INITIAL ESTIMATE OF FRINGE FIELD EFFECTS IN HL-LHC USING FREQUENCY MAP ANALYSIS

S. Jones*, D. Newton, A. Wolski, University of Liverpool, UK and The Cockcroft Institute, UK

Abstract

The planned High Luminosity upgrade to the LHC will require stronger focusing of the beam in the interaction regions. To achieve this, the inner triplet quadrupoles will be replaced with new magnets having larger gradient and aperture. In this new focusing regime the quadrupole fringe fields are expected to have a greater effect on the beam dynamics, due to their large aperture, as compared to the nominal LHC. In this preliminary study, simplified models are used in a tracking code to assess the impact of the fringe fields on the dynamics using frequency map analysis.

INTRODUCTION

The High Luminosity upgrade to the Large Hadron Collider (HL-LHC) aims for an increase in luminosity by an order of magnitude. This will be achieved by decreasing the β function (and hence the beamsize) at the interaction points. The inner triplet (IT) quadrupoles are to be replaced with stronger magnets to achieve greater focusing, but they will also require a large physical aperture to accommodate the beam which will have a large β function at these points [1]. The large aperture and increased strength of these magnets means the fringe field may have an detrimental effect on the dynamic aperture of the HL-LHC lattice. An initial estimate of the impact of the fringe fields could be determined by Frequency Map Analysis (FMA) [2]. This paper introduces a technique used to calculate a semi-analytical description of the fringe fields, and describes how the HL-LHC lattice was modified to include the fringe fields whilst keeping the linear dynamics as close as possible to the nominal lattice. The results of a preliminary FMA are presented, and we make an initial assessment of the possible impact of the fringe fields on the dynamic aperture. Further work required to fully assess the impact of the inner triplet fringe fields is discussed.

IT QUADRUPOLE TRANSFER MAPS

Realistic magnetic field maps have been calculated numerically for the IT quadrupoles fringe field region [3]. By taking the radial component of this numerical field, on the surface of a cylinder, an analytical description of the magnetic field at all interior points can be calculated using the method of generalised gradients [4]. This approach has several advantages; any numerical errors on the cylinder surface are exponentially damped within the cylindrical volume. Additionally the analytic field is consistent with Maxwell's equations.

The generalised gradients for the IT quadrupole magnets were calculated giving an analytic description of the fringe field region. This description was used with the Wu-Forest-Robin symplectic integrator [5] to generate a transfer map, for the fringe field, in the form of a truncated Taylor map to sixth order in the dynamical variables. Fringe field Taylor maps representing either side of the magnet (entrance and exit) can be produced by reflection of the magnetic data.

In order to generate a Taylor map to describe the IT quadrupole body, the fringe field description at the boundary with the body can be extended, to give a Taylor map describing a hard edge magnet of arbitrary length. Therefore a complete set of unique Taylor maps describing entrance and exit fringe fields as well as a quadrupole body of any length can be calculated.

The quadrupole strength, k_1, is proportional to the on-axis, quadrupole generalised gradient, $C_{2,s}^{[0]}$ [4]:

$$k_1 = \frac{1}{B\rho}\left.\frac{\partial B_y}{\partial x}\right|_{x,y=0} = \frac{2}{B\rho}C_{2,s}^{[0]}(z).$$

It is possible to calculate the integrated quadrupole strength across any section of the magnet (for each Taylor map above) by numerically integrating the generalised gradient.

The numerical field data can be scaled to give a Taylor map with a desired integrated quadrupole strength. The integrated quadrupole strength of the hard edge can be determined by the length of the map also. Using these methods to scale the fringe field and hard edge maps appropriately will allow matching of the lattice tunes.

INTEGRATING THE TAYLOR MAPS INTO THE HL-LHC LATTICE

For the work described in this paper, a MAD-X description of the sLHC v3.1b lattice was used with a set of random multipole errors applied. The lattice is described with thin-lens multipole kicks and drifts, with each magnet split into a set of thin lenses. The lattice was imported into SAMM (Simple Accelerator Modelling in Matlab) [6], a code under development at Liverpool University, which allowed for easier modification of the lattice and implementation of the Taylor map.

The set of multipoles describing the innermost IT quadrupole magnets either side of IP1 (MQXC.1L1/1R1) were chosen as the sections to be replaced with the Taylor maps. Two variations of the lattice were created. First, a "hard edge" model where each multipole section is replaced with a single Taylor map representing a hard edge quadrupole with a quadrupole gradient and length equal to that of the multipole kicks. And second, a "fringe field" model where each section has three Taylor maps, one each to represent the entrance and exit fringe fields and one to represent the main

* sam.jones@liv.ac.uk

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body of the magnet, with a quadrupole gradient matched to the multipoles, but with a shortened length to account for the integrated quadrupole strength of the fringe fields. Drifts with negative length were also added in both cases to keep the length of each lattice consistent with the nominal case. It was found that replacing the multipoles with these matched Taylor maps and negative drifts did not affect the linear dynamics of the lattice significantly.

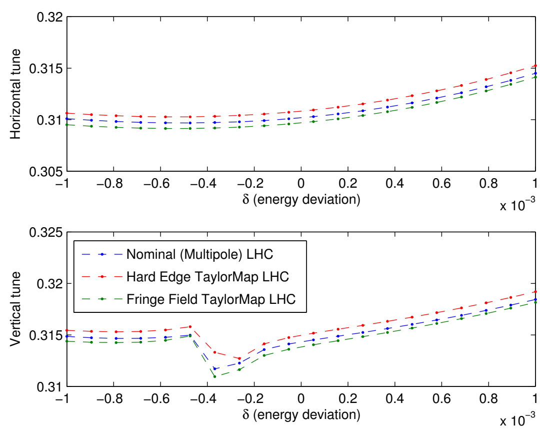

Figure 1: Chromaticity plots for each model of HL-LHC lattice.

Fig. 1 there is a reasonable agreement in the tunes and chromaticity; despite having the same integrated quadrupole strength the hard edge and fringe field cases have slightly different tunes, but the discrepancy is at the level of 10^{-4}. In the vertical tune space it appears that a resonance is crossed around $\delta = -0.4 \times 10^{-3}$.

The Numerical Analysis of Fundamental Frequencies (NAFF) algorithm can be used to calculate the tune to a greater precision than a traditional discrete Fourier transform [7]. The tune can then be calculated accurately with fewer turns, which means that less computationally-expensive tracking is needed.

Further improvement can be made to the NAFF when used in conjunction with a Hanning Window, which minimizes side-lobes in the Fourier spectrum resulting in a more stable and accurate calculation of the tune.

Fig. 2 shows how the tune changes for particles with differing initial horizontal offset from the closed orbit. As before, there is a reasonable agreement in tune between the three cases. Particles that are lost over 128 turns are not displayed; in this case particle loss in the region crossing a sextupole resonance around 8σ, as well as the unstable region beyond 12σ.

FREQUENCY MAP ANALYSIS

The frequency map analysis was based on tracking over 128 turns, this provides a balance between accuracy and speed of the calculation of the tune variation. The tune variation was calculated by the differences between the tune for the first 64 turns and the second 64 turns. These tune

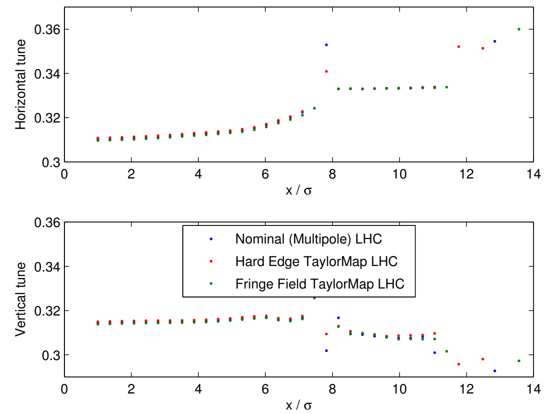

Figure 2: Tune as a function of betatron amplitude for each model of HL-LHC lattice.

variations can then be combined in quadrature to give an overall measure of the tune diffusion rate:

$$D = \log\left(\sqrt{\Delta\nu_x^2 + \Delta\nu_y^2}\right).$$

The diffusion rate, D, is indicated by a colour scale in Figs 3 - 6.

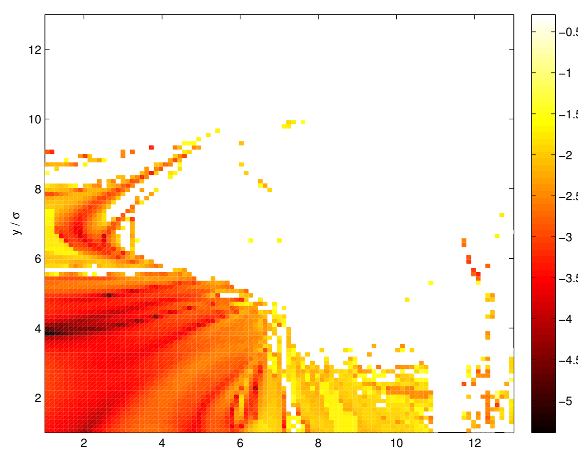

Figure 3: Amplitude map for the nominal (Multipole) HL-LHC.

Figures 3 and 4 are the amplitude maps for the nominal and fringe field lattices respectively. The particles were launched on a grid between one to 13 σ horizontal and vertical deviations away from the closed orbit, with no deviation from the closed orbit momenta. Particles that did not survive the tracking, and tended to infinity, are also shown in white on the plot. The two amplitude plots have minor differences but largely similar features.

Figures 5 and 6 are the frequency maps for the nominal and fringe field lattices respectively. Similar to the amplitude maps, there are only small differences between the lattices.

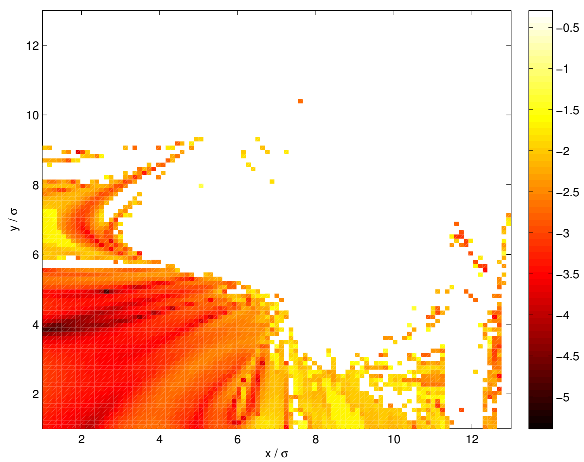

Figure 4: Amplitude map for the fringe field (Taylor Map) HL-LHC.

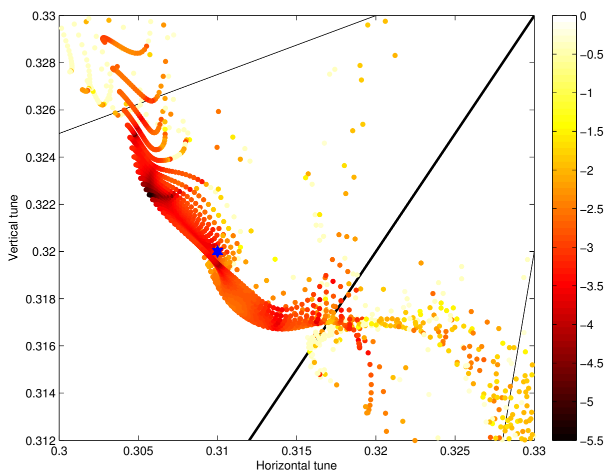

Figure 5: Frequency map for the nominal (Multipole) HL-LHC. The working point of the machine is indicated by the blue marker.

Given the similarities between the above plots, this initial study therefore suggests that the fringe fields do not have a significant detrimental effect on the HL-LHC beam.

CONCLUSION AND FUTURE PLANS

Beginning with a fringe field, modelled using an electromagnetic code, an accurate non-linear Taylor map can be constructed. By applying the correct scaling a Taylor map description can constructed that is consistent with an existing beamline element which has no nonlinear components, such as the thin-lens multipole kick in this version of the HL-LHC lattice. The nonlinear effects of the fringe field can be estimated by comparing the results of the frequency map analysis for each case. With this initial framework set up, we will be able to make various improvements with the aim of a more comprehensive study of the low-β^* quadrupole fringe

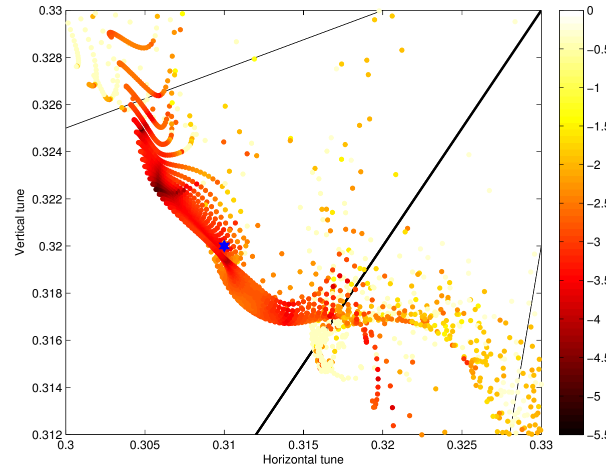

Figure 6: Frequency map for the fringe field (Taylor Map) HL-LHC. The working point of the machine is indicated by the blue marker.

fields within the HL-LHC, undertaken with the Sixtrack tracking code [8].

Each of the IT magnets could be represented by its unique set combination of hard edge and fringe field Taylor maps. The models could then be extended to include the fringe field effects of each of the IT magnets combined, and to investigate if there is a larger effect on the frequency map than observed here.

The Taylor maps are calculated with the use of a differential algebra (DA) code developed within the Cockcroft Institute. The code is being improved to handle DA series up to any order; this improvement will allow for more accurate Taylor maps and magnetic field maps, as both descriptions use this DA code.

A finite-order symplectic transfer map generated from the generalised gradients would be preferable to the Taylor map; this is something we plan to implement using a symplectic integrator [9] [10]. Symplectic maps would mean that tracking could be done for many turns using the same framework, allowing a long-term particle survival study to compare with frequency map analysis.

A symplectic thin lens model derived from the generalised gradients would allow us to move this technique from the SAMM code to the Sixtrack tracking code. An implementation of our maps within Sixtrack would allow fringe field effects to be studied using well-established protocols for LHC dynamic aperture calculations [8].

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