ON NUCLEON STRUCTURE

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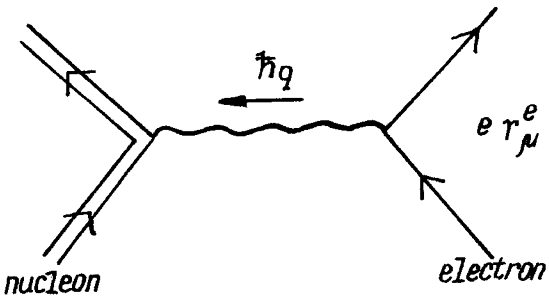
One of the primary aims of a study of proton structure is the determination of the phenomenological form factors $F_1(q^2)$ and $F_2(q^2)$. These form factors appear in electron scattering experiments and are well known. They are associated with the vertex operator shown below.

$$eF_1(q^2)\gamma_\mu + \frac{e\hbar k}{2Mc}F_2(q^2)\gamma_\mu\gamma_\nu q_\nu$$



The form factors are thought to be functions exclusively of the invariant q^2 where $\hbar q$ is the four-momentum-energy transfer of the virtual photon involved in the scattering process. Similar form factors arise in the case of the neutron and are also functions of q^2 alone. The differential cross-section of the electron scattering process deduced from the above vertex operator by standard methods of electrodynamics is given by the Rosenbluth formula

$$\frac{d\sigma}{d\Omega}=\left(\frac{e^2}{2E_o}\right)^2 \frac{\cos^2\vartheta/2}{\sin^4\vartheta/2} \left\{\frac{1}{1+\frac{2E_o}{Mc^2}\sin^2\frac{\vartheta}{2}}\right\} \times$$
$$\times\left\{F_1^2(q^2)+\frac{\hbar^2q^2}{4M^2c^2}\left[2\left(F_1(q^2)+KF_2(q^2)\right)\tan^2\frac{\vartheta}{2}+K^2F_2^2(q^2)\right]\right\} \quad (1)$$

where:

$$q=\frac{2E_o}{\hbar c}\sqrt{1+\frac{2E_o}{Mc^2}\sin^2\frac{\vartheta}{2}} \quad (2)$$

and where the other symbols are well known. In the past the F_1 and F_2 values have been derived separately from the experimental data owing to their greater importance at small and large angles respectively.

One can see that if the cross-section is measured at a pair (1,2) of energy- angle conditions (E_1, ϑ_1) and (E_2, ϑ_2) such that the q-value is the same for each member of the pair, two measurements determine both $F_1(q^2)$ and $F_2(q^2)$ for that particular value of q. This behaviour is shown in Fig. 1. The curved lines are portions of ellipses in the F_1 F_2 plane and correspond to the same cross-section for given values of E and ϑ .

The equation of the basic ellipse is:

$$a_{11}F_1^2 + a_{12}F_1F_2 + a_{22}F_2^2 = \text{constant} \quad (3)$$

where a_{11}, a_{12} and a_{22} are simple functions of q and ϑ deducible from equation (I). In other words one single determination of a cross-section means that the (F_1, F_2) point must be somewhere along the arc of an ellipse for the q-value associated with the pair of values E and ϑ.

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For example, the set of lines at 1000 MeV, 45° ($q^2 = 11.46 \times 10^{26}$ cm^{-2}) show values of the F_1, F_2 combination for three values of the cross-sections, $\frac{d\sigma}{d\Omega}$ and $\left(\frac{d\sigma}{d\Omega}\right) \times (1.0 \pm 0.10)$. The figure 0.I0 in the bracket indicates the present magnitude of experimental error. The average line in the middle is drawn for the case thought to be approximately true experimentally. The condition $F_1 = F_2$ is also thought to be true approximately and is indicated by the diagonal straight line so marked in the figure. Another set of lines is also shown for the experimental conditions: $E = 500$ MeV and $\vartheta = 135^{\circ}$. The intersection of the 500 MeV set with the 1000 MeV set determines uniquely the value of the pair (F_1, F_2). Allowing for experimental error there is of course a spread in the allowed values of F_1 and F_2 as indicated approximately by the figure resembling a parallelogram.

A similar set of values of F_1, F_2 and their experimental tolerance is shown in the upper part of the figure where $q^2 = 2.67 \times 10^{26}$ cm^{-2}. Of course, other sets of measurements at the same E and ϑ can also be obtained and give more information so that the solution is, in a sense, overdetermined. However, such measurements do serve to check that F_1 and F_2 are functions of q^2 only. This type of consistency test, if made more severe, say 1%, could perhaps lead to further information on the validity of quantum electrodynamics or perhaps other fundamental aspects of the vertex operators involved in electron-proton scatterings. We have made such tests in the past and
3176 I propose to make such more severe tests in the future.

Thus we can imagine many elliptical arcs passing through the region of the intersections shown in Fig. 1. Taken with their experimental errors, they will determine a small area in which the actual point F_1 F_2 lies. We have made many such determinations in the past but none so far that involves an energy greater than 650 MeV. The largest q^2 values extend approximately to $17 \times 10^{26} cm^{-2}$. (The value $22.6 \times 10^{26} cm^{-2}$ corresponds to the reciprocal of the square of the reduced nucleon Compton wave-length.)

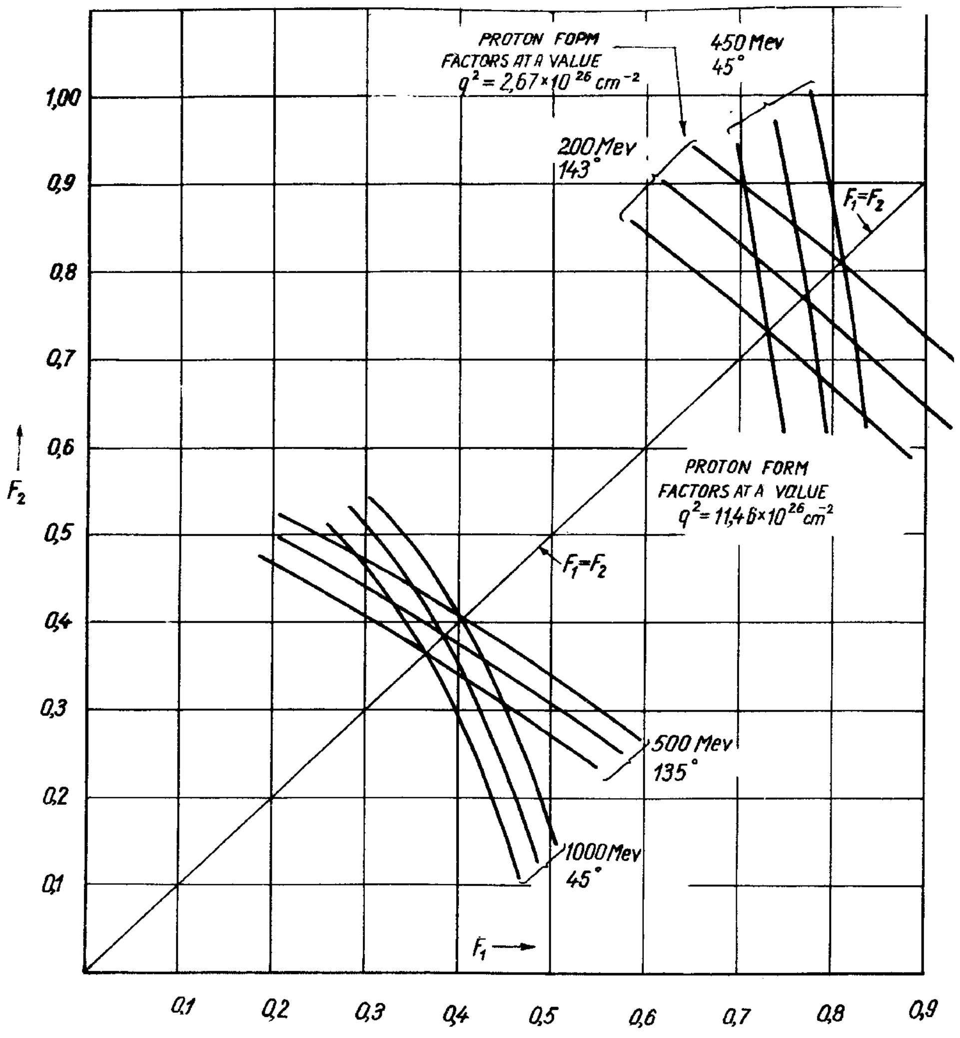

Fig.1.

Another example lying within the measured range is shown in Fig.2. In this case, because we have not been able to use the small angle region (45°) at high energy (1000 MeV),

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we have had to use a pair at E=600 MeV and $\vartheta = 91.6^{\circ}$ together with the other pair E = 500 MeV and 135°.

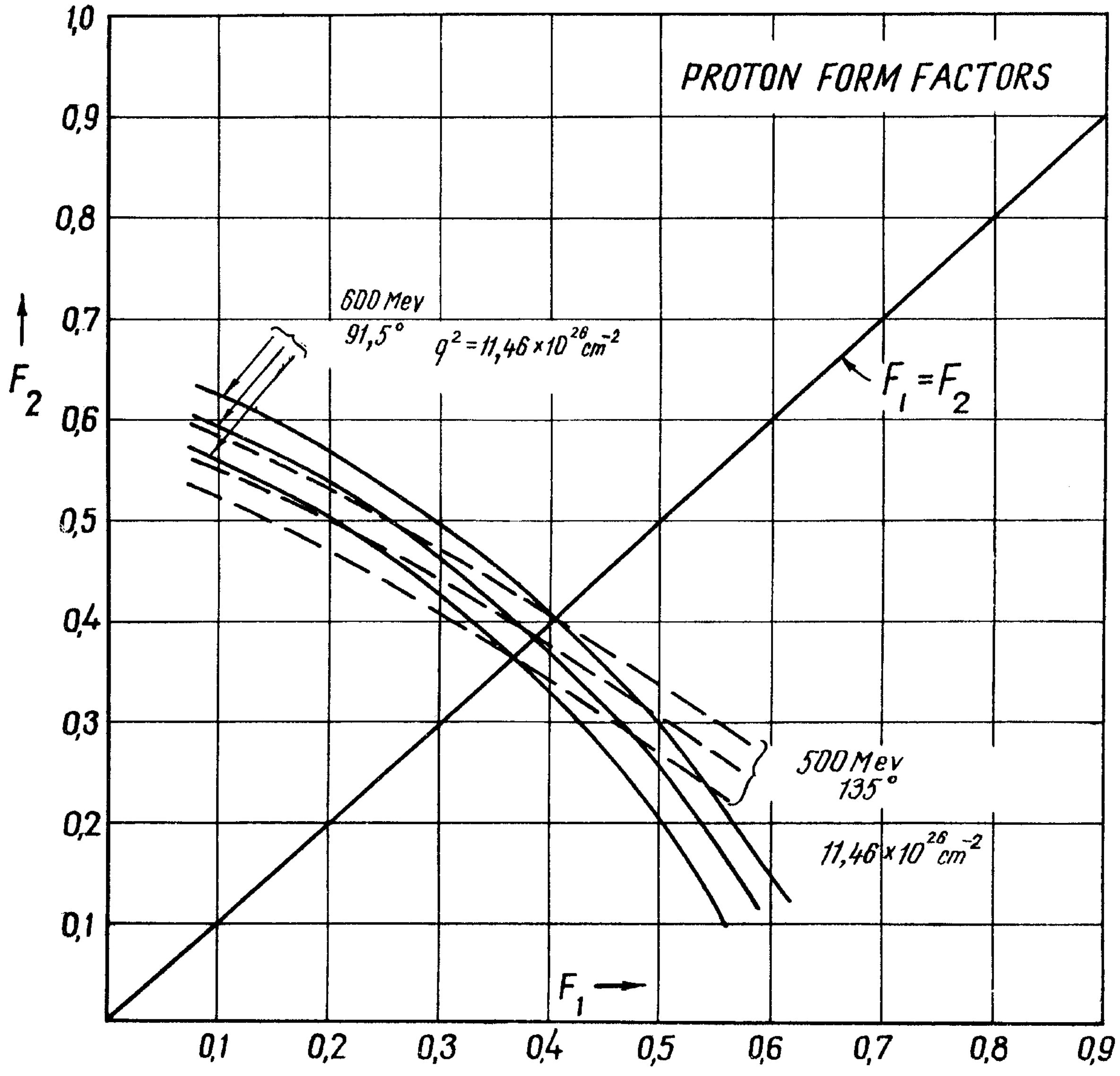

Fig.2

Here one observes that the intersection is not nearly so well defined and in fact it is relatively poorly defined for small values of F_1. Nevertheless, even from this relatively poor example, one sees that F_1 has clear upper limits. The lower limits of F_1 are less well defined. It must be emphasized once again that there are other experimentally measured points implying elliptical arcs running through the region at $q^2 = 11.4 \times 10^{26} cm^{-2}$ such as e.g. corresponds to 650 MeV and 75°. I have not constructed such

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complete sets of curves and cannot therefore put definite limits on F_1 and F_2 separately for the proton. But I think our published papers have given rather generous estimates of the tolerances permitted by experiment.

Some time ago R. Karplus[1] made a similar estimate from the early data of Chambers and Hofstadter[2] with the results shown in Fig.3. The approximate tolerance limits

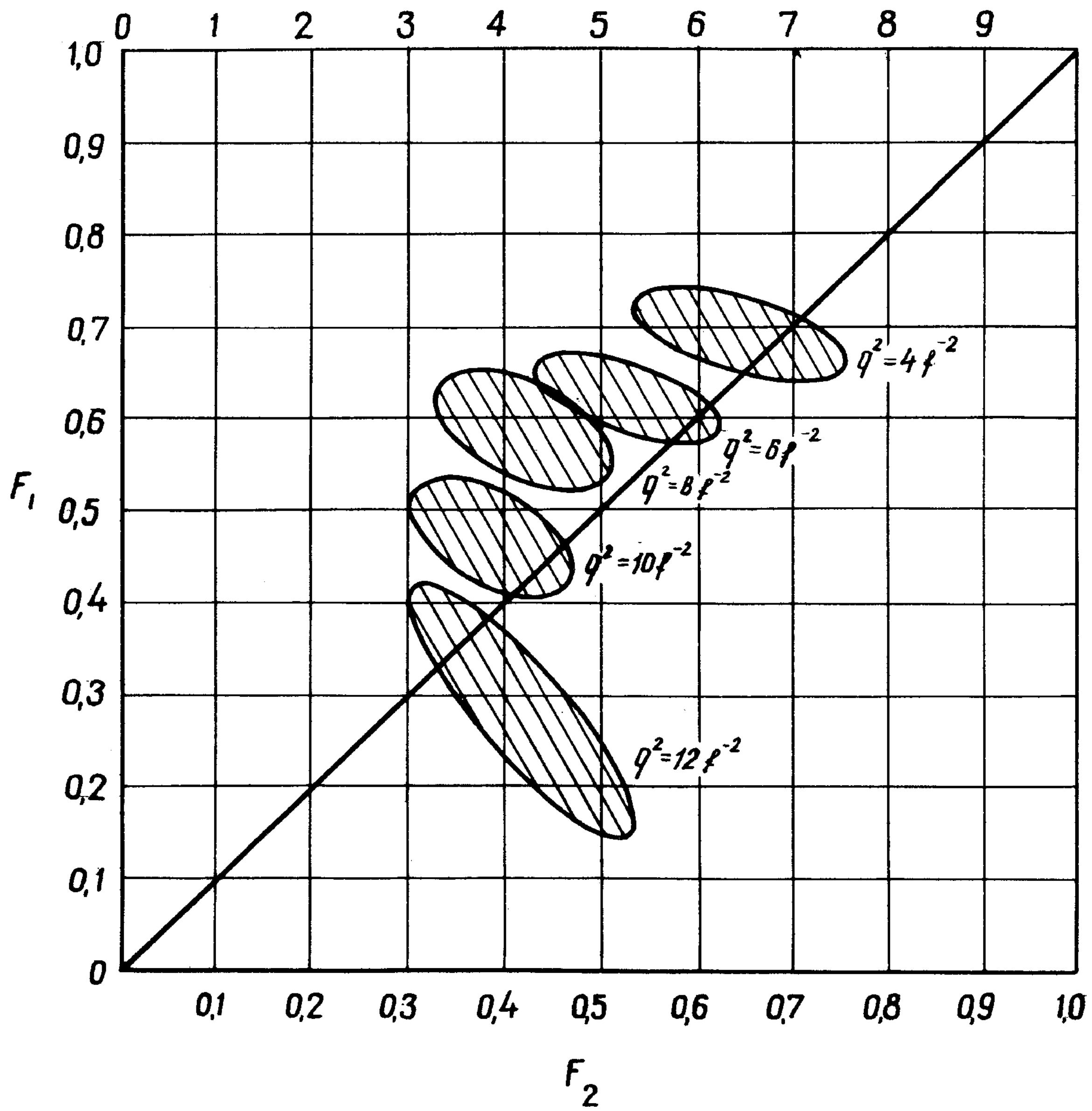

Fig.3.

are given by the equal probability ellipses (note that the F_1 F_2 axes are intercharged with respect to those in Figs. 1 and 2). 3176

It is also possible to measure a <u>ratio</u> of cross sections at the same q^2 and from the ratio to deduce a single number, namely F_1/F_2. These results have always been in agreement with the results quoted in references [2] and [3] and indicate the ratio $F_1/F_2 \cong 1.18 \pm 0.20$ at values of q^2 up to 9.3×10^{26} cm^{-2}. Further experiments of this type are clearly needed.

For the above and other obvious reasons we have always employed the simplifying assumption that $F_1 = F_2$. However, it is now worth while to look definitely for departures from this assumption.

I do not propose to make a statistical study of the ΔF_1, ΔF_2 values allowed by experiment in this report since I feel that very shortly (if not already) there will be new good measurements of F_1 and F_2 at energies up to 1000 MeV. Under such conditions the intersections shown in Figs. 1 and 2 become quite definite. An increase in experimental accuracy will also result in large reductions in the present uncertainties in the ratio of F_1 to F_2.

Many studies have shown that the $F_1 = F_2 = F$ values for the proton can be represented approximately in terms of a model by phenomenological exponential charge and magnetic moment density distributions. Since such exponential models appear unattractive from a theoretical point of view because of their behaviour at large radii we have investigated the matter a bit furhter theoretically. It proved very easy to show [4] that a behaviour at large radii corresponding to mesontheoretical models will give the same form factors as the exponential provided only that the singularities in the
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typical repulsive core-radius. This idea of having finite electromagnetic densities at the centre of the proton must have some analogue in the language of dispersion relations and it would be interesting to find the spectral representation corresponding to this finite core.

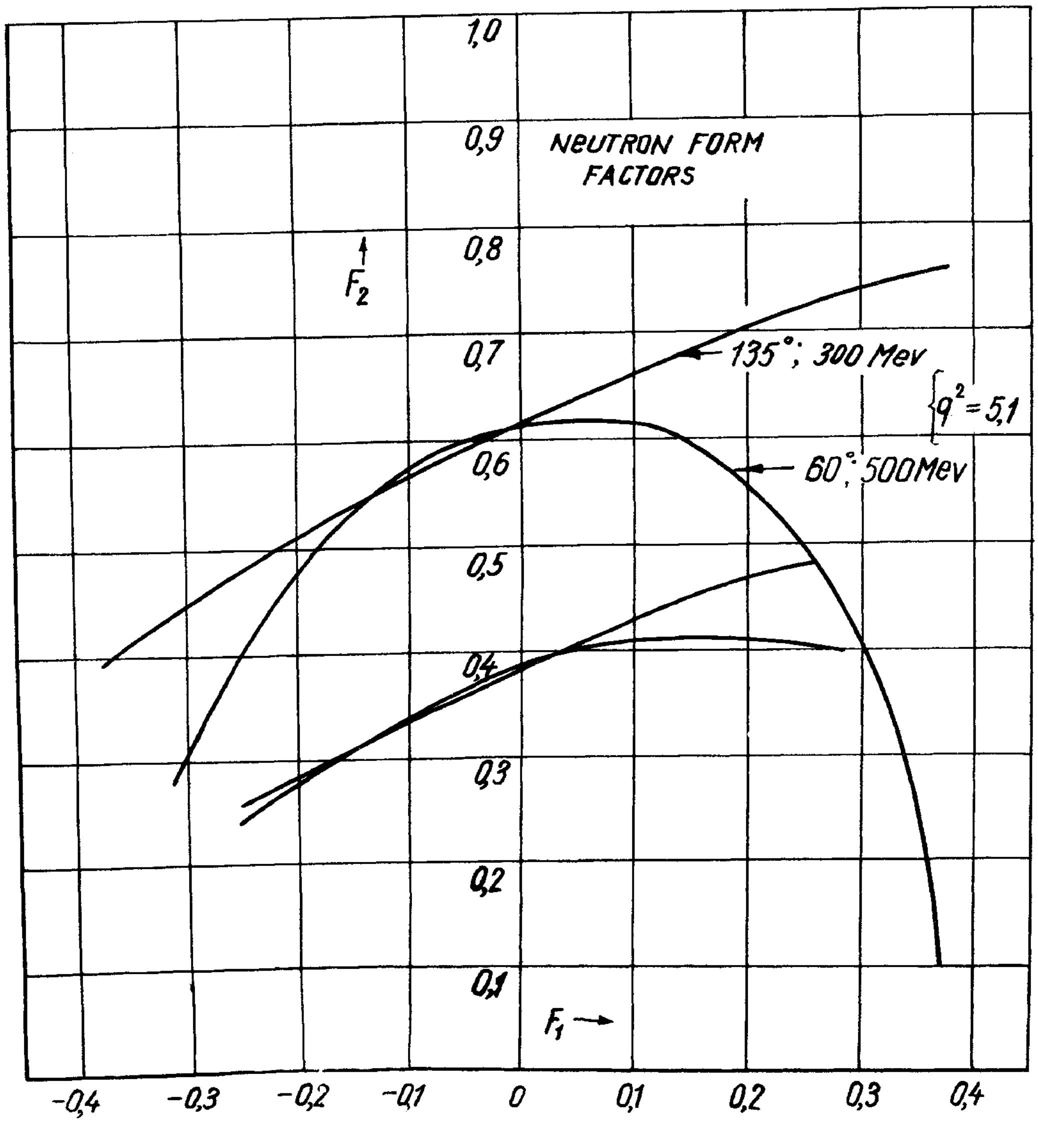

Fig.4.

Next we come to the problem of the neutron and some new experimental material. In this case it is known that $F_{1N} \cong 0$ at small values of q^2. The experimental determinations of neutron form factors [3,5] are consistent with $F_{1N} \cong 0$ and $F_{2N} \cong F_{1P} = F_{2P}$. Fig. 4 shows a plot 3176 representing the elliptical arcs for the neutron and is similar to Figs. 1 and 2 for the proton. In this case the condition $F_1 = 0$ places the (F_{1N} F_{2N}) points in the proximity of the vertical axis. The intersections of the two different ellipses at a given q^2 are not now so clear as in the case of the proton. In fact the behaviour is not so different from that shown in Fig.2 where the intersection lies almost along a line and is rather indefinite. One can also see from this figure that if the neutron s F_1 is positive then it is rather easily determined but if it is negative (which seems more reasonable from a physical point of view) the exact determination of its value is difficult at presently available energies, because the intersection of the two ellipses is not well defined. On the other hand if ixperimental error can be reduced the determination is easier. If the incident electron energy can be increased to 1000 MeV fixing of the F_{1N} values again becomes much easier.

I would like now to discuss some new measurements on the neutron by S. Sobottka, a graduate student at Stanford University. Before I do this I wish to note a few remarks affecting the interpretation of the experiments.

there are many small difficulties associated with the theory of the deuteron which make their appearance when it is desired to deduce the neutron form factors from experiment. These have been discussed many times and in the past[3,5-10]. Slow but definite progress is being made in removing these difficulties..

3176 I shall only mention in this talk uncertainties in the final results as they are affected by the small uncertainties of the theory.

Sobottka has repeated measurements of a type made previously[3,5] on the inelastic scattering of electrons from deuterons at energies up to 650 MeV and extending between small (45^o-60^o) angles and large angles (135^o).

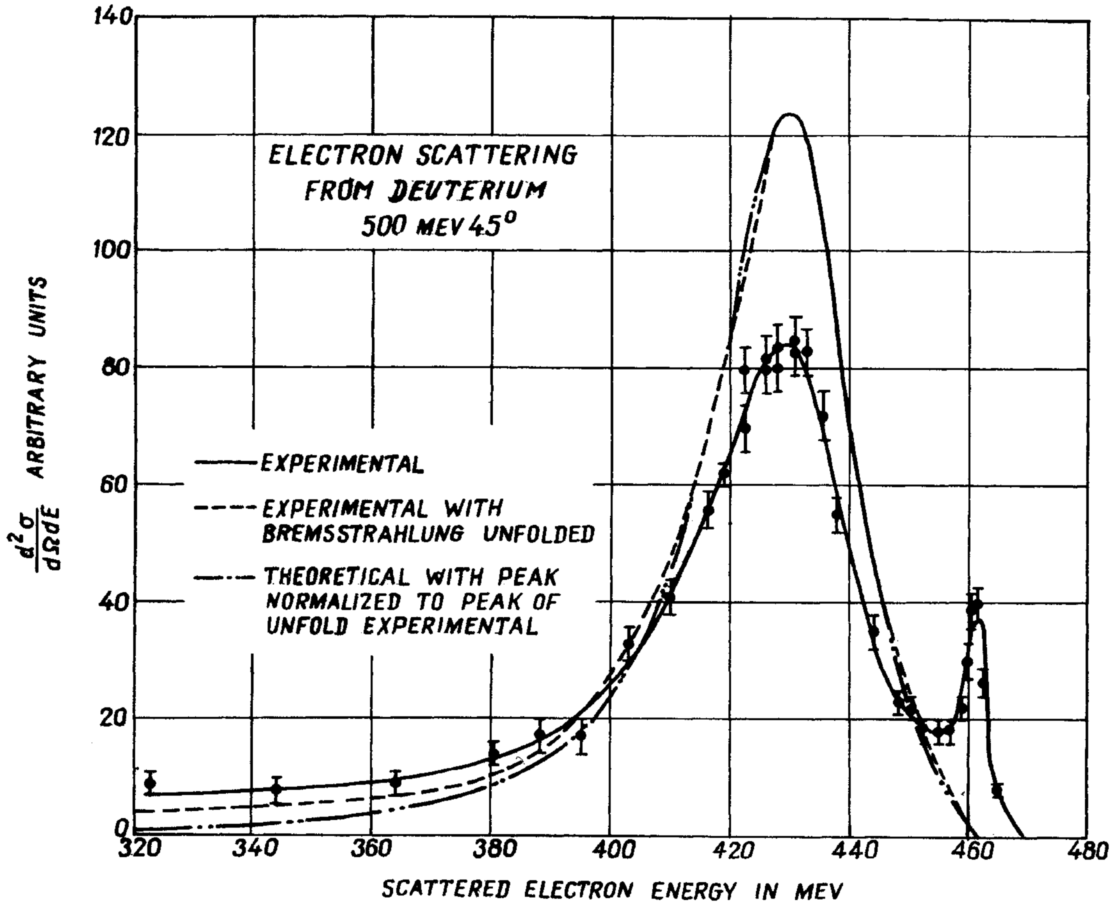

Fig.5.

Two examples of his data are shown in Figs. 5 and 6. In Fig. 5 one sees both the elastic and inelastic scattering peaks at 500 MeV and 45^o. Absolute cross- 3176 sections have been determined from a comparison with proton peaks in a well-known way. The inelastic data are corrected for radiative effects by an unfolding procedure and when so corrected give the dashed curve. The dash-dot curve is a result of a modified Jankus calculation[5,6] and exhibits approximate agreement with the corrected experimental curve when the two curves are adjusted to the same peak values. The experimental curve has a higher tail at lower energies. Fig. 6 shows another example

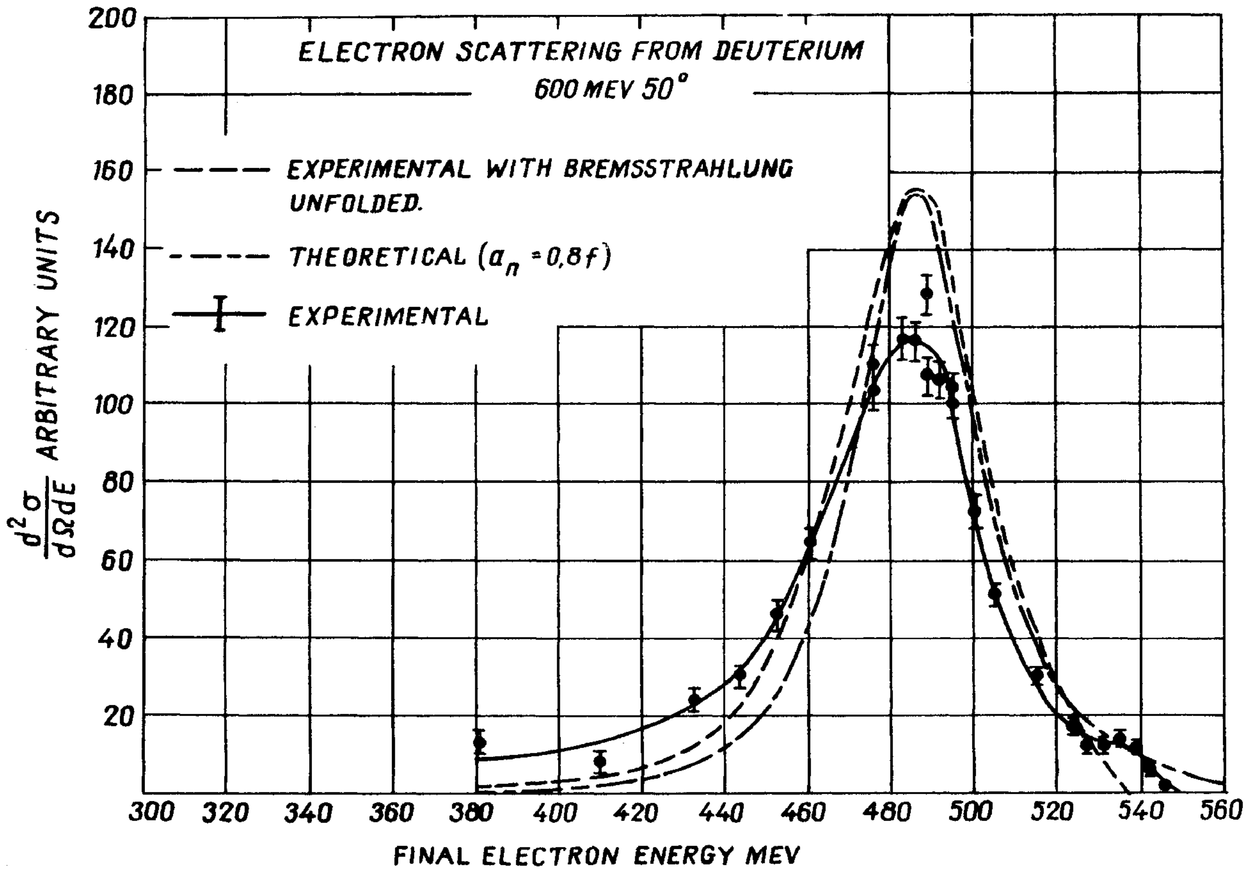

Fig. 6.

at 600 MeV and 50° where now the corrected experimental curve is definitely wider than given by the simple theory. The wider experimental curves were also found previous-

3176 ly and were commented upon in references [3,5,11]

A satisfactory explanation of these small differences has not yet been definitely assigned although recent work by Durand[8] with an improved theory shows that the interaction of proton and neutron in the final state may be responsible for part of the discrepancies. One might think that a possible cause of the difference could be the substitution by Yearian and Hofstadter of the 4-q into the calculations of Jankus in place of the latter's 3-q. Durand[8] has now shown that this modification is correct and is accurate at the peak but it is slightly incorrect on the sides of the peaks in Fig. 5 and 6. R.Herman and the author[12] have recently made calculations with and improved value of q suggested by Durand. This value may be substituted in the Jankus formula in place of his 3-q. The Durand-q is given by:

$$q_D = \frac{1}{E_D} s \qquad (4)$$

when s is the 3-q value and

$$E_D = \left[1 + \frac{E_o}{M} - \frac{E}{M} - \frac{\epsilon}{M} - \frac{E_o E}{M^2} \sin^2 \vartheta/2\right]^{1/2} \qquad (5)$$

and E_o and E are the energies of the incident and scattered electron respectively, ϵ is the deuteron's binding energy and M the mass of a nucleon. When this is done the results (Fig. 7) are very similar to the originally modified Jankus formula although there is an improvement in the neighbourhood of the low energy tail. Thus the additional width of the experimental curves probably cannot be ascribed to this cause and may be due to final state interactions or mesonexchange effects. However the 3176 approximation corresponding to Eq. (4) still needs further improvement.

In Sobottka s work the peak of the curve, rather than the area, has been the actual quantity compared with theory. As has been pointed out previously[3,10] this choice makes the interpretation of the experimental result less sensitive to meson-exchange effects. However, a dependence of the peak value on i) the choice of the n-p potential, ii) the effects of interaction in the final state and iii) the D-state contribution is not eliminated.

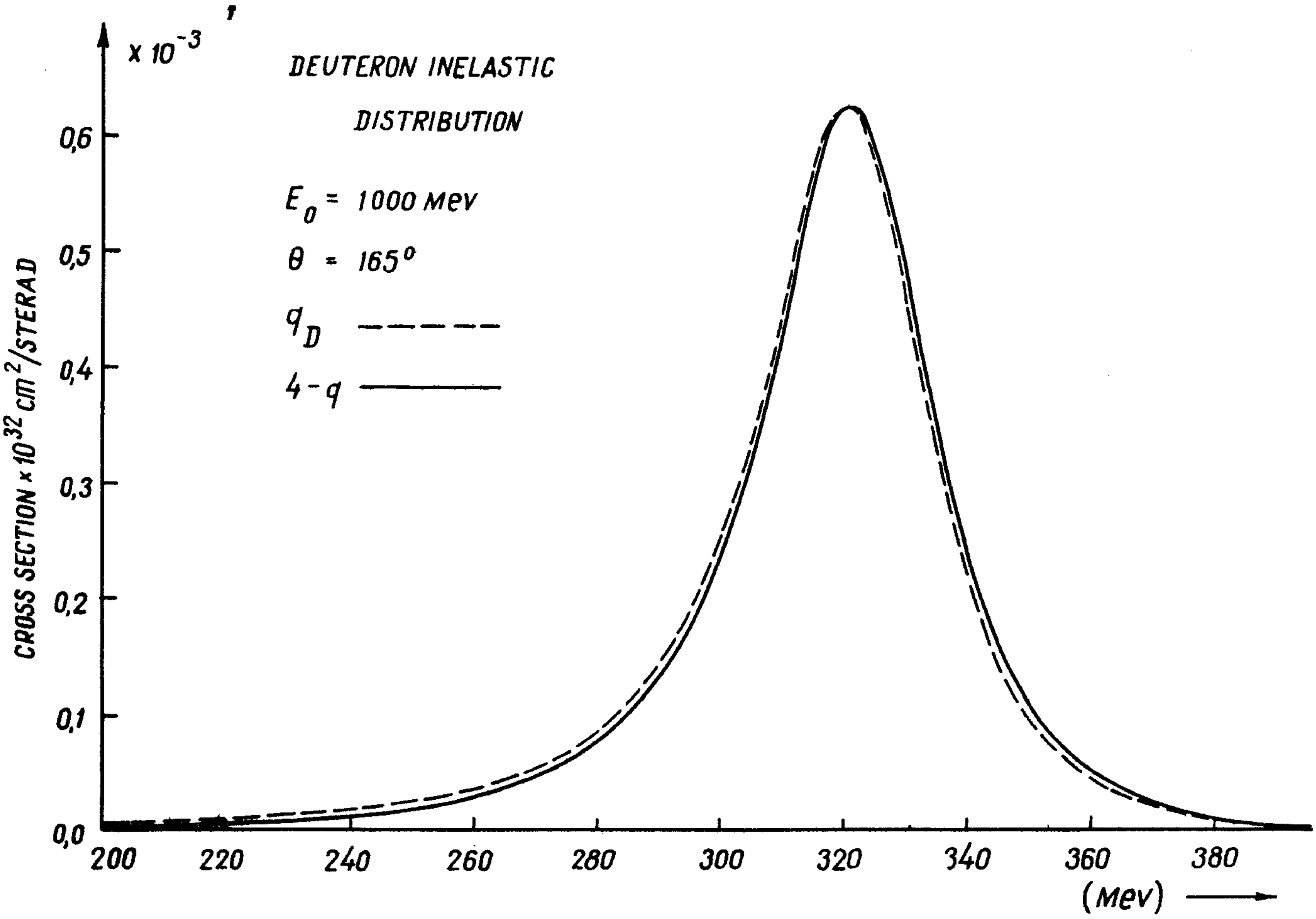

Fig.7.

Using the peak value of the cross-section Sobottka has employed the Goldberg[13] impulse-approximation to interpret the results. This theory provides results at larger q-values very similar to those of the modified

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Jankus theory at the peak of the inelastic deuteron distribution. The results for F_{2N} are shown in Fig. 8. To obtain those results Sobottka assumed $F_{1N} = 0$ and thus the values of F_{2N} (or F_{2N}^2) are obtained directly since the latter is the only unknown, aside from the

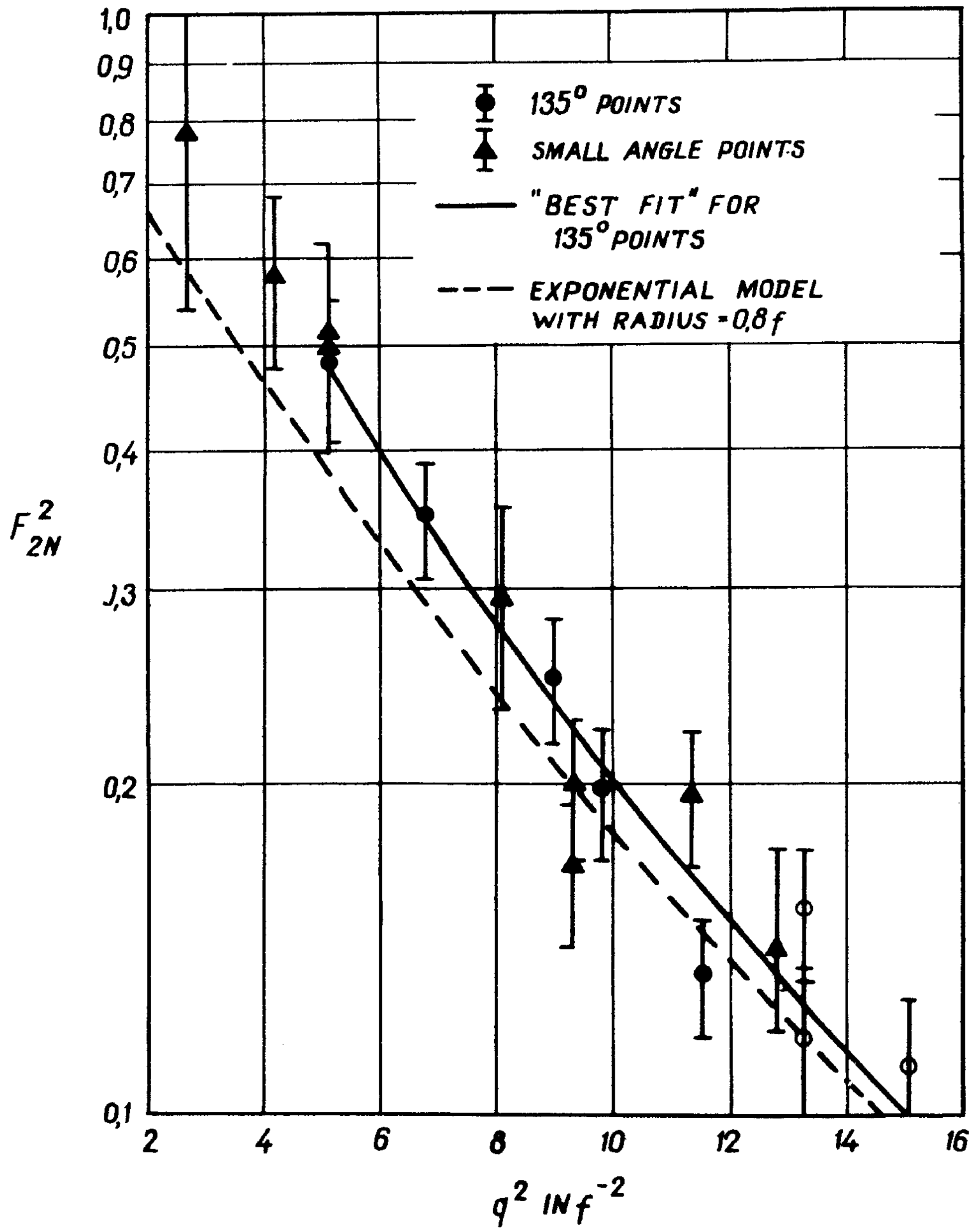

Fig.8.

theoretical uncertainties mentioned above. In the figure one also sees the "proton" line corresponding to the proton F^2 values given by an exponential or a cut off model with rms radius about 0.80×10^{-13} cm. The

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Sobottka results are considerably more accurate than the earlier results of Yearian and Hofstadter[3] which are shown in Fig. 9 for comparison. All the results of both sets of measurements are in good agreement with each other within experimental error.

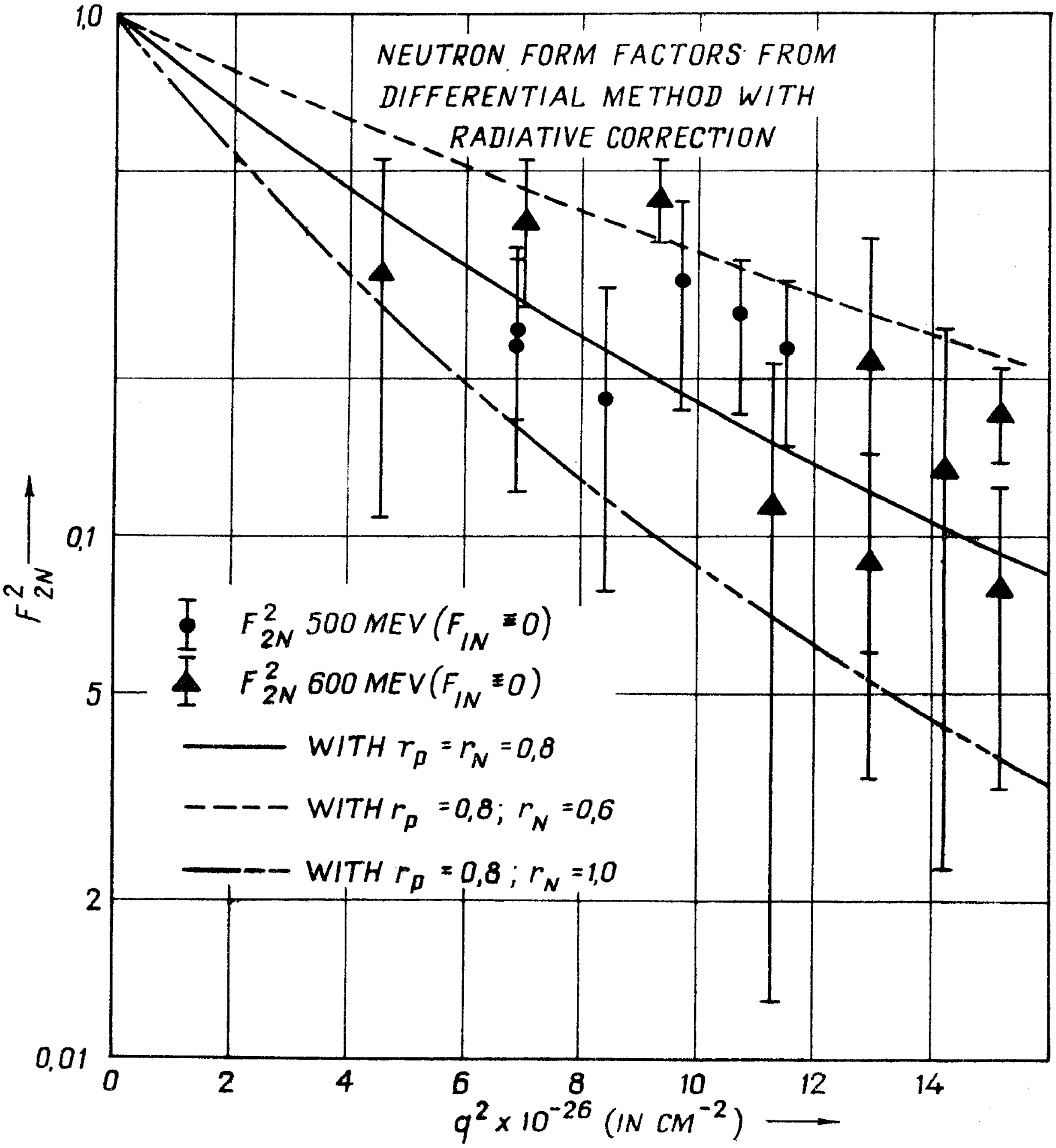

Fig.9.

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Now let us relax the condition that $F_{1N} = 0$. By choosing to make measurements on the deuteron peak at small angles and at large angles for the same value of

q^2 we may hope to observe the departures of F_{1N} from zero. This procedure is already indicated by the different curves of Fig. 4. Notice that in this figure the primary intersections are taken to be at $F_{1N} = 0$ which seems to be approximately true. Sobottka s investigation was carried out in a closely related manner, now to be described. He used Goldberg s impulse-approximation theory to find ${F_{2N}}^2$ when $F_{1N} = 0$. He then used the Rosenbluth formulation of the neutron cross-section to find an F_{1N} which gave equivalent experimental results. In this procedure one ignores a small interference term between neutron and proton form factors.

Let us define F^o_{2N} to be the neutron's magnetic form factor when F^o_{2N} is taken to be zero. In this case the ratios of the measurements of ${F_{2N}}^2$ at small angles and at large angles should be unity since they are measured at the same value of q. If the experimental ratio indicates a value different from unity, say R, where:

$$R = \frac{\left(F^o_{2N}\right)^2 \text{(for small angle points)}}{\left(F^o_{2N}\right)^2 \text{ (for large angle points)}} \text{ at the same q} \quad (6)$$

and if:

$$\alpha = \frac{F_{1N}}{F_{2N}} \quad (7)$$

then it is easy to show that the Rosenbluth equation gives:

$$R = \frac{\left\{\alpha + \frac{\hbar^2 q^2}{4M^2c^2}\left[2(\alpha+K)^2\tan^2\vartheta_s/2+K^2\right]\right\}\left[2\tan^2\vartheta_\ell/2+1\right]}{\left\{\alpha^2 + \frac{\hbar^2 q^2}{4M^2c^2}\left[2(\alpha+K)^2\tan^2\vartheta_\ell/2+K^2\right]\right\}\left[2\tan^2\vartheta_s/2+1\right]} \quad (8)$$

ϑ_s and ϑ_ℓ are the small and large values of ϑ. 3176

In the Sobottka experiments ϑ_ℓ was taken as 135^o and ϑ_S as 60^o, 75^o or 90^o. In such cases a rough diagram of R, plotted against F_1/F_2 is shown in Fig. 10. From the measured cross-section it is possible to find R and compare it with unity. This procedure amounts to looking for the intersections shown in Fig. 4. In that figure we have already pointed out that the intersections are taken

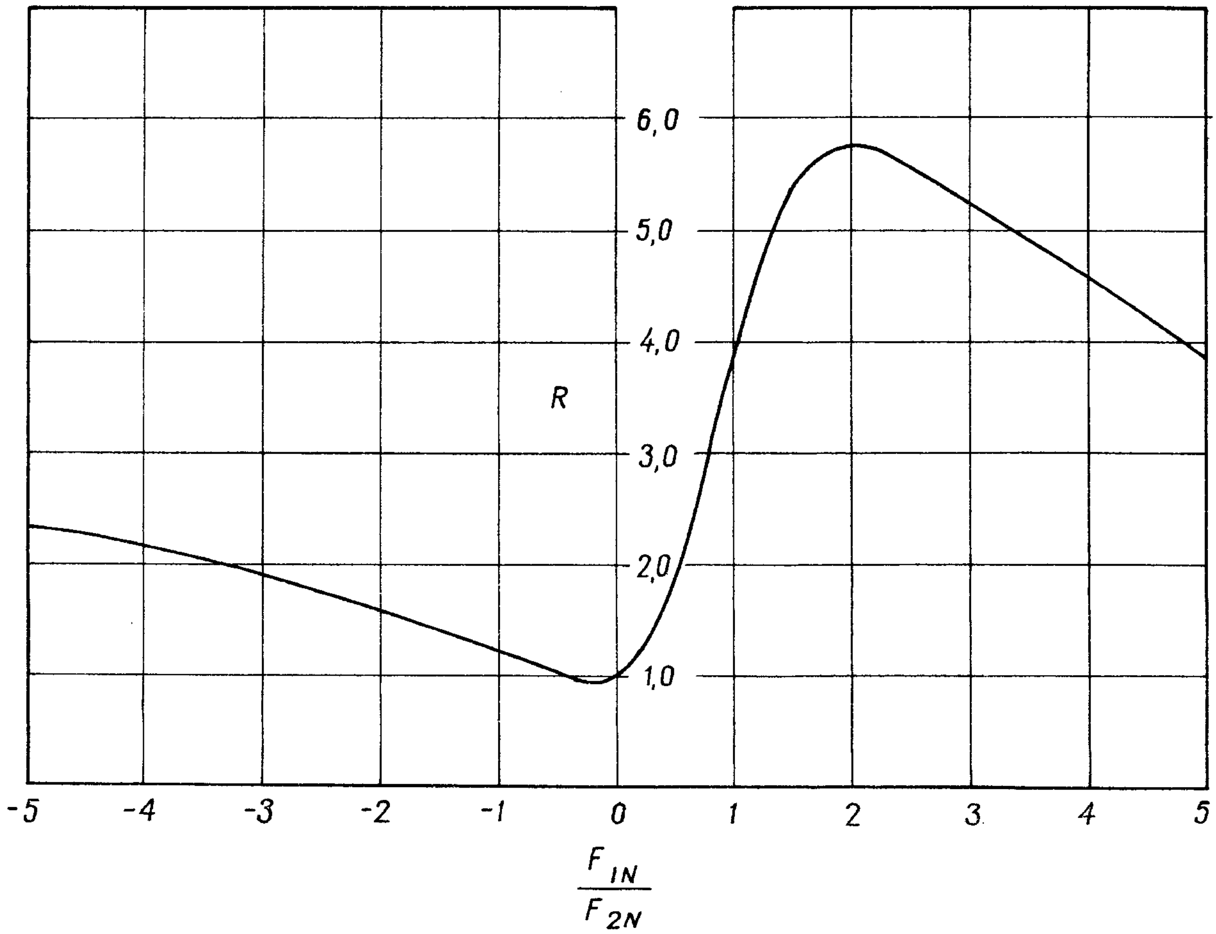

Fig. 10

at $F_1 = 0$. In this way Sobottka has been able to find values of α from the measured values of R. His results are shown in Fig. 11. Two types of points are shown corresponding to a positive ratio (solid circles) F_1/F_2 or a negative ratio (solid triangles). The sign cannot be determined. Both Figs. 4 and 10 show that experiment determines smaller positive limits of F_{1N} than negative ones. It may be seen that if the ratio is positive F_{1N} is very close to zero even at large values of q^2 (q^2 =

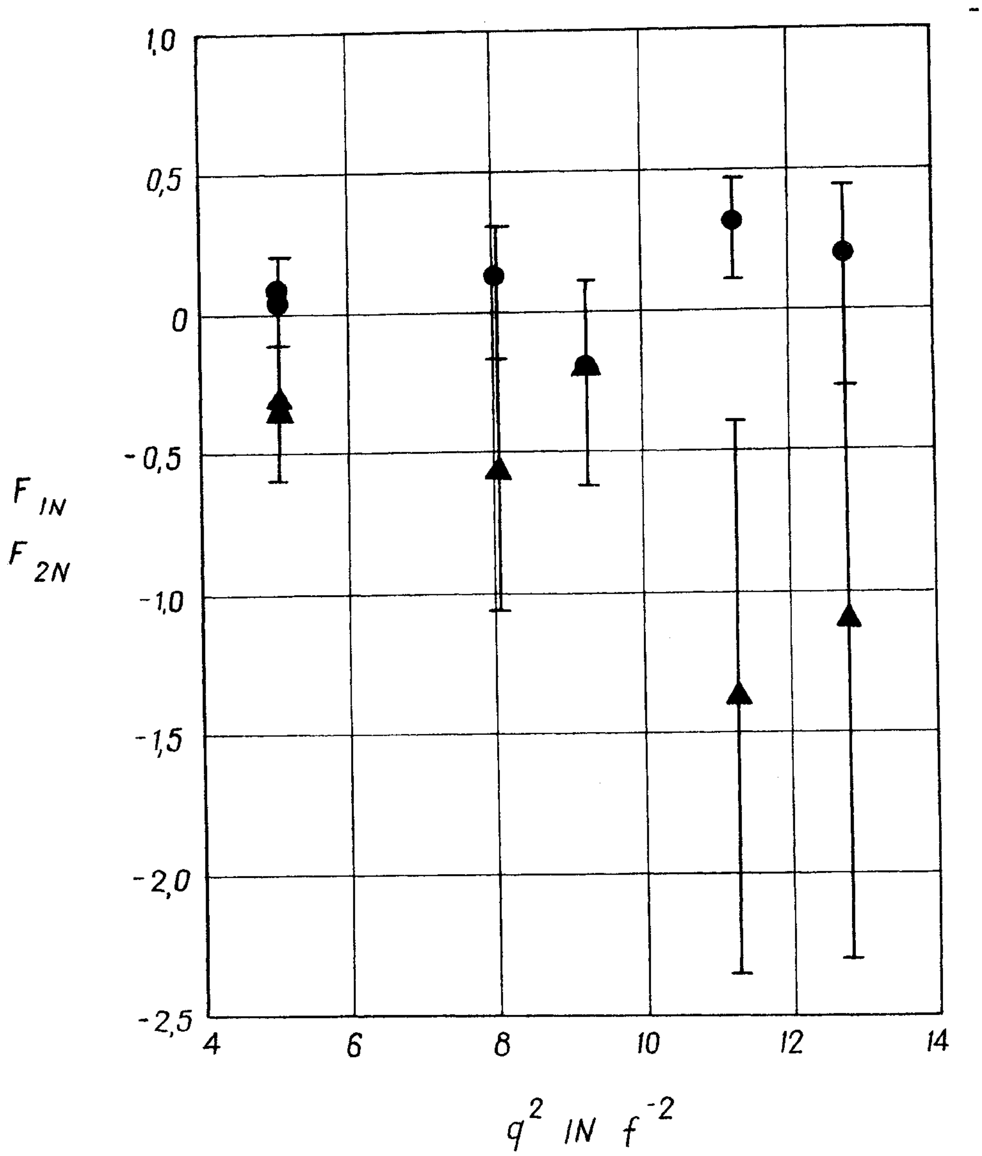

Fig.11.

= 12.5×10^{26} cm^{-2}). If F_{1N} is negative the value of F_{1N}/F_{2N} may be as large as unity or even larger but the errors are also quite large as we see from Figs.4 or 11. In some cases the experimental errors are joined together as at q^2 = 8 or 9.3. This is because at the limits of

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experimental error the solution for α becomes imaginary.

Now it is clear that this experiment still leaves much to be desired but even so it is a very difficult one. Nevertheless it points to a definite way obtaining better data on F_{1N}. The conclusion is still that the experiments are consistent with $F_{1N} = 0$ but the limits of error are such that $-2.5 < \frac{F_{1N}}{F_{2N}} < 0.5$ for $5.1 < q^2 < 12.8 \times 10^{26}\ cm^{-2}$.
The earlier limits of McIntyre[14] and Schiff[15] give smaller values for this ratio but do not reach such large values of q^2.

B i b l i o g r a p h y

1. R. Karplus (private communication).
2. E.E. Chambers and R. Hofstadter, Phys. Rev., 105, 1454 (1956).
3. R. Hofstadter, F. Bumiller and M.R. Yearian, Rev. Mod. Phys., 30, 482, (1958).
4. R. Hofstadter, Nuovo Cimento, Serie X, 12, 63, (1959).
5. M.R. Yearian and R. Hofstadter, Phys.Rev., 111, 934 (1958).
6. V.Z. Jankus, Phys.Rev., 102, 1586 (1956).
7. R. Blankenbecler, Phys. Rev., 111, 1684 (1958).
8. L. Durand III (to be published).

3176

9. R. Hofstadter, Annual Review of Nuclear Science, VII, 231 (I954).

I0. S. Drell, (1958) Ann. Int. Conf. on High Energy Physics at CERN.
W.K.H. Panofsky, Ann.Int.Conf.on High Energy Physics at CERN.

11. M.R. Yearian and R. Hofstadter, Phys.Rev., 110, 552 (1958).

12. R. Herman and R. Hofstadter (to be published).

13. A. Goldberg, Phys.Rev., 112, 618 (I958).

14. J.A. McIntyre and G.R. Burleson, Phys.Rev., 112, 2077 (I958) and other references given in this paper.

15. L.I. Schiff, Rev.Mod.Phys., 30, 462 (I958).