CLASSIFICATION AND SYMMETRIES OF STRONGLY INTERACTING PARTICLES

B. d'ESPAGNAT, Rapporteur

CERN, Genève

Assuming Yukawa interactions between baryons and mesons — and charge independence — the most general strong interaction Lagrangian is, as is well-known,

$$\begin{aligned}\mathfrak{L} &= g_1 \bar{N} \boldsymbol{\tau} N \boldsymbol{\pi} + g_4 \bar{\Xi} \boldsymbol{\tau} \Xi \boldsymbol{\pi} \\ &+ g_2 \bar{\Lambda} \boldsymbol{\Sigma} \boldsymbol{\pi} + \frac{1}{i} g_3 \bar{\boldsymbol{\Sigma}} \times \boldsymbol{\Sigma} \cdot \boldsymbol{\pi} \\ &+ g_5 \bar{N} K \Lambda + g_6 \bar{N} \boldsymbol{\tau} K \boldsymbol{\Sigma} \\ &+ g_7 \bar{\Xi} \dot{K} \Lambda + g_8 \bar{\Xi} \boldsymbol{\tau} \dot{K} \boldsymbol{\Sigma} \end{aligned} \tag{1}$$

with

$$\dot{K} = -i \tau_2 K^* = \begin{pmatrix} -K^{0*} \\ K^{+*} \end{pmatrix}$$

γ matrices being omitted. Several authors have attempted to make this $\mathfrak{L}$ more definite by reducing the somewhat impressive amount of independent coupling constants and we should review now some of their most recent proposals. As you know already, from former attempts, two approaches can be used: either the cautious one, which assumes as little as possible and tries to stick to experiment at every step, or the ambitious one which starts from an idea, deduces many symmetries from it and then compares with actual data. The first one is unquestionably the better, except that unfortunately its yield is very poor. At any rate we shall begin our review by the most cautious approaches and then go step by step to the more ambitious models.

I feel that in this spirit I should begin with Pais' recent remarks. Pais' first paper on this subject is, however, already published [1] so that I shall only recall the essential results of it, which are as follows. Let us assume that

$$g_2 = \pm g_3; \quad g_5 = \pm g_6; \quad g_7 = \pm g_8 \tag{2}$$

with correspondence of signs, then Pais shows that, to the extent that the $\Lambda - \Sigma$ mass difference is small compared to baryon mass, this assumption is incompatible with the data on ΣK, ΛK pair production with pions on nucleons.

In a second paper, Pais [2] shows that unfortunately none of the other possible assumptions lead to relations between the various production amplitudes (except of course to triangular inequalities): in particular, one cannot exclude in this way all the symmetries (2) which would not have the given sign correspondence $+\,+\,+$ or $-\,-\,-$.

Inequalities may, however, have their importance: in this connection I should like to mention here a paper by Amati and Vitale [3] who consider K^-, p reactions with the only assumption that $g_2 = \pm g_3$ (renormalized) and that K interactions are rather weak.

For a review of this paper the reader is referred to Dalitz' report p. 197.

Returning to Pais' work, its great value is obviously that it postulates a priori really very little. It follows, however, from Pais' second paper that this cautious approach cannot, unfortunately, lead us very far: in fact all it could do was to exclude case (2) out of infinitely many possible cases. We might, of course, stop here but if we want to go further we must necessarily resort to assumptions.

Now one assumption that has been proposed by several people is that the baryon bare masses are all equal. Let us have a little glance at this. In fact we shall only need the weaker assumptions

$$\begin{aligned} a)\ m_N &= m_\Xi \\ b)\ m_\Lambda &= m_\Sigma \end{aligned} \tag{3}$$

Under assumption *b*) case (2) is of course trivially excluded, because the whole Lagrangian is then 4-dimensionally invariant with $\boldsymbol{\Sigma}, \Lambda$ as a 4-vector, and thus no $\Lambda\Sigma$ mass splitting can occur; but *a*) also gives us some indications. As a trivial example let us mention the simple cases

$$g_1 = g_4, \quad g_5 = \varepsilon g_7, \quad g_6 = \varepsilon g_8 \tag{4}$$

with

$$\varepsilon = \pm 1 \tag{5}$$

These can be most simply excluded using the transformation

$$N \to \Xi, \quad \pi \to \pi \quad K \to \dot{K},$$

$$\begin{pmatrix} \Sigma \\ \Lambda \end{pmatrix} \to \varepsilon \begin{pmatrix} \Sigma \\ \Lambda \end{pmatrix} \tag{6}$$

which leaves the whole $\mathfrak{L}$ invariant while changing N into Ξ and conversely. $\mathfrak{L}$ cannot therefore induce any N, Ξ mass difference.

Nothing, of course, tells us that (4) holds. Now another use of transformation (6) and of similar substitutions could be simply to classify the strong interactions into a part that remains invariant under them and a part that changes its sign: this for further theoretical use. Before we embark on this however, I shall make the trivial remark that instead of transformation (6) we might just as well have used its product with any isotopic spin rotation—which leaves the Lagrangian invariant anyhow—and in particular with a 180° rotation around the I_2 axis. This gives

(B)
$$N \to -i\tau_2\Xi, \quad K \to -i\tau_2\dot{K} = -K^*,$$
$$\pi_i \to (-1)^i \pi_i \quad \text{i.e.} \quad \pi \to -\pi^*, \ \pi^\circ \to \pi^\circ$$
$$\Lambda \to \Lambda, \quad \Sigma^+ \to -\Sigma^-, \text{ etc...}$$

For practical purposes (B) is quite equivalent to (6): its one formal advantage is that the two bosons, π and K, behave similarly under it. We already know that the full Lagrangian cannot be invariant under (B) if bare masses are equal but as I said we want to use (B) for a classification of the interaction terms, hoping that this will lead us to natural assumptions.

Of course, such a classification would look more highly promising for our purpose if, instead of being just a mathematical substitution, some physical interpretation could be given to it. This is precisely what is provided by the contribution of Budini, Dallaporta and Fonda [4]. They consider to that end a kind of compound model where we have as fundamental particles

a baryon Λ_0 with no isotopic spin, hypercharge, and charge

$$I = U = Q = 0$$

the K meson

the π meson

From these the known particles are obtained by clothing with K and π: for instance they could use

$$N = \Lambda_0 K, \quad \Xi = \Lambda_0 \dot{K}, \quad \boldsymbol{\Sigma} = \Lambda_0 \boldsymbol{\pi}, \quad \Lambda = \Lambda_0$$

In such a scheme it appears natural to split the charge conjugation C into a product of two operations, one, B, being roughly speaking a charge conjugation of the boson field only, the other, S, being a particle antiparticle conjugation acting on Λ_0 only. Mathematically one chooses

$$K \to -K^*, \quad \pi \to -\pi^*, \quad \Lambda_0 \to \Lambda_0$$

then

(B)
$$\Sigma^+ \to -\Sigma^- \text{ etc.}$$
$$N \to -\Lambda K^* = -\Lambda i\tau_2\dot{K} = -i\tau_2\,\Xi$$

so that this is indeed just our former (B), but now with a physical interpretation. Then B · S = C gives

(S)
$$\Lambda_0 \to \Lambda_0{}^c, \quad K \to -K, \quad \pi \to -\pi$$
$$N \to -\Lambda_0{}^c K = -i\tau_2\Lambda\,\dot{K}^* = -i\tau_2\,\Xi^c$$

(With the general definition $\chi^c = C^{-1}\,\bar{\chi}^T$).

In the author's idea, their compound model should just be used in order to introduce the B and S (boson conjugation and spinor conjugation) in a natural and so to speak physical way. Once they have thus entered the picture it becomes believable that they play a role in nature, though what this role exactly is we, of course, do not know. Quite tentatively they suggest that the Lagrangian (1) should be invariant under B and S separately. Then the N-Ξ mass-splitting should be attributed to some non-Yukawa interaction, for instance to a (baryon, baryon, K, K) direct interaction.

At this point I would tentatively insert a small remark[5]. It was shown by Zel'dovich, Feynman and Gell-Mann and others that the representation

$$\chi = a\,N$$
$$\hat{\chi} = \bar{a}\,N$$

with
$$a = \frac{1+\gamma_5}{2}, \qquad \bar{a} = \frac{1-\gamma_5}{2}$$

is particularly suited to the description of *weak* interactions, because with *V-A* coupling, $\hat{\chi}$ does not enter. The same of course holds if we change χ, $\hat{\chi}$ to

$$\chi = a\,N + \bar{a}\,i\tau_2\,\Xi^c$$
$$\hat{\chi} = \bar{a}\,N + a\,i\tau_2\,\Xi^c$$

provided we request invariance under $\chi \to e^{i\theta\gamma_5}\chi$, i.e. baryon conservation. Now in terms of χ the " boson conjugation " B takes the simple form

(B)
$$\chi \to -\chi^c$$

in full analogy with what it is for bosons. The product of the " spinor conjugation " S with ordinary parity P takes also a simple form

(S.P)
$$\chi \to -i\gamma_4\chi, \quad \pi \to \pi, \quad K \to K \tag{7}$$

(if K is pseudoscalar).

From (7) it is obvious how in the new representation χ one should write the B and S conserving terms; they are just those which do not involve a γ_5. (It may further be pointed out that a PC invariant Lagrangian, see below, could similarly be split into B and (S.P) separately conserving and non-conserving terms.) Thus with the χ representation both weak and strong interactions take rather simple forms while B and S conjugations are straightforward and endowed with a kind of physical interpretation as reported. Whether or not these facts are an indication of any deep-lying symmetry, it is of course much too early to judge.

Votruba and Lokajíček [6] have followed a different line. They consider two sets of vector matrices

$$\boldsymbol{\omega} \text{ and } \boldsymbol{\lambda}$$

in a 3-dimensional isospace, which have close analogies with the $\gamma\,(\beta)$ and σ matrices in ordinary space time. They subject them to a matrix algebra which is too complicated to be transcribed here, but which has the property that its *only* representations that involve irreducible non-zero $\boldsymbol{\omega}$ matrices are

a) $\boldsymbol{\lambda} = \frac{\boldsymbol{\tau}}{2}$ $\qquad U = \frac{2}{3}\ \boldsymbol{\omega} \cdot \boldsymbol{\lambda} = 1$

b) $\boldsymbol{\lambda} = \frac{\boldsymbol{\tau}}{2}$ $\qquad U = -1$

c) $\boldsymbol{\lambda} = \begin{pmatrix} \mathrm{T} & 0 \\ 0 & 0 \end{pmatrix}$ $\qquad U = 0$

T being the spin one matrix. These, of course, they put in correspondence respectively with

a) $N;K$

b) $\Xi;\dot{K}$

c) $\boldsymbol{\Sigma}, \Lambda;\ \boldsymbol{\pi}, \pi_0'$

and, these representations of their matrix algebra being the only non-pathological ones, they are thus able to limit the number of possible baryons and mesons. This, I think, is their essential result. As far as interactions are concerned I would, however, mention the fact that they also are led to transformations which are practically identical with B and S in a very natural way.

Finally, the negative most recent Berkeley results on backward-forward asymmetry in Λ-decay make it unnecessary that we should dwell much on possible P and C non-conservation in strong K interactions. However, I would like to mention the fine theoretical point made both by Soloviev [7] and Drell [8] that, with non-gradient couplings, the assumption of PC conservation, together with that of charge independence, unambiguously lead to separate P and C conservation but in the π interaction terms only, not in the K interaction terms. A theory based on such premises would therefore accommodate, and according to Drell even predict, a polarization of the Λ in the production plane and, therefore, a front-back or left-right asymmetry of the decay pions.

LIST OF REFERENCES

1. Pais, A. Phys. Rev., *110*, p. 574, 1958.
2. Pais, A. (to be published)
3. Amati, D. and Vitale, B. (to be published in Nuov. Cim.)
4. Budini, P., Dallaporta, N. and Fonda, L. (to be published in Nuov. Cim.)
5. d'Espagnat, B. (to be published in Nuov. Cim.)
6. Votruba, V. and Lokajíček, M. (to be published in Nuclear Physics)
7. Soloviev, V. G. Nuclear Physics, *6*, p. 618, 1958.
8. Drell, S. D., Frautschi, S. C. and Lockett, A. M. Stanford Univ., Physics Dept., technical report No. 30, June 1958 (unpublished).