a small dee for a large synchro-cyclotron to use stochastic acceleration as injection. Hopefully you can get perhaps this factor of 10, but the nice thing about it is that if it does not work, you merely take it out in half a day and you are right back where you started, which you cannot do if you try to modify a synchro-cyclotron for a spiral ridge. Eventually, of course, synchro-cyclotrons will have this problem of how do they keep up with these ever bigger accelerators. When our machine was built, it was the most energetic in the world; now it is just a little baby. The question is : how do you keep the physicists interested in using the machine, and if you can increase the intensity at not much risk to ruining the machine, I think is a very interesting piece of information.

Vorob'ev : It might be perhaps of interest to discuss all kinds of measures that can be taken in order to increase the intensity of the beams in the present accelerators. It might be of interest to say a few words about the capturing of the particles in the betatron regime. They have used all kinds of improvements of the capture in the betatron regime (strong and weak focusing electrostatic and magnetic lenses), capturing at the front and back edges of the electron pulses at injection, and so on.

If the machine is not operated in the optimum regime, the use of these measures can increase the intensity of the beam by several orders of magnitude. If the machine is in optimum regime, all these measures do not help.

MULTIPLE FREQUENCY ACCELERATION

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A type of accelerator is investigated which consists of a number of radio-frequency voltages, each of a different and constant frequency, in conjunction with a constant magnetic guide field.

For purposes of analysis, we consider the accelerating system to be composed of an infinite number of radio-frequency accelerating voltages, with the separation between adjacent frequencies being ω_o. We assume that only one harmonic order of each frequency is effective. The component of the accelerating voltage of interest may then be written

$$V = V_N \sum_{n=-\infty}^{\infty} e^{i[(\omega_M + n\omega_o)t + \phi_1 n + \phi_2 n^2 + r]} \cdot e^{ik(\theta - \theta_1 n)}$$

$$V = V_N e^{i\omega_M t} \sum_{n=-\infty}^{\infty} e^{i[n\omega_o t + (\phi_1 + \theta_1)n + \phi_2 n^2 + \gamma - k\theta]} .$$

V_N is the magnitude of the voltage of each frequency, which are assumed to be equal.

ϕ_1 and ϕ_2 determine the time phase relation of the frequencies, and θ_1 represents the relation in azimuth of the radio-frequency voltages. K is the harmonic order. This accelerating voltage is equivalent to an infinite number of radio-frequency accelerating voltages, each modulated at a linear rate, which may be written in the form