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# Gauged Supergravity Vacua in String Theory

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**Abstract**

We present a first instance of an exact supersymmetric string vacuum in curved space-time. It corresponds to the Freedman-Gibbons electrovac solution to one version of N=4 supergravity, with N=2 space-time supersymmetry. The conformal theory includes a Wess-Zumino-Witten model on the SU(1,1) group manifold.

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The stable supersymmetric solutions of String Theory that have been discussed in the literature so far always include Minkowski space in some number of dimensions. Accordingly, the effective supergravities that describe low-energy fluctuations around them have scalar potentials with minima at a zero value of the cosmological constant. On the other hand, there exists a large class of (gauged) supergravity models that admit maximally symmetric (anti)de Sitter, rather than Minkowski, space-times, as they typically involve scalar potentials with critical points of non-zero cosmological constant. Although the scalar potentials are generally unbounded from below, there are stability criteria [1] that make these vacua viable. There are also other models whose potentials have no critical points at all, but whose scalars may be stabilized by turning on background gauge fields. This, however, breaks the maximal symmetry, and the resulting solutions may possess maximal symmetry only in a lower number of dimensions.

Historically, the first instance of the latter phenomenon was provided by the Freedman-Gibbons electrovac solution [2] to one version of gauged N=4 supergravity [3]. The purpose of this letter is to point out that the Freedman-Gibbons vacuum is also an exact solution of String Theory, to all orders in the Regge slope expansion, and to identify the corresponding two-dimensional conformal model. This, we believe, is of some interest, being the first example of unbroken supersymmetry in curved space-time in String Theory.

## Gauged Supergravity from Strings

In order to see how gauged supergravities may arise in String Theory, consider the heterotic string compactified from 10 to 4 dimensions on the product of two (large) 3-spheres, parallelized by appropriate torsion backgrounds [4]. The left-moving string excitations then include an $SU(2)_A \otimes SU(2)_B$ Kac-Moody algebra [5], with $k_A$ and $k_B$ the corresponding (large) integer anomalies. As the gravitini transform in the (2,2) representation of $SU(2)_A \otimes SU(2)_B$[1], the corresponding six gauge bosons may be identified with non-Abelian graviphotons.

Consider next a 2D sigma model for the remaining four space-time coordinates, with arbitrary backgrounds for the metric $G_{\mu\nu}$, dilaton $\Phi$, axion b, and graviphoton $\mathbf{A}_\mu$ and $\mathbf{B}_\mu$ fields. The $\beta$-function equations follow from variations of the 4D action [6,7]

[1] In this case there is also a right-moving $SU(2) \otimes SU(2)$ current algebra, in addition to the usual $E_8 \otimes E_8$ or SO(32). These, however, will play no role in our arguments, since the gravitini do not transform under them. These current algebras could be partially or completely eliminated in more general, left-right asymmetric, constructions.



$$S = \int d^4x \sqrt{-g} \Big( R - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi \partial_\nu \phi + e^{2\phi} \partial_\mu b \partial_\nu b) - \frac{e^{-\phi}}{4} (\mathbf{A}_{\mu\nu} \mathbf{A}^{\mu\nu} + \mathbf{B}_{\mu\nu} \mathbf{B}^{\mu\nu})$$

$$- \frac{b}{4} ({}^*\mathbf{A}_{\mu\nu} \mathbf{A}^{\mu\nu} + {}^*\mathbf{B}_{\mu\nu} \mathbf{B}^{\mu\nu}) - \frac{1}{3} \delta c \, e^{\phi} \Big) + \text{higher-derivative terms} \quad . \qquad (1)$$

Here $\mathbf{A}_{\mu\nu}$ and $\mathbf{B}_{\mu\nu}$ are the field strengths for the two SU(2) groups, and $g_{\mu\nu} = e^{-\phi} G_{\mu\nu}$ is the "physical" space-time metric, corresponding to a standard form for the gravity action, with no dilaton factors in front. Moreover, $\delta c$ is the central-charge deficit, namely the difference between the central charge of the super-conformal system describing the six compactified dimensions, and its "flat" value ($3/2 \times 6 = 9$): it plays the role of a cosmological term [7]. In our case

$$\frac{1}{3} \delta c = - 2 \left( \frac{1}{k_A + 2} + \frac{1}{k_B + 2} \right) \approx - 2 \left( g_A^2 + g_B^2 \right) \quad , \qquad (2)$$

where $g_{AB} = 1/\sqrt{k_{AB}}$ are identified with the gauge coupling constants at zero dilaton, and the above equation holds to leading order in $1/k_{AB}$. The observation that the lowest-order action of eq. (1) is precisely the bosonic part of the version of N=4 supergravity with $SU(2)_A \otimes SU(2)_B$ gauging considered in refs. [3,2] was a motivation for the present work[2]. It should be appreciated that the exponential scalar potential, a puzzle in gauged supergravity, arises quite naturally in this context.

Besides having no critical points, the dilaton potential is unbounded from below, since $\delta c$ is negative. Still, it can be stabilized by turning on background graviphoton fields, as shown by Freedman and Gibbons [2]. More generally, this could also be achieved with backgrounds for the axion and/or other gauge fields. Here, however, we shall confine our attention to one of the Freedman-Gibbons solutions, obtained in the limit of $U(1)_A^3 \otimes SU(2)_B$ gauging, *i.e.* for $g_A = 0$. This solution leaves N=2 space-time supersymmetry unbroken, and is classically stable [2]. The non-vanishing fields are :

$$ds^2 = \frac{1}{K\cos^2\rho}(-dt^2 + d\rho^2) + dx^2 + dy^2 \quad , \qquad (3a)$$

$$\phi = \log\left(\frac{K}{2g_B^2}\right) \quad , \qquad (3b)$$

[2] Our conventions differ from ref. [4] by factors of 2, 2 and -1/4 for the dilaton, axion and metric fields, respectively.

$$A_\mu = (\frac{t g \rho}{g_B}, 0, 0, 0) \quad . \tag{3c}$$

Space time is thus the direct product of two-dimensional anti-de Sitter space[3] and two-dimensional Euclidean space. The covariantly constant electric field is along one of the three $U(1)_A$ directions, whereas the uncontracted $SU(2)_B$ is left unbroken in the vacuum.

We shall now prove that this solution fits in a 2D conformal model. It is therefore the first instance of a classical string solution with unbroken supersymmetry in a curved space-time.

## Alternative Interpretation of WZW Models

Or next task will be applying the coset decomposition of groups to Wess-Zumino-Witten (WZW) models. These can be nicely reinterpreted as sigma models on coset manifolds, with antisymmetric-tensor and gauge-field backgrounds that only depend on coset coordinates.

For a compact group G, the WZW model is defined by the action [4]

$$S_g = -\frac{k}{16\pi}\left(\int_{M_2} \mathrm{Tr}\left(g^{-1}\, dg\, g^{-1}\, dg\right) + \frac{2}{3}\int_{M_3} \mathrm{Tr}\left(g^{-1}\, dg \wedge g^{-1}\, dg \wedge g^{-1}\, dg\right)\right) , \tag{4}$$

where the field $g \in G$, $M_3$ denotes a three-dimensional manifold with boundary $M_2$, and k is a positive integer. Given a subgroup H of G, a generic g may be written as

$$g = h\, \omega \quad , \tag{5}$$

where $h \in H$, and $\omega$ parametrizes the (right) coset G/H. It is convenient to distinguish three terms in the resulting decomposition of the action (4), writing

$$S_g = S_h + S_\omega + S' \quad , \tag{6}$$

where

$$S' = -\frac{k}{8\pi}\left\{\int_{M_2} \mathrm{Tr}(h^{-1}\, dh\, d\omega\, \omega^{-1}) + \int_{M_3} \mathrm{Tr}(h^{-1}\, dh \wedge d\omega\, \omega^{-1} \wedge d\omega\, \omega^{-1} + h^{-1}\, dh \wedge h^{-1}\, dh \wedge d\omega\, \omega^{-1})\right\}$$
$$= -\frac{k}{8\pi}\int_{M_2} \mathrm{Tr}(h^{-1}\, dh\, d\omega\, \omega^{-1} - h^{-1}\, dh \wedge d\omega\, \omega^{-1}) \quad . \tag{7}$$

The last equality follows from Stokes' theorem, since the 3D integrand is a closed three-form. We may now interpret the three terms in eq. (6) as an H-current algebra, a sigma model on the coset manifold with antisymmetric-tensor background, and an H gauge-field background, respectively. Indeed, if in the differential $d\omega\, \omega^{-1}$ one distinguishes two components, one in the Lie algebra of H and one in its complement in G,

$$d\omega\, \omega^{-1} = d\omega\, \omega^{-1}|_H + d\omega\, \omega^{-1}|_{G/H} \quad , \tag{8}$$

only the first contributes to eq. (7). It couples to the left (vector+axial) H-currents, and can be reinterpreted as a background gauge field A valued in the Lie algebra of H and depending only on coset coordinates[4]. The same decomposition, when used in the quadratic part in $S_\omega$, yields

$$S_\omega = -\frac{k}{16\pi}\int_{M_2} \mathrm{Tr}\left((d\omega\, \omega^{-1}|_{G/H})^2 + A^2\right) + \text{cubic terms} \quad . \tag{9}$$

The first term is the invariant metric on the coset manifold. The second is familiar from Kaluza-Klein theory, where the correct parametrization of a (D+1)-dimensional metric that yields D-dimensional gravity and electromagnetism is of the form

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} + \frac{1}{4}A_\mu A_\nu & \frac{1}{2}A_\mu \\ \frac{1}{2}A_\nu & 1 \end{pmatrix} \quad . \tag{10}$$

The decomposition follows easily in the language of vielbeins, and ensures that $g_{\mu\nu}$ is properly inert under gauge transformations of the vector field. Eq. (9) involves a standard extension of this result to the non-Abelian case. Finally, the cubic part of $S_\omega$ is again a closed form, since it is the Wess-Zumino term of G, minus that of H, minus the

[3] It is customary to refer to the cover, parametrized by $t \in (-\infty, +\infty)$, $\rho \in (-\pi/2, \pi/2)$.

[4] In general, the gauge field has Dirac string singularities. This is indeed the case for SU(2), though not for SU(1,1).

cross term in eq. (7), all of which are closed. It may therefore be associated to the field strength of an antisymmetric-tensor background that, again, depends only on coset coordinates. In the particular case of G/H=SU(2)/U(1), this last term is clearly absent, since the coset is two dimensional. The conclusion is that the WZW model on SU(2) can be reinterpreted as a sigma model on the two-sphere with only an Abelian gauge-field background.

## The String Vacuum

We shall now use the above decomposition to prove that the electrovac solution, eq. (3), considered as a background for string propagation, results in a WZW model on the non-compact group SU(1,1). Since SU(1,1) ≈ SO(2,2)/SO(2,1), this can be also thought of as three-dimensional anti-de Sitter space with background torsion, showing that the Freedman-Gibbons solution possesses an even larger effective symmetry. More precisely, the full string ground state is described by a (1,0) super-conformally invariant sigma model for $SU(1,1)_{-k} \otimes R^2 \otimes R^2_\Lambda \otimes SU(2)_{k_B}$, with -k and $k_B$ denoting the corresponding current anomalies. Here SU(1,1) contains $AdS_2$, as well as one of the (non-compact) U(1)'s originating from the contraction of $SU(2)_\Lambda$, whereas the other two are responsible for the $R^2_\Lambda$ factor. The first $R^2$ is the Euclidean two-space of the Freedman-Gibbons solution.

In order to cancel the conformal anomaly, one must demand that

$$\frac{3k}{k-2} + 4 + \frac{3k_B}{k_B+2} = 10 \quad , \tag{11}$$

which implies that

$$k = 4 + k_B \quad . \tag{12}$$

It should be noticed that the SU(1,1) anomaly coefficient, -k, is negative. This is fortunate, since it allows one to associate the single compact generator in SU(1,1) with the time coordinate. Moreover, eq. (12) forces k to be an integer, although this is not required a priori by topological or unitarity arguments as in the compact case.

The electrovac solution of eqs. (3) is finally recovered by resorting to the following parametrization of the (cover of the) SU(1,1) group manifold:



$$g = \begin{pmatrix} \cosh(\phi/2) & -i\sinh(\phi/2) \\ i\sinh(\phi/2) & \cosh(\phi/2) \end{pmatrix} \begin{pmatrix} e^{it/2}\sqrt{\frac{1+\cos\rho}{2\cos\rho}} & e^{-it/2}\sqrt{\frac{1-\cos\rho}{2\cos\rho}} \\ e^{it/2}\sqrt{\frac{1-\cos\rho}{2\cos\rho}} & e^{-it/2}\sqrt{\frac{1+\cos\rho}{2\cos\rho}} \end{pmatrix} \quad . \tag{13}$$

This form corresponds to the (right) coset decomposition $SU(1,1)/R^1 \otimes R^1$, and is closely related to a suitable analytic continuation of the standard parametrization of SU(2) in terms of Euler angles. The WZW action then becomes

$$S = \frac{k}{4}\int\frac{d^2\xi}{4\pi}\left(\frac{-\partial_+ t\partial_- t + \partial_+\rho\partial_-\rho}{\cos^2\rho} + tg^2\rho\, \partial_+ t\partial_- t + \partial_+\phi\partial_-\phi + 2\, tg\rho\, \partial_+\phi\partial_- t\right) \quad . \tag{14}$$

From the preceding analysis, it is straightforward to identify the various terms in this action: $\sqrt{k}/2\,\phi$ is a normalized field giving rise to a (non-compact) Abelian gauge symmetry. Moreover, the gauge-field and physical-metric backgrounds are precisely those given in eq. (3), since $g_B = 1/\sqrt{k_B} \approx 1/\sqrt{|k|}$, where the second correspondence follows from the anomaly cancellation condition of eq. (12).

Some doubt was recently cast [8] on the possibility of attaining unitary string propagation on SU(1,1), a rather surprising claim in view of the fact that the solution (14) has no Dirac string singularity. Clearly, our analysis gives us no final say on this point, as we are not displaying a fully consistent modular-invariant theory. However, a preliminary analysis [9] shows that there is no problem if one restricts the spin j of the highest-weight representations so that $0 \geq j \geq -k/2$, in complete analogy with the compact SU(2) case.

## Space - Time Supersymmetry

The Freedman-Gibbons solution is particularly interesting, since it is compatible with N=2 space-time supersymmetry. Two points are particularly worth stressing. First of all, the negative sign of the cosmological constant is crucial, since only anti-de Sitter space-time groups are known to admit supersymmetric gradings. Moreover, the reduction from the maximal N=4 supersymmetry to N=2 supersymmetry is induced, in the supergravity analysis of ref. [2], by a projection on the supersymmetry parameters that involves both the space-time gamma matrices and a U(1) charge of the fully contracted $SU(2)_\Lambda$. The restriction comes from the requirement that the supersymmetry parameters be covariantly constant spinors in the background.

Framing all this in the 2D language results, as usual, in a simpler and more illuminating picture. To this end we note that, in String Theory, unbroken space-time supersymmetry demands that the zero mode $G_0^{(tot)}$ of the 2D supercurrent $T_F^{(tot)}$ annihilate the Ramond vacuum. This condition is equivalent to the existence of a holomorphic dimension (1,0) primary field, whose contour integrals generate space-time supersymmetry transformations [10]. In our case, the left-moving currents have free fermionic superpartners, and $T_F^{(tot)}$ is a sum of terms, one for each of the factor groups, of the form

$$T_F = J^a \psi^a + f_{abc} \psi^a \psi^b \psi^c \quad . \tag{15}$$

The cubic term, only present for non-Abelian groups, plays an important role in what follows.

From the super-Virasoro algebra one finds that, for any super-conformal model,

$$G_0^2 = L_0 - \frac{c}{24} \quad , \tag{16}$$

where c denotes the central charge of the model. In particular, the contribution of a free super-coordinate to the right-hand side vanishes: 1/16 - 3/2 1/24 = 0, where 1/16 originates from the fermionic zero mode. For $SU(2)_B$, on the other hand, one finds

$$G_0^2(SU(2)) = \frac{3}{16} - \frac{1}{24}\left(\frac{3k_B}{k_B+2} + \frac{3}{2}\right) > 0 \quad . \tag{17}$$

The inequality holds for all compact Lie groups, and shows that a compactification on curved internal manifolds gives masses to fermions and breaks supersymmetry in Minkowski space-time [10]. In our case, we may circumvent the argument because

$$G_0^2(SU(1,1)) = \frac{3}{16} - \frac{1}{24}\left(\frac{3k}{k-2} + \frac{3}{2}\right) < 0 \quad , \tag{18}$$

*i.e.* SU(1,1) gives a negative contribution, that precisely cancels the one of $SU(2)_B$, once the conformal anomaly condition of eq. (11) is taken into account.

Although $(G_0^{(tot)})^2$ vanishes identically in the Ramond vacuum, this turns out not to be the case for $G_0^{(tot)}$. Rather, because of the trilinear couplings in $T_F$, one finds



$$G_0^{(tot)}|0\rangle_{Ram} = \left(\gamma_B^1 \gamma_B^2 \gamma_B^3 + \gamma^0 \gamma^1 \gamma^2\right)|0\rangle_{Ram} \quad . \tag{19}$$

In eq. (19), $\gamma_B$ and $\gamma$ denote gamma matrices representing the Dirac algebra for $SU(2)_B$ and SU(1,1), respectively. As the standard GSO projection forces the Ramond vacuum to be an eigenvector of the product of all ten gamma matrices, demanding that $G_0^{(tot)}$ annihilate the vacuum yields

$$\left(1 + \gamma_A^2 \gamma_A^3 \gamma^x \gamma^y\right)|0\rangle_{Ram} = 0 \quad . \tag{20}$$

This is precisely the condition of ref. [2], and reduces the number of space-time supersymmetries from N=4 to N=2.

In conclusion, we have displayed a first instance of a classical ground state for the heterotic string, with unbroken supersymmetry in a non-trivial space-time. This solution could be generalized by trading the $SU(2)_B \otimes R^2$ algebra for some other superconformal model, but the conditions for unbroken supersymmetry need in this case to be reconsidered. It is worth stressing that the solution does not pertain to the type-I superstring. The reason lies in the very nature of the supersymmetric WZW model, where the internal background torsion couples to an operator in the NS sector. This is to be contrasted with the proper internal torsion for the type-I superstring, that would couple to an operator in the R⊗R sector of the closed string[5].

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## References


[1] P. Breitenlohner and D.Z. Freedman, *Phys. Lett.* **115B** (1982) 197;
*Ann. Phys.* **144** (1982) 249.

[2] D.Z. Freedman and G.W. Gibbons, *Nucl. Phys.* **B233** (1984) 24.

[3] E. Cremmer, J. Scherk and S. Ferrara, *Phys. Lett.* **74B** (1978) 61;
D.Z. Freedman and J.H. Schwarz, *Nucl. Phys.* **B137** (1978) 333.

[4] E. Witten, *Comm. Math. Phys.* **92** (1984) 455.

[5] P. Di Vecchia, V.G. Knizhnik, J.L. Petersen and P. Rossi, *Nucl. Phys.* **B253** (1985) 701;
P. Goddard and D. Olive, *Nucl. Phys.* **B257**[FS14] (1985) 226.

[6] D. Friedan, *Phys. Rev. Lett.* **45** (1980) 1057;
C. Lovelace, *Nucl. Phys.* **B273** (1986) 413;
E.S. Fradkin and A.A. Tseytlin, *Phys. Lett.* **158B** (1985) 316;
C. Callan, D. Friedan, E. Martinec and M. Perry, *Nucl. Phys.* **B262** (1985) 593;
A. Sen, *Phys. Rev.* **D32** (1985) 2102.

[7] I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, *Phys. Lett.* **211B** (1988) 393, and preprint **CERN-TH.**5231 (1989), to appear in *Nucl. Phys.* **B.**

[8] J. Balog, L. O'Raifeartaigh, P. Forgacs and A. Wipf, preprint **MPI-AE/PTh** 66/88 (1988).

[9] M. Petropoulos, Ecole Polytechnique preprint **A935.1089** (1989).

[10] D. Friedan, Z. Qiu and S. Shenker, *Phys. Lett.* **151B** (1985) 37;
D. Friedan, E. Martinec and S. Shenker, *Nucl. Phys.* **B271** (1986) 93.