A SPACE-CHARGE COMPATIBLE "TOMOGRAPHY" OF BEAM PHASE-SPACE DISTRIBUTIONS

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Abstract

The well-known 3-gradient method allows accessing to a beam RMS emittance and Twiss parameters at a position *A* by measuring its rms size at a downstream position *B* with at least 3 different transport conditions from *A* to *B*. We suggest extending this method to access to a beam phase-space distribution model at *A* from beam profiles measured at *B*. We propose to use an iterative method which consists in:

- defining a parametric model describing the beam distribution in 4D transverse phase-space at a position *A*,
- adjusting iteratively the model parameters by minimizing the difference between beam profiles measured at *B* and those obtained by transporting the beam generated according to the model with TraceWIN code from *A* to *B*.

This method allows taking into account space-charge and other transport non-linearities.

INTRODUCTION

A beam is fully characterised by its distribution in phase-space. Measuring this distribution is then a key-point in order, either to give correct initial conditions to beam simulation codes, or/and to validate these codes.

The **direct measurement** of this distribution, consisting in measuring the angular distribution of beam samples at a given position, fully interceptive and room and time consuming is very complicated with high current or high energy beam.

A solution consists in measuring the beam profile at a **point *B*** for various transports from an **upstream point *A*** where the beam phase-space distribution can be reconstructed.

In case of **linear forces**, this reconstruction uses **tomography** technics, needing a large number of regularly varying beam transport conditions, and assuming that:

- the beam profile is a projection of the phase space distribution along angular direction and
- the beam transport is a simple linear transformation of beam phase-space distribution [1].

Nevertheless, in case of **non-linear forces** (especially with space-charge), the transformation depends on the unknown initial distribution. Tomography technics cannot be directly applied.

METHOD PRINCIPLE

General Algorithm

The solution we proposed is the following:

- Step 0: make **beam profile measurements** Pm_i ($1 \leq i \leq N$) at a point *B* of an accelerator in *N* various transport conditions from an upstream point *A*.
- Step 1: assume that beam phase-space **distribution description** at *A*, depends on *n* parameters: π_i.
- Step 2: generate and **transport** this beam with a transport code (TraceWIN [2]) to *B*, in the *N* transport conditions.
- Step 3: compute a **distance** *D* between the simulated beam profiles Ps_i and the measured ones, and vary iteratively the beam model parameters and **iterate** step 1 and 2 until *D* is minimum.

Each step is described below.

Step 0: Experiment

At step 0, usually, the beam profiles Pm_i are measured at position *B* for *N* various transport conditions between points *A* and *B*.

Step 1: Beam Parametric Model

In a **continuous transport channel**, the phase-space distribution of a perfectly matched* beam is a function of the motion Hamiltonian:

$$f(\vec{r},\vec{r}') = F(H(\vec{r},\vec{r}')),$$

We modelled the phase-space distribution by a **2-parameter** function of *H*:

$$f(\vec{r},\vec{r}') = F_0 \exp\left(-\left(\frac{H(\vec{r},\vec{r}')}{\beta_1}\right)^{n_2}\right).$$

with, the Hamiltonian modelled by a **4-parameter** function:

$$H(\vec{r},\vec{r}',\pi_{1-4}) = \frac{1}{2}r'^2 + \pi_1^2\left[\frac{r^2}{2} - 2\pi_3\int_0^r \frac{1}{v}\int_0^v u\exp\left(-\left(\frac{u}{\pi_2}\right)^{\pi_4}\right)du\,dv\right]$$

and:

$$r^2 = x^2 + y^2,\ r'^2 = x'^2 + y'^2,\ \text{and}\ x' = \frac{dx}{ds}$$

In a **regular (~periodic) transport channel**, one assumes here that a perfectly matched beam is a linear deformation of the perfectly matched beam in its equivalent# continuous focusing channel. The couplings in (x, x') and (y, y') phase-spaces, is then modelled by a **4-parameter** transformation:

* a beam is said ***perfectly matched*** to a continuous focusing channel if its phase-space distribution is invariant along the channel.
\# two regular or periodic transport channels are said ***equivalent*** if they have the same phase-advances per unit length.

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$$\begin{pmatrix} X \\ X' \end{pmatrix} = \begin{pmatrix} 1-\frac{L_x}{f_x} & L_x \\ -\frac{1}{f_x} & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}, \begin{pmatrix} Y \\ Y' \end{pmatrix} = \begin{pmatrix} 1-\frac{L_y}{f_y} & L_y \\ -\frac{1}{f_y} & 1 \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}$$

The model is based on the following assumptions:

- the x and y coordinates, as well as the velocities *x'* and *y'* are supposed to be **axisymmetric**,
- the kinetic energy *T(r')* is obtained assuming a **paraxial approximation** for velocities[!].

The model uses then **10 different parameters**.

Step 2: Beam Transports

The beam transport is simulated with the TraceWIN code [2].

The first stage consists in **generating a multiparticle distribution** from the parametric function of step 1.

The second stage consists in **transporting this beam from A to B** in the same N transport conditions as in step 0. A batch version of TraceWIN with PICNIC3D space-charge routine [3] is used and encapsulated in a Matlab code for an automatic process. The associated beam profiles Ps_i at B are computed and stored.

Step 3: Profile Comparison, Iterative Process

The measured profiles Pm_i and the simulated ones P_{si} are compared. The chosen **distance D is the sum of the quadratic deviation of these normalized profiles**.

The algorithm minimizing D is the Matlab fminsearch function, iterating steps 1 to 3 and adjusting the parameters until a minimum is found.

As a profile is obtained from a N_p multiparticle distributions, it carries a statistical noise. Even with a no noise measurement, the minimized D would not converge to 0 but to $D_{stat} = \sqrt{1/N_p}$. If the model does not describe perfectly the distribution, an error contribution D_{model} is added.

It is necessary to underline that the number of experiments N needs to be large enough in order to have enough information on the initial distribution.

SIMULATION RESULTS

Before applying this technic to a set of experimental profiles, it has to be **benchmarked** by processing a set of data obtained on a "**numerical experiments**". Three types of input distributions have been used:

- (a) The first is generated from the model with known values of the parameters.
- (b) The second is the result of simulations of an IFMIF-EVEDA distribution at an early stage of development.
- (c) The third is the result of simulations of the entrance of the HEBT for the IFMIF-EVEDA accelerator for a more recent version.

! the transverse velocities are small with respect to the longitudinal one, which is considered to be constant.

Two kinds of benchmarks are done:

- the model is directly fitted to the (b) and (c) distributions to **test the accuracy of the model**.
- a numerical experiment is simulated with (a) distribution, and its processing is made following the steps described previously to **test if the process converges to the expected initial values**.

Model: Benchmark with Distribution (b)

The effect of space charge on equilibrium distribution is to transform the elliptic shape to a more square shaped one [4]. The (b) distribution showed in Fig. 1 is a good exercise to test the model as it has a regular shape, but yet is affected by space charge as it does not have a pure elliptic shape.

The best fit and the deviation with the original distribution are showed in Fig. 1. The final distances are $D_x = \sqrt{1/N_p}$ and $D_y \approx 1.58 \cdot 10^{-3}$, while the statistical noise (10^6 particles) is $D_{stat} \approx 1.4 \cdot 10^{-3}$. The error coming from the model is small, and we can see in lower part of Fig. 1 that there is no particular pattern in the deviation.

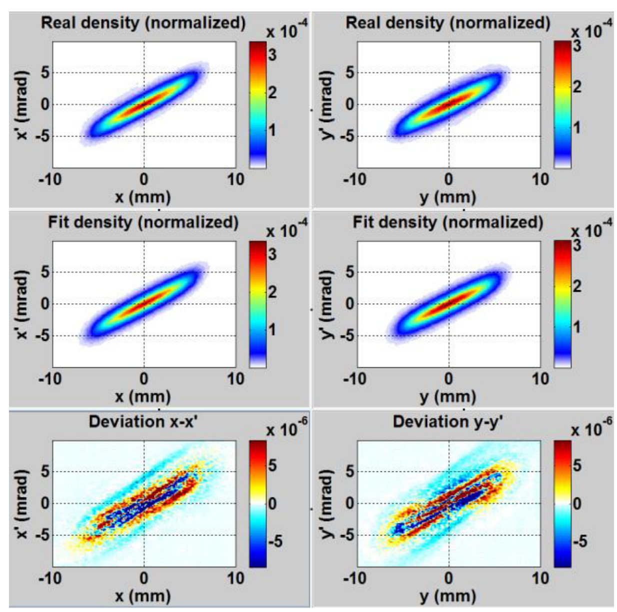

Figure 1: Phase-space distributions in the *(x,x')* (left) and *(y,y')* (right) planes, fit for the (b) distribution

Model: Benchmark with Distribution (c)

For this distribution, the shape has a "butterfly" pattern, (Fig. 2). The final distances are $D_x = 2.77\ 10^{-3}$ and $D_y = 2.73\ 10^{-3}$, which shows that the error coming from the model cannot be assumed to be small with respect to the statistical noise. We observe on Fig. 2 that the global shape of the distribution is obtained, but the model is not able to reproduce the "butterfly" shape of the distribution.

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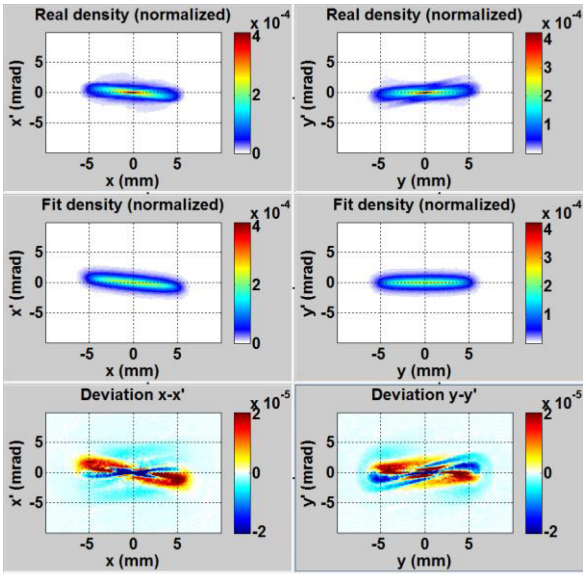

Figure 2: Phase-space distributions in the *(x,x')* (left) and *(y,y')* (right) planes, fit for the (c) distribution

Another approach is to assume that rotation term in the expression of the kinetic energy in the Hamiltonian model is then not negligible : $T(r,r') = r'^2 + \left(P_\theta / r\right)^2$.

We can than introduce a bias in the distribution of P_θ as a function of r introducing **two more parameters** to the model. This adds a correlation between *x'* and *y'*.

The model fist better the distribution, as seen in Fig. 3. The distances become $D_x = 1.77\ 10^{-3}$ and $D_y = 1.95\ 10^{-3}$ and the error due to the model is reduced. Unfortunately, increasing the number of parameters reduces the algorithm stability.

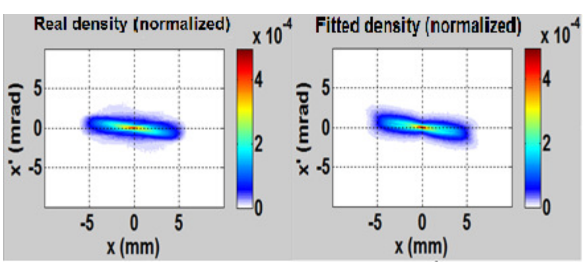

Figure 3: Phase-space distributions in *(x,x')* plane and fit for the (c) distribution assuming correlation between *x'* and *y'*.

Tomography: Numerical Experiment with the Model (a)

Finally, we process a numerical experiment using a distribution at point *A* described by the model. In step 1, the initial parameters used for the minimisation are changed randomly in order to simulate an error with respect to what was expected in the simulations. 4 transports are used in order to have the projection on the main axis of the distribution and two in between.

In these conditions, even if the distance between the measured and simulated profiles is small, the parameters did not perfectly converge to their initial values (Fig. 4). By adding other transport conditions (and equivalent projection angles), the results are closer and closer to the expected one. However, the convergence is still dependent on the initial conditions. A criterion has to be determined in order to generate more adapted experimental conditions, including space-charge.

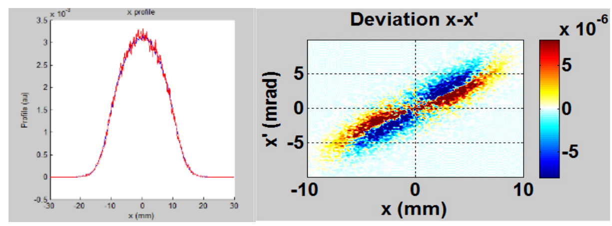

Figure 4: results obtained if the experiment does not give enough information. *Left*: comparison of profiles obtained from measurements and simulations after minimisation. *Right*: deviation between the *(x,x')* input distribution and the reconstructed one.

CONCLUSION

A physic-based parametric model was developed in order to represent the distribution in phase space. This model was then used in a minimisation method to reconstruct in a distribution in an upstream point A, from profiles measurements in a downstream point B.

The model was showed to be able to model distributions coming from de IFMIF-EVEDA simulations, with some limitations ("butterfly" pattern). Some more work needs to be done also to extend the model and see if it can represent the "butterfly" pattern seen in the (c) distribution.

The model has also to be simplified in order to reduce the number of parameters, for which some of them appears to be correlated, in order to improve the stability of the code.

Applied in the complete method, this model is able to reconstruct the distribution. The main limitation is the number of experiment sets needed, to have enough information for the reconstruction.

REFERENCES

[1] K. M. Hock, A. Wolski, *Tomographic reconstruction of the full 4D transverse phase space*, Nuclear Instruments and Methods in Physics Research A,vol. 726, p. 8-16 (2013).

[2] R. Duperrier, N. Pichoff, and D. Uriot, Proceedings of the ICCS, Amsterdam (2002).

[3] N. Pichoff et al., *Simulation Results with an Alternate 3D Space Charge Routine PICNIC*, Linac98, Chicago.

[4] N. Pichoff et al., Transverse Profile equilibrium in a space-charge dominated beam, APAC98, Tsukuba.

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