NPA/Int. 68-19
25.7.68

INTENSITY DISTRIBUTION OF FOCUSSED MULTIMODE LASER BEAMS

During the course of preliminary work [1,2] on the application of focussed laser beams to studies of particle beam separators and high voltage impulse generators carried out in NPA Division in 1965 photographs were obtained of the breakdown plasma created in air at the focus of the lens. These photographs revealed the presence of several closely spaced but distinct regions of plasma along the optic axis (Figure 1).

These observations were subsequently confirmed by several workers [3,4] but no satisfactory explanation of the phenomenon of multiple collinear laser produced sparks has been proposed [5]. However, it is clear that, in common with other forms of ionisation and breakdown in gases, laser-induced breakdown requires the production of an initiatory electron and its subsequent amplification by electron-atom collisions in an electric field. In the case of laser beam-induced discharges the laser radiation field itself appears to provide the initial electron in a very short ($\sim 10^{-9}$ sec) time, but the precise mechanism of liberation is not well understood [6,7].

The amplification occurs as a result of energy transfer in collisions between electrons and atoms in which the electrons draw energy from the optical frequency electromagnetic field created by the laser. The photographs of separate regions of ionisation along the optic axis of the laser beam thus suggests that the spatial variation of the intensity of the electromagnetic field has regions of maximum and minimum lying along the beam axis and it is clearly of interest to examine in detail the intensity distribution in the neighbourhood of the Gaussian focal spot of the lens.

Such a study is relevant to the question of assessing the magnitude, not only of the electric field but also the electron diffusion length [8] Λ. This is a crucial parameter in understanding the mechanism by which the electron density increases and it is a measure of the average distance travelled by an electron from its point of liberation until it reaches a position where the electric field is too low for it to undergo further ionising collisions.

There has been considerable uncertainty and a great deal of speculation concerning the precise form and magnitude of Λ which appears in the criterion for the onset of laser induced breakdown, viz,

$$\nu_i \gg D_- / \Lambda_-^2$$

where ν_i is the electron ionisation rate and D_- is the electron diffusion coefficient.

For these reasons we have investigated theoretically the detailed nature of the focal region and the purpose of this paper is to present the results of an analysis which was carried out in order to calculate the three-dimensional intensity distribution in the focal region in the form of contours of constant intensity - isophotes.

Numerical Solution of the Kirchoff-Fresnel-Huygens Equation

The starting point in our analysis is the well known Kirchoff treatment of the Fresnel-Huygens development of optical diffraction theory. The intensity distribution in the region of the Gaussian image point of a perfect lens is given by the Fresnel diffraction pattern of the converging spherical wavefront by the lens aperture. However, if the lens is not perfect then the wavefront becomes distorted from the perfect spherical shape i.e. it undergoes aberrations. This is illustrated in Figure 2 which shows the exit pupil of a converging lens and P(r,z) the geometrical image point of a wavefront incident on the lens at an arbitrary angle.

If the lens were perfect the emergent wavefront would be a perfectly spherical wave W centred at P. However, in the presence of lens aberration the wavefront becomes distorted to some arbitrary shape W'. The deformation of the wavefront at the exit pupil can be described in terms of an aberration function Φ defined as the number wavelengths displacement between the perfect wavefront W and the distorted wavefront W' at any point on the exit pupil surface. For a given aberration function the wave amplitude in the image space in the converging wave is represented, in accordance with the Huygens principle as a double integral over the surface of the exit pupil [9].

Thus the amplitude at any point P (u, v) in image space in the focal region is given by

$$U(P) = \frac{-i}{\lambda} \frac{a^2}{R^2} e^{i\left(\frac{R}{a}\right)^2 u} \int_0^1 \int_0^{2\pi} A(\rho) \exp\left[i\left\{k\Phi(\rho) - v\rho\cos\theta - \frac{1}{2} u \rho^2\right\}\right] \rho \, d\rho d\theta$$

where $A(\rho)$ is the radial variation in amplitude across the incident wavefront at the aperture of radius $\rho = a$, and where u and v are the so-called optical coordinates

$$u = \frac{2\pi}{\lambda} \left(\frac{a}{R}\right)^2 z \qquad v = \frac{2\pi}{\lambda} \left(\frac{a}{R}\right),$$

and $k = 2\pi/\lambda$ is the wavenumber, R is the focal length of the lens, i.e. the distance from the lens to the Gaussian image point. The intensity distribution is given by the relation

$$I(P) = |U(P)|^2 = \left(\frac{a^2}{R^2}\right)^2 \left| \int_0^1 \int_0^{2\pi} A(\rho) \exp\left[i\left\{k\Phi(\rho) - v\rho\cos\theta - \frac{1}{2} u\rho^2\right\}\right] \rho d\rho d\theta \right|^2$$

In the case of a focussed laser beam the radius a is of course the beam radius at the lens.

In the absence of aberrations the intensity will be a maximum at the Gaussian image point. However in the presence of aberrations this will not be the case and the displaced point of maximum intensity is then called the diffraction focus [9].

For lenses with small aberrations (less than one wavelength) the integral has been solved analytically by Zernike and Nijboer [10] by expanding the integrand as a power series and neglecting all but the first few terms in the expansion. Their method of analysis is not suitable for computing the intensity when the aberrations are not small compared to a wavelength since the integrand oscillates very rapidly.

In the present investigation the integral has been evaluated numerically firstly for the case of a single mode-laser ruby laser in which the intensity distribution $A(\rho)$ has been assumed, for simplicity, to vary parabolically across the aperture. The beam is considered to be focussed by a relatively short focal length ($\lesssim$ 10 cm) simple lens when there will be relatively large amounts of primary spherical aberration present, amounting to several wavelengths. (We note that for a very good telescope or microscope objective lens the aberration is of the order of $\lambda/2$).

For a simple lens of focal length f the aberration function (9) is given by

$$\Phi(\rho) = -\frac{1}{4}\rho^4 \left[\frac{n^2}{8 f^3 (n-1)^2} - \frac{n}{2(n+2)} \frac{K^2}{f^3} + \frac{2(n+1)^2}{n(n+2)} \frac{K^2}{f^3} \right]$$

where $K = -\frac{1}{2f}$ for the case of plane waves. The coefficient n is the refractive index of the lens material.

A computer program for these equations was prepared and, as a check, was first used to reproduce the results of Linfoot and Wolf (11) and of Zemike and Nijboer (10). Excellent agreement was obtained for these cases of plane waves and zero and very small lens aberration between the numerical solution and the analytical solution. The more complicated case of large aberration wavefront was then examined with the additional complication of the parabolic beam profile included in the numerical analysis. The results obtained are shown in Figure 3 which reveals distinct regions of intense electric field along the optic axis. In these regions, electrons, once liberated can increase rapidly as a result of ionising collisions to form the distinct regions of highly ionised gas.

The breakdown threshold intensity value of the laser radiation field will be reached firstly at the diffraction focus. At subsequent times this value will be reached at the other points of maximum intensity, which, for the particular case of a short focal length lens (f = 10 cm) and a single mode laser, lie successively nearer to the lens. In this way successive distinct regions of plasma are created one after the other and they will give the appearance of a plasma moving towards the lens. The number of such regions will be determined by the extent to which the beam intensity exceeds the threshold value. The time between the appearance of successive plasmas is governed by their distance apart. The apparent propagation velocity V towards the lens is governed by the rise time of the pulse of laser radiation and the distance d between the maxima in the intensity distribution. It can readily be shown that this apparent propagation velocity V_a is given by

$$V_a = \frac{2\,N\,d}{\tau} \quad \frac{1}{1 - \ell}$$

where N is the ratio of the laser peak intensity to the breakdown threshold intensity, τ is the duration of the laser flash and ℓ is the ratio of the intensities at two successive maxima.

For the particular case of a 10 cm lens the maxima are separated by about 2 mm and with a typical Q-switched laser pulse of 30 nanoseconds duration and peak intensity of twice the breakdown threshold the apparent propagation velocity towards the lens is 5×10^{7} cm sec^{-1}. This is in very good agreement with measured values.

Extension to Multi-Mode Laser Operation

The above numerical analysis applied specifically to a laser operating in a single axial mode and while it is now possible to mode lock lasers most studies of laser induced breakdown have been carried out with multi-mode lasers where the laser cavity simultaneously sustains oscillations in many axial modes. Each transverse mode is a plane wave, either a standing or travelling wave, propagating in the cavity at small angles to the axis of the resonator. Thus the output of a multi-mode laser consists of a number of plane waves with slightly different directions of propagation.

We now assume that these plane waves are coherent in themselves but incoherent with each other. In this way we can extend the above analysis to compute the intensity distribution in the focal region simply by adding the contributions to the intensity distribution at a given point due to all the separate modes. Thus if the i^{th} axial mode makes a small angle α_i with the axis of the laser resonator it can easily be shown that the intensity distribution at a point (u,v) near the Gaussian image point is given by

$$I(P) = \sum_{i=1}^{M} \left\{ f_i(\alpha_i) \left| \int_0^1\!\!\int_0^{2\pi} \rho \sqrt{1-\rho^2}\, \exp\left[k\,\Phi - (v - u\alpha_i)\rho\cos\theta - \frac{1}{2} (u + v\alpha_i)\rho^2 \right] d\rho\, d\theta)^2 \right|\right.$$

PS/6694

where $f_i\ (\alpha_i)$ is a factor which is introduced to account for the fact that the power in each mode of propagation may not be the same. If P_i is the power in the i^{th} mode then

$$f_i\ (\alpha_i) = P_i / P_o$$

where P_o is the power in the fundamental mode.

There is a dearth of reliable data on mode structure of Q-switched lasers and on how the energy is distributed between the various modes. Consequently it is not possible to assign a precise value to $f_i\ (\alpha_i)$. However, a simplified theory of mode structure **in** passive Q-switched systems (13) indicates that the ratio P_1/P_2 of the powers in two modes, 1 and 2, is given by

$$\frac{P_1}{P_2} = \left(\frac{1 - a_1}{1 - a_2}\right)^m$$

where a_1 and a_2 are the power losses per transit for the two modes and m is the number of transits the light makes to build up from noise level to its peak value. Since m is normally large (e.g. 1500 transits (13)) in passive Q-switched lasers, then a small difference $(a_1 - a_2)$ will produce a very large difference in the relative powers between modes.

In extending the numerical analysis to multi mode operation we have initially assumed that the relative power in the modes is of a Gaussian nature. This is in reasonably good agreement with the experimental results of many observers. However, the numerical computation would take a considerable time, even on the CERN CDC 6600 and it was not considered feasible to perform the calculation for more than 5 axial modes whereas in general there may be as many as 30 or more modes of propagation present.

On this basis the computed intensity distribution near the focus of a 10 cm lens when 5 modes are present and when the intensity distribution across each mode is parabolic over each mode and each wavefront, for a laser whose beam divergence is 1 milliradian has been obtained and is shown in Figure 4 .

The distribution is far more complicated than for the single mode case and, in addition to the maxima lying along the axis between the Gaussian focus and the lens, there is evidence of the existence of maxima and minima lying on the axis to the right of the diffraction focus. Consequently an apparently moving plasma should appear on either side of the Gaussian focus. This is in fact found to be the case experimentally with longer focal length lenses than that considered here and very much more powerful lasers. Nevertheless these preliminary results for short focal length lenses and only 5 modes suggest a feasible explanation of the distinct regions of plasma and apparently moving discharges extending over several metres along the optic axis observed with gigawatt lasers. The analysis must be extended to examine longer regions of the optic axis than hitherto studied.

General Conclusions

A feasible explanation of the appearance of multiple collinear laser produced sparks in gases has been found in terms of the appearance of regions of intensity maxima and minima along the optic axis of the laser beam. These regions are produced by interference of the focussed electromagnetic wave of the beam.

Lens aberration plays a very important part in governing the intensity distribution at the lens foci. This is further complicated by multi-mode operation.

The analysis provides evidence in support of the view that a reasonably good approximation to the diffusion length Λ is that corresponding to a cylinder, i.e. the assumption that electrons diffuse out of a "cylindrical" focal volume is a better approximation than from a spherical volume.

Although each region of plasma is an absorbing medium along the optic axis and thus in the path of the laser beam, these regions do not prevent the development of further more distant plasmas. The reason for this is probably Fresnel diffraction of the beam around the opaque plasmas.

L.R. Evans and C. Grey Morgan

Footnote

This report is an abridged version of a lecture given by the authors at N.P.A. Division on Monday 15th July 1968.

References

1) J. Bleeker and C. Grey Morgan - NPA/Int. 65-30 (1965).

2) J. Bleeker and C. Grey Morgan - NPA/Int. 65-31 (1965).

3) M. Young, M. Hercher and Chung-Yiu Wu - J. Appl. Phys. 37, 4938 (1966).

4) N.G. Basov et al. - Soviet Physics, Doklady 12, 248 (1967).

5) L.R. Evans and C. Grey Morgan - Nature (Land) in the press (1968).

6) H.B. Bebb and A. Gold - Phys. Rev. 143, (1966).

7) P.M. Davidson - Proc. Roy. Soc. A, 191, 542 (1947)

8) C. Grey Morgan - "Fundamentals of Electric Discharges in Gases" Part II Vol. I, Handbook of Vacuum Physics, Edit. Beck. Pergamon Press. Oxford, (1965).

C. Grey Morgan - NPA/Int. 67-7 (1967).

9) M. Born and E. Wolf - "Principles of Optics", Pergamon Press, Oxford, (1959).

10) F. Zernike, and B.R.A. Nijboer - Contribution in "La Théorie des Images Optiques, (Paris, Revue d'Optique (1949)),232, (1949).

11) E.H. Linfoot and E. Wolf - Proc. Phys. Soc. B 69, 823 (1956).

12) A.J. Alcock et al. - Phys. Rev. Letters 20, 1005, (1968)

13) W.R. Sooty - Applied Phys. Letters 7, 36, (1965).

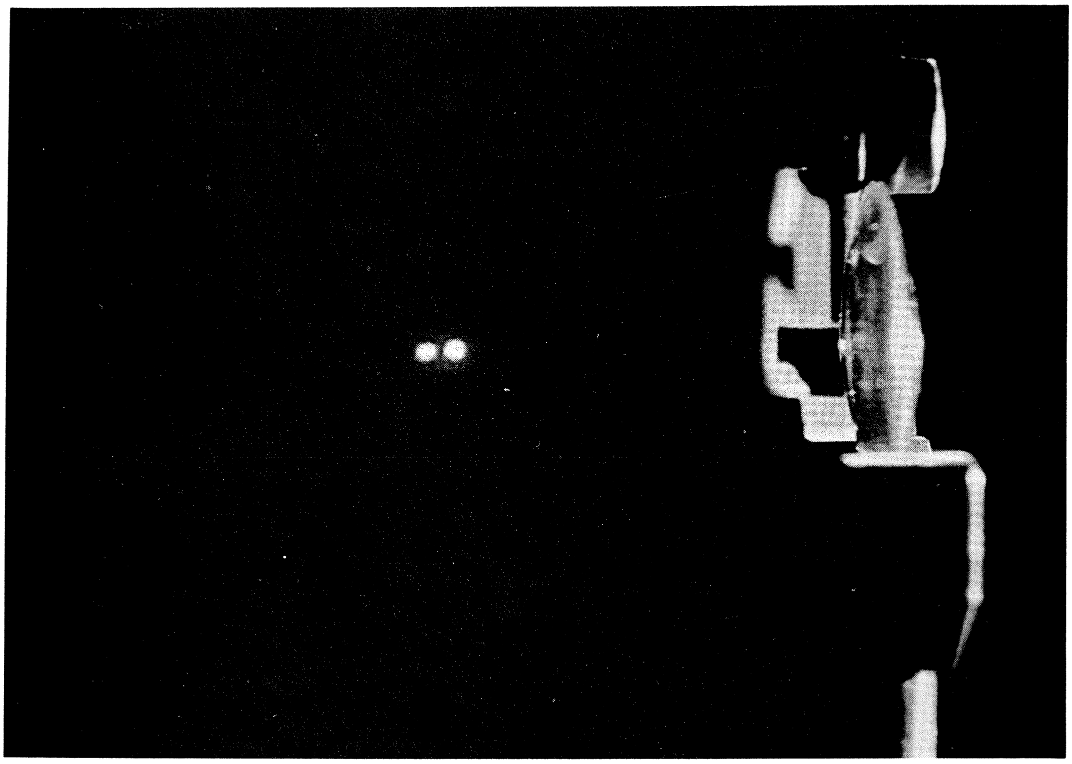
Legends

Figure 1 Multiple collinear breakdown in air at atmospheric pressure.

Figure 2 Distortion of a wavefront. Reference system and notations.

Figure 3 Intensity distribution in the neighbourhood of the Gaussian focus of a 10 cm lens for a single mode laser. The distribution is symmetrical about the optic axis.

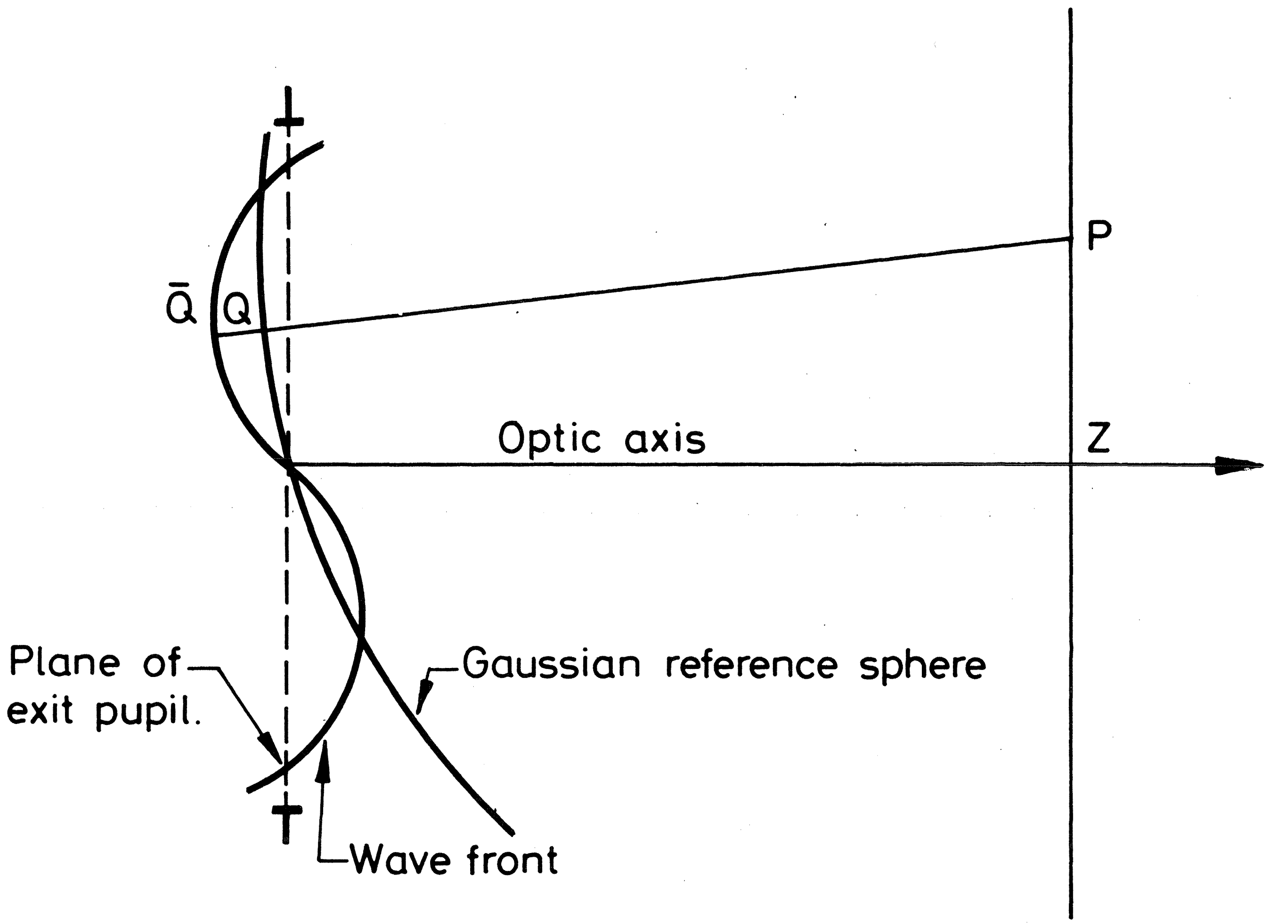
Figure 4 Intensity distribution in the neighbourhood of the Gaussian focus of a 10 cm lens for a multi-mode laser. The distribution is symmetrical about the optic axis.



Optical frequency breakdown in air at atmospheric pressure

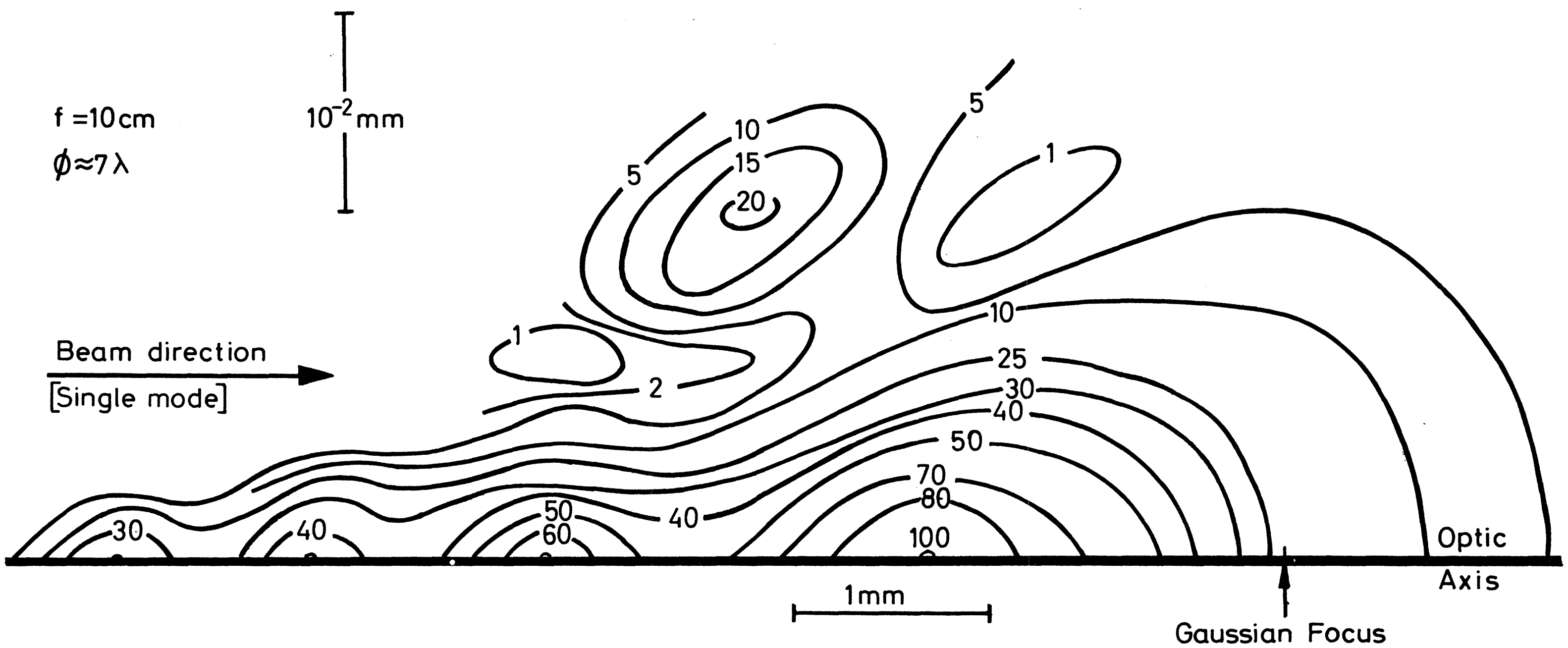
— Fig. 1 —

— Fig. 2 —



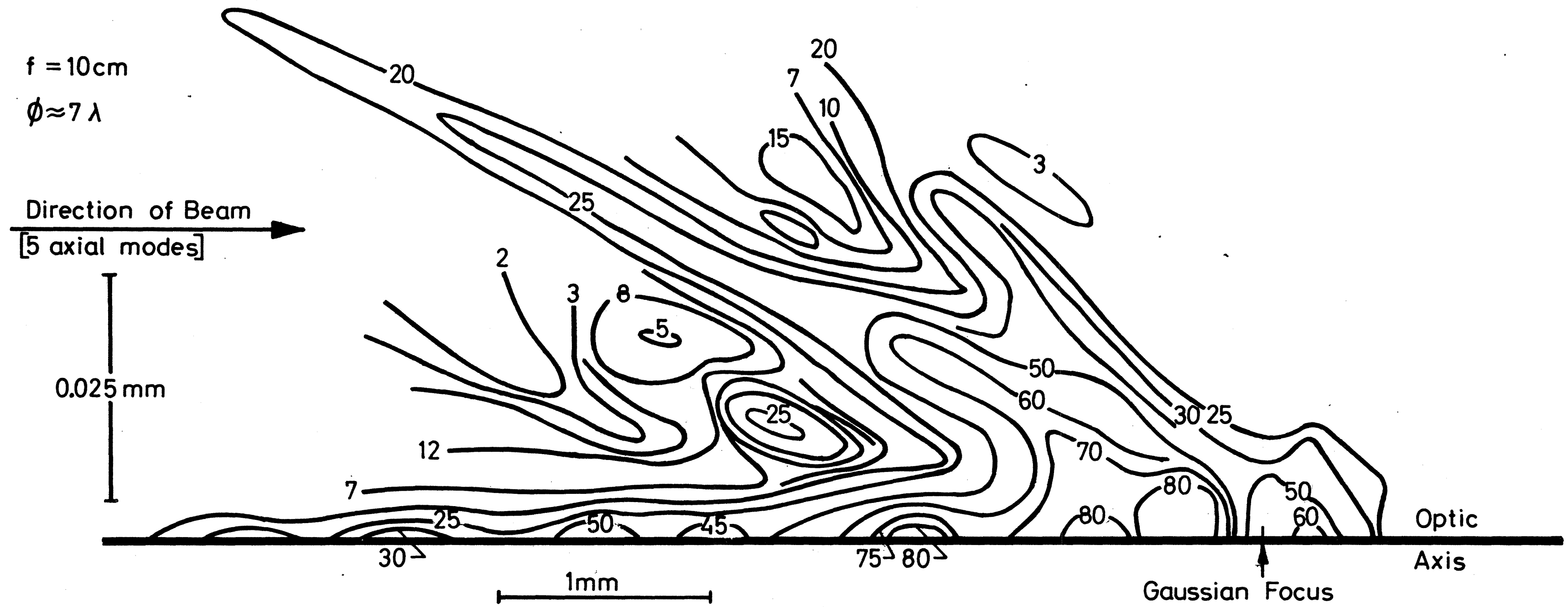
P
Q̄
Q
Optic axis
Z
Plane of exit pupil.
Gaussian reference sphere
Wave front

— Fig. 3 —



f =10cm
$\phi \approx 7\lambda$
10^{-2} mm
Beam direction
[Single mode]
1
2
5
10
15
20
5
1
10
25
30
40
50
70
80
100
30
40
50
60
40
Optic
Axis
1mm
Gaussian Focus

—Fig. 4—



f = 10cm
$\phi \approx 7\lambda$
Direction of Beam
[5 axial modes]
0.025 mm
1mm
Optic
Axis
Gaussian Focus
20
25
2
3
8
5
12
7
25
30
50
45
75
80
7
20
10
15
3
50
60
70
30
25
80
80
50
60