HIGH-POWER, HIGH-EFFICIENCY FELs

Andrew M. Sessler

Accelerator and Fusion Research Division, LBL, Berkeley, CA, USA*)

ABSTRACT

High power, high efficiency FELs require tapering, as the particles loose energy, so as to maintain resonance between the electromagnetic wave and the particles. They also require focusing of the particles (usually done with curved pole faces) and focusing of the electromagnetic wave (i.e. optical guiding). In addition, one must avoid transverse beam instabilities (primarily resistive wall) and longitudinal instabilities (i.e. sidebands).

1. INTRODUCTION

Free Electron Lasers have attracted a great deal of attention because they operate across a wide range of the electromagnetic spectrum, because they are easily tunable, and because they are capable of high power operation. The range of operating wavelengths has already been demonstrated to a considerable degree, as has the tunability. Considerable effort (many hundreds of millions of dollars) has been put into the development of high power lasers, with resultant significant theoretical and technological advance, but only small experimental advance.

In this article we shall focus upon the high power operation of FELs. We shall, almost exclusively, as is appropriate for the literature, concentrate upon the theoretical concepts required for high power operation of an FEL. We shall say almost nothing about technological and experimental matters.

In Section 2, we shall discuss the concept of tapering which is necessary to overcome the inevitable saturation behavior of untapered wigglers. This is the crucial idea which has led physicists to contemplate the high power operation of FELs. It has, in fact, been adequate for the first demonstration of high power operation in the microwave range.

Tapering, alone, is not adequate, in anything but the microwave range of frequencies, to make a high power FEL. One needs to focus the electrons and one needs to focus the electromagnetic wave. This is because high power inevitably (except for very long wavelengths) requires long wigglers. We turn our attention, in Section 3 to the subject of particle focusing, which I hasten to point out is not trivial, for the focusing must be designed in such a way as to not degrade the performance of the FEL. (The "obvious" methods are not acceptable.)

^{*}Work performed under the auspices of the U.S. Department of Energy by the Lawrence Berkeley Laboratory under contract No. DE-AC03-76SF00098.

In Section 4 we discuss the focusing of the electromagnetic radiation ("light"). This is far from trivial, as the wigglers we shall desire are long compared to the natural diffraction length of the light. Remarkably enough, the FEL action itself tends to focus the light and in this regard, it is appropriate to think of the FEL as an optical fiber.

In Section 5 we consider one of the things that might preclude long wigglers, and hence high power FELs; namely, transverse instabilities of the electron beam. The worst offender is the resistive wall instability, which we show can, nevertheless, be controlled.

In oscillators particularly, sidebands (i.e., longitudinal instability of the FEL), can be a problem. We very briefly touch upon this subject in Section 6. In Section 7 we mention, but only that, some of the technological problems which must be overcome in high-power operation of FELs. Finally, in Section 8 we present conclusions.

2. TAPERING

The analysis which is required to understand tapering is based upon the basic FEL Equations, first derived by Kroll, Morton, and Rosenbluth [1]. These equations (the KMR equations) which surely have been derived earlier in the School, and hence which I shall give without derivation, are:

$$\begin{split} \frac{d\gamma_{j}}{dz} &= -\frac{\omega a_{s} a_{w}}{2c} f_{B} \sin \psi_{j} ,\\ \frac{d\psi_{j}}{dz} &= k_{w} + \frac{d\varphi_{s}}{dz} - \frac{\omega}{2c\gamma_{j}^{2}} \left[1 + \frac{a_{w}^{2}}{2} \right] ,\\ \frac{da_{s}}{dz} &= f_{B} \frac{\omega_{pe}^{2} a_{w}}{k_{s} c^{2}} \left\langle \frac{\sin \psi}{\gamma} \right\rangle ,\\ \frac{d\varphi_{s}}{dz} &= f_{B} \frac{\omega_{pe}^{2} a_{w}}{a_{s} k_{s} c^{2}} \left\langle \frac{\cos \psi}{\gamma} \right\rangle . \end{split} \tag{1}$$

The equations are in a form given by Szoke, Neil and Prosnitz [2]. The subscript j refers to the j-th particle. The notation employed is given in Table 1.

In a waveguide (i.e., for microwaves) these equations are modified with the only change being an addition of the term δk_s in the $\frac{d\psi_j}{dz}$ equation and the replacement of ω_{pe}^2 by $\omega_{p'eff'}^2$. The new expressions are:

$$\delta k_s = k_s - \omega/c, \tag{2}$$

which is non-zero in a waveguide, and

$$\omega_{p/eff}^2 = \frac{4\pi Ie}{m_e c a b}, \qquad (3)$$

Table 1. Notation

Bw = peak magnetic field of a linear wiggler

 λ_w = wiggler period

$$k_w = 2\pi/\lambda_w$$

$$a_w = \frac{e B_w}{\sqrt{2} mc^2 k_w}$$

 ω = angular frequency of the signal field

 λ_{s} = wavelength of the signal

$$k_s = 2\pi/\lambda_s\,E_s = \text{peak electric field of a linearly polarized signal}$$

$$a_s = \frac{eE_s}{\sqrt{2}\ mc^2k_s}$$

 ϕ_s = electric field phase with respect to a vacuum plane wave

 γ_i = Lorentz factor

c = velocity of light

$$y_j = (k_s + k_w)z - w_s t + \phi_s$$

$$\omega^2_{pe} = \frac{4\pi \text{ ne } e^2}{\text{m e}}$$

 n_e = density of electrons

 m_e = mass of electrons

⟨⟩ = average over particles j

$$f_B = J_o(\xi) - J_1(\xi)$$

where
$$\xi = \frac{a_W^2}{2(1+a_W^2)}$$

where I is the beam current and a x b are the dimensions of the waveguide. (The effective beam density for microwaves is given by spreading the beam out over a mode volume.)

Tapering is important because the condition for FEL synchronism (i.e., that $d\psi/dz = 0$ for a central particle) is:

$$\lambda_{\rm s} = \frac{\lambda_{\rm w}}{2\gamma^2} \left(1 + a_{\rm w}^2 \right) \,, \tag{4}$$

and as an electron loses energy synchronism can not be maintained if λ_{w} and a_{w} are constants (i.e. if the wiggler is untapered). To keep the electron bunch in synchronism one can decrease λ_w , decrease B_w (and hence a_w) or some combination of the two. Usually B_w is varied since that can readily be done for a particular wiggler realization and so one can easily study performance of the FEL as a function of taper.

The effect of taper has been studied in hundreds of numerical simulations. The non-tapered case has been put into scaled form [3], and this proves a convenient starting point of the study of the effect of taper. Here, I will show the result of tapering studies using particular parameters; namely those of the Electron Laser Facility (ELF) at Livermore. The parameters are given in Table 2. The result of a 1-D numerical simulation for an un-tapered wiggler is given in Fig. 1 and Fig. 2. For a tapered wiggler the results are shown in Fig. 3. [Figs. 1, 2 and 3 are due to Efrem Sternbach.]

Although the effect of tapering has been studied in many hundreds of simulations, it has only been studied in a few experiments. In Fig. 4 we show the dramatic results achieved on ELF [4]. The extracted power and efficiency was as high as 1.86 GW and 45% in the best case.

An experiment at LANL has achieved 5% extraction at 10.6 μ m, while an experiment at Stanford, at 1.57 μ m, has achieved a few percent extraction [5].

The concept of tapering has been shown to work in experiments and the numerical simulations are in good agreement with the experiments. Thus, the way would seem to be open to high efficiency FELs. However, high efficiency requires long wigglers and this requires focusing the electron beam, to which subject we now turn.

3. PARTICLE FOCUSING

Even for short wigglers such as the original ELF which had a length of 3 meters, particle focusing is required. For ELF horizontal focusing was provided by quadrupoles while the FEL itself provided vertical focusing. The FEL action was so robust that the quadrupoles reduced it negligibly. Not so for long wigglers where one must be very much more careful. The study of this subject was pioneered by E. T. Scharlemann who, as a result of his studies, showed how to achieve electron beam focusing and yet minimally effect FEL performance [6].

Linear wigglers focus the beam in the vertical direction (the direction in which the wiggler field points) spontaneously. The focusing force is provided by the cross-product of the wiggle motion with the z-component of magnetic field that accompanies the periodic wiggler field.

For a linear wiggler the field is:

$$\vec{B}_{w} = \hat{y} B_{0} \sin k_{w} z \cosh k_{w} y + \hat{z} \cos k_{w} z \sinh k_{w} y, \qquad (5)$$

Table 2. ELF Parameters

Beam energy 4.14 MeV
Beam energy in γ 8.1
Beam current 800 A
Wiggler wavelength 9.8 cm
Radiation frequency 34.6 GHz
Waveguide width 9.8 cm
Waveguide height 2.9 cm

Input radiation power 60 kWatts/TE⁰¹ mode

Wiggler peak field 4.67 k GaussEnergy spread $\pm 1 \text{ MeV}$

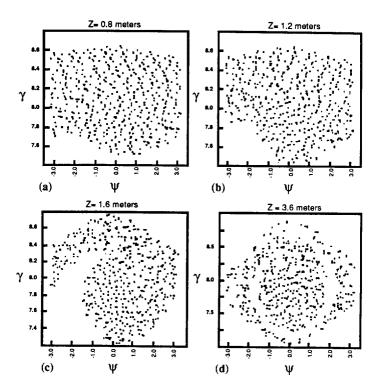


Fig. 1. Numerical simulations of ELF (Table 2). Particle energy vs. phase is shown for four different distances down the wiggler.

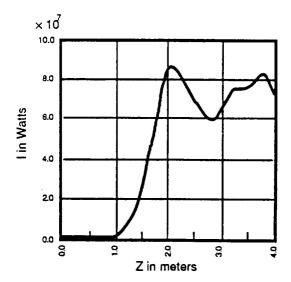


Fig. 2. Numerical simulation for ELF (Table 2) showing microwave power as a function of distance down the wiggler.

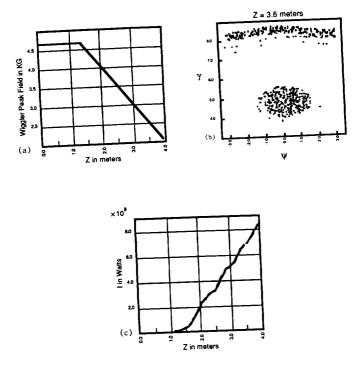


Fig. 3. Numerical simulations for a tapered ELF. The tapering is shown in (a); the resultant trapping of particles in (b) and the output microwave power in (c).

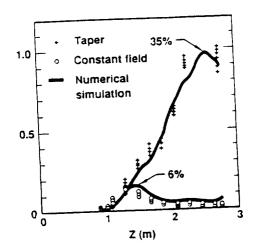


Fig. 4. Power vs wiggler length as measured experimentally at ELF, with comparison with numerical simulations. The numerical simulation was performed with a code (FRED) that has space charge and 3-D effects included. Tapering enhanced the extraction efficiency from 6% to more than 35%.

which results in wiggle motion:

$$\frac{\vec{v}_w}{c} = -\hat{x} \frac{eB_w}{\gamma mc^2 k_w} \cos k_w z \cosh k_w y_o.$$
 (6)

The motion of Eq. (6) interacts with B_z to focus the beam:

$$\frac{d^2y}{dz^2} = \frac{e}{\gamma mc^2} \frac{v_x}{c} B_z$$

$$= -\left(\frac{eB_0}{\gamma mc^2}\right)^2 \frac{1}{k_w} \cos^2 k_w z \sinh k_w y \cosh k_w y.$$
(7)

An average over a wiggler period converts $\cos {}^{2}k_{w}z$ to a factor of 1/2, and an expansion of the sinh and cosh to first order in $k_{w}y$ yields

$$\frac{\mathrm{d}^2 y}{\mathrm{d}z^2} = -k_{\beta y}^2 y,\tag{8}$$

with

$$k_{\beta y}^2 = \frac{1}{2} \left(\frac{eB_0}{\gamma mc^2} \right)^2 = \frac{a_{w0}^2 k_w^2}{\gamma^2},$$
 (9)

Eq. (9) is the equation of motion for a particle in a harmonic potential well. The solution is

$$y(z) = y_{\beta} \cos(k_{\beta y}z + \phi_y), \qquad (10)$$

where $\varphi_{\boldsymbol{y}}$ is an arbitrary phase determined by initial conditions.

Both the betatron motion and the off-midplane increase in the wiggler field affect FEL resonance. The FEL synchronism condition can be written

$$\frac{\omega}{c} = (k + k_w)\beta_{\parallel} \tag{11}$$

obtained from setting $d\phi/dz = 0$ (see Table I), or, approximately,

$$\beta_{\parallel} \cong 1 - \frac{1}{2\gamma^2} \left(1 + \gamma^2 \beta_{\perp}^2 \right). \tag{12}$$

The changes in $\gamma^2 \beta_{\perp}^2$ are of two types: from the increase in wiggle motion away from the wiggler midplane, and from the variation in transverse betatron velocity through the betatron orbit of an electron. For natural wiggler focusing these two components of $\gamma^2 \beta_{\perp}^2$ combine to remain constant (averaged over a wiggler period) over the betatron orbit of any electron. From Eq. (6),

$$\begin{split} \gamma^2 \beta_\perp^2 \text{ (wiggle)} &= 2 \ a_{w0}^2 \cos^2 k_w z \cosh^2 k_w y \\ & \equiv \ a_{w0}^2 \left(1 + k_w^2 y^2 \right) \text{ (averaged and expanded),} \end{split} \tag{13}$$

with a_{w0} the value of a_w on the midplane. Eq. (10) can be used to rewrite this expression further:

$$\overline{\gamma^2 \beta_{\perp}^2} \text{ (wiggle)} \cong a_{w0}^2 \left[1 + k_w^2 y_{\beta}^2 \cos^2 \left(k_{\beta y} z + \phi_y \right) \right]. \tag{14}$$

Also, it is clear that

$$\overline{\gamma^2 \beta_\perp^2} \text{ (betatron)} = \gamma^2 k_{\beta y}^2 y_\beta^2 \sin^2 \left(k_{\beta y} z + \phi_y \right), \tag{15}$$

$$\overline{\gamma^2 \beta_\perp^2} \text{ (betatron)} = a_{w0}^2 k_w^2 y_\beta^2 \sin^2 \left(k_{\beta y} z + \phi_y \right). \tag{16}$$

The sum is

$$\overline{\gamma^2 \beta_\perp^2} \text{ (total)} = a_{w0}^2 \left(1 + k_W^2 y_\beta^2 \right), \tag{17}$$

which is constant for each electron (i.e., there is no dependence of β_{\perp}^2 upon z). Hence the FEL synchronism condition does <u>not</u> change as an electron moves along its trajectory. The increase of the wiggle motion away from the midplane is just compensated by the decrease of betatron oscillations: a most remarkable feature!

If horizontal focusing is provided by quadrupoles then one can show [6] that β_{\perp}^2 depends upon z and hence the FEL resonance condition varies along a trajectory.

If, however, horizontal focusing is provided by a sextapole channel then β_{\perp}^2 is again a constant of the motion [6]. Such a channel is provided by curved pole faces and thus one can see immediately why this wiggler configuration is good, for now the magnetic field not only increases vertically, but also horizontally.

Experiments on an FEL have shown that a sextopole channel is quite effective in transporting a beam through a long wiggler.

4. WAVE FOCUSING

For a high power FEL, and therefore usually a long FEL, the optical beam, as well as the electron beam, must be focused. Quite remarkably, and most fortunately, the very FEL action provides such focusing. The term "optical guiding" is employed to describe this phenomena,

which was realized at an early stage [1,2,7,8]. An optical fiber model, and a clear demonstration of the importance of the phenomena for long lasers, was given later [9]. A convenient formalism for analysis was given by Moore [10].

That an FEL acts to focus light can be seen immediately from the KMR equations. The phase of the signal wave is ever changing as one goes through an FEL, as is described by the last of the KMR equations. Outside the electron beam there is not change and thus the electron beam acts as a fiber having an effective index of refraction. Furthermore, under usual circumstances, the light is increasing in intensity as one goes through an FEL. Thus there will be "gain guiding" (i.e., the ever increasing signal tends to compensate, often can compensate) for the outward diffraction of the light. Clearly the index of refraction is given by [9]:

$$Re(n) -1 = \frac{1}{k_s} \frac{d\phi_s}{dz} = \frac{f_B \omega_{pe}^2 a_w}{a_s k_s^2 c^2} \left\langle \frac{\cos\psi}{\gamma} \right\rangle ,$$

$$Im(n) = \frac{1}{k_s a_s} \frac{d a_s}{dz} = \frac{f_B \omega_{pe}^2 a_w}{a_s k_s^2 c^2} \left\langle \frac{\sin\psi}{\gamma} \right\rangle .$$
(18)

These expressions can be employed to yield considerable insight into the phenomena. In the exponential gain regime the whole subject can be reduced to solving a cubic algebraic equation [9,10].

Numerical study can be done and in Fig. 5 we show the results of employing the 2D particle simulation code FRED, using the parameters listed in Table 3. In this single simulation one sees the effect of gain guiding and then the optical fiber effect (called "refractive" guiding in the literature) for very many meters of wiggler and about 60 Rayleigh lengths.

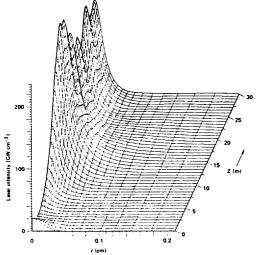


Fig. 5. A three-dimensional plot of laser intensity vs r and z inside the wiggler.

Table 3. Simulation Parameters

Current (1) Electron beam radius in the wiggler (a) Electron Lorentz factor (v) Fractional electron energy spread	270A 0.01 cm 2000
$(rms \Delta \gamma/\gamma)$ Laser wavelength $(2\pi/k)$ Input laser power Dimensionless rms wiggler vector	1.2x10 ⁻³ 2500Å 30 MW
potential (a_w) Wiggler length Wiggler period $(2\pi/k_w)$	4.352 30 m 10 cm

Experimental demonstration of optical guiding has not yet been achieved in the optical range. Work has been done in the microwave range, but there waveguide and space charge effects are important, and although the experiments are consistent with the concept of optical guiding, clear and unambiguous effects are neither predicted nor observed. Because of optical guiding one can construct very long wigglers (hundreds of meters in length) and hence extract a significant fraction of the electron beam energy. Thus the concept is vital for (optical) high-power FELs.

5. RESISTIVE WALL

The electron beam in an FEL is subject to transverse instabilities. Of the various sources for instabilities, the finite resistance of the vacuum chamber walls is the most important. Analysis of the phenomena has been given by Neil and Whittum [11].

One can start with the well-known equation for the transverse behavior of the beam, of current I, in a resistive pipe:

$$\frac{d^{2}\xi(z,t)}{dt^{2}} + k_{0}^{2}c^{2}\xi(z,t) = \Omega^{5/2} \int_{-\infty}^{t} \frac{\partial \xi(z,t')}{\partial t'} \sqrt{t-t'} dt' , \qquad (19)$$

where

$$\Omega^{5/2} = \frac{4eI}{\pi \gamma m \, b^3 \sigma^{1/2}}$$
(20)

The beam pipe has radius b and wall conductivity σ . The focusing is characterized by the quantity k_{σ} .

This equation can be easily solved, asymptotically, for the case of constant beam energy, constant initial beam displacement d, and constant focusing. The maximum displacement of the beam after traveling a distance z down the FEL, and for a position in the beam characterized by the time back from the beam head, τ , is

$$\xi(z,\tau) = \frac{d}{(3\pi)^{1/2}} \left(\frac{2}{A}\right)^{1/3} \exp\left\{1.5\left(\frac{A}{2}\right)^{2/3}\right\}. \tag{21}$$

The quantity A is given by:

$$A = \frac{I(kA)}{17} \left(\frac{\tau}{\pi\sigma}\right)^{1/2} \frac{cz}{\gamma k_0 b^3} . \tag{22}$$

The strong dependence on pipe radius (exp $1/b^2$) is clearly seen. Since in an FEL the pipe radius must be less than the wiggler wavelength, one can easily engender a severe effect.

Numerical study has been made in Ref. [11] of the basic equation. For an oscillator there is no problem, but for long amplifiers the growth can be significant. Using parameters typical of a high-power amplifier they find 5 e-folds. By increasing the beam pipe as one moves down the FEL (which is possible because the field is reducing because of tapering), and making the wall quite conductive (aluminum or copper rather than [say] titanium), one can bring the e-folding down to close to unity, which should be acceptable.

6. SIDEBANDS

Sidebands refer to a longitudinal instability of the FEL. Of course, the FEL works by means of a longitudinal instability, but a controlled instability involving only one wave of a particular frequency. If other frequencies than the central one are also growing, and becoming significant, then the single mode analysis of KMR is no longer appropriate.

There have been hundreds of papers on this subject and I only intend here to present the basic equations describing the phenomena, and then some numerical results showing the importance of attention to the phenomena for high power FELs. The basic equations are really conceptually quite simple. They are [12].

$$\frac{\partial \gamma_{j}}{\partial z} + \frac{1}{\nu_{\parallel}} \frac{\partial \gamma_{j}}{\partial t} = -\frac{\omega}{2c} \frac{a_{s} a_{w} f_{B}}{\gamma_{j}} \sin \psi_{j} ,$$

$$\frac{\partial \psi_{j}}{\partial z} + \frac{1}{\nu_{\parallel}} \frac{\partial \psi_{j}}{\partial t} = k_{w} + \frac{d\phi_{s}}{dz} - \frac{\omega}{2c\gamma_{j}^{2}} \left[1 + \frac{a_{w}^{2}}{2} \right] ,$$

$$\frac{\partial a_{s}}{\partial z} + \frac{1}{\nu_{g}} \frac{\partial a_{s}}{\partial t} = f_{B} \frac{\omega_{pe}^{2} a_{w}}{k_{s} c^{2}} \left\langle \frac{\sin \psi}{\gamma} \right\rangle ,$$

$$\frac{\partial \phi_{s}}{\partial z} + \frac{1}{\nu_{g}} \frac{\partial \phi_{s}}{\partial t} = f_{B} \frac{\omega_{pe}^{2} a_{w}}{a_{s} k_{s} c^{2}} \left\langle \frac{\cos \psi}{\gamma} \right\rangle .$$
(23)

It can be seen that these equations are a simple, and almost obvious, generalization of the KMR equations. The quantity v_{\parallel} is the average longitudinal velocity of the electron beam down the wiggler, while v_g is the group velocity of the electromagnetic signal. That the group velocity, and not the phase velocity, is the correct quantity can be seen by carefully examining the ekonal approximation (upon which the KMR equations are based). An experiment, coming after the theoretical analysis, has, in fact, demonstrated that the group velocity is correct.

The partial differential equations (2N+2 of them), not surprisingly, describe a world of richness of phenomena. They have been studied both analytically and numerically. For high-power FELs the consequences can be important [13]. In Fig. 6 are shown the results obtained using the 2D particle simulation code GINGER. The parameters are those of a high power microwave fusion plasma application; namely an 8.5 MeV beam of 3 kA going through an 8 meter wiggler of 9 cm wave length. The initial design was made with the particle simulation code FRED (i.e., modeling the KMR equations) and predicted 8 GW of output power. As can be seen in the Figure, the design had a fatal flaw in that much of the power was in sidebands and, futhermore, there was not as much power as expected.

Re-design [13] of the tapering scheme resulted in the Fig. 7 in which the sidebands now have very little power and the main line has the desired 8 GW. The story is clear, one must pay attention to the possibility of sidebands i.e., it is not adequate to design on the basis of the KMR equations (even when these equations are extended to include waveguide and space charge effects).

7. TECHNOLOGICAL CONSIDERATIONS

Construction of a high-power FEL requires careful attention to many technological considerations. I do not intend to cover these matters here, but I have designated the subject by a separate Section as I want to make it very clear the importance of these elements. Lectures in this

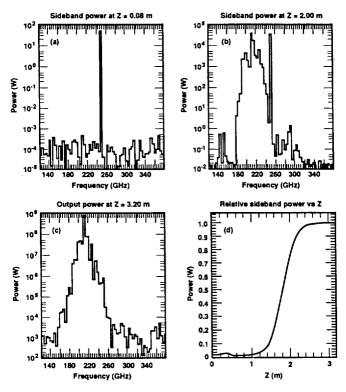


Fig. 6. Power spectrum and relative sideband power with synchronous taper showing (a) output power at z = 0.08 m, with the initial shot noise distribution; (b) output power at z = 2 m; (c) output power at z = 3.2 m; and (d) relative sideband power as a function of axial position.

school have been devoted to the matter of Optical Cavities and to the matter of Wigglers, so I needn't repeat these considerations here. Suffice it to say that these elements are very important in a high-power FEL, and that the high power puts special requirements on their design and construction. Finally, let me note that high power requires intense beams and so beam control and beam steering (besides being necessary for proper FEL performance) is particularly important; one doesn't want to make radioactive, or destroy, the FEL.

8. **CONCLUSION**

In order to produce a high-power, high-efficiency FEL requires tapering, curved pole faces (i.e., a sextapole channel for electron beam focusing), optical guiding (for light focusing), attention to various instabilities (so as to prevent them) and careful attention to many technological details. Nevertheless, it appears possible to build high-power FELs at least down to $1/2~\mu m$.

There are very large programs to make FELs, operating at close to 1 μ m, in place in the US with LLNL, TRW, MSNW, LANL and Boeing being the major players. So far a peak output power

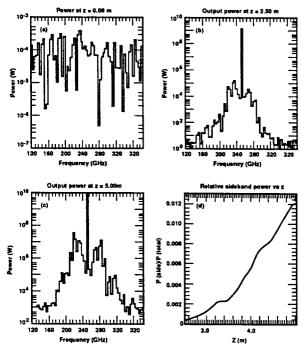


Fig. 7. Power spectrum and relative sideband power with "Raman taper" for the 7 MeV electron beam FEL design: (a) output power at z=0.08 m showing the initial shot noise distribution; (b) output power at z=2.5 m position; (c) output power at the end of the wiggler at the z=5 m position; and (d) plot of relative sideband power as a function of axial position.

of 50 MW has been reported, [14], but the motivation is to develop an ASAT weapon, or an SDI weapon for which the requirement is about 1 GW, of average power, or peak powers of 0.1 TW to 1.0 TW [15].

At longer wavelengths the task of making high-power FELs is much reduced. A significant program is underway at Livermore to produce plasma heating (in a tokamak) at 140 GHz and 250 GHz [13]. The goal is 10 GW peak power and 2 MW average power. There has been much talk, but no experimental work, to use an efficient FEL as a power source for a linear collider [16]. The requirement -- now still easier at 17 GHz -- is 3.9 TW of peak power and 35 MW of average power.

In this brief article we have attempted to present a balanced exposition of the primary elements required for producing high-power, high-efficiency FELs. We have not gone into details; there are hundreds of papers in the literature on each of the subjects we have covered. The reader wishing more details may want to start with the review article by Schlarleman [17], then go onto the FEL Handbook [18] and from there into the original literature.

REFERENCES

- [1] N. M. Kroll, P. L. Morton, M. N. Rosenbluth, IEEE J. Quant. Elect., QE-17, (1981) 1436.
- [2] A. Szoke, V. K. Neil, and D. Prosnitz, Physics of Quantum Electronics, Vol. 7, Addison-Wesley, Reading, MA (1980) 175.
- [3] A. M. Sessler and M. Kumada, "Scaling of the FEL-1D Equations", ELF Note 128, Lawrence Livermore National Laboratoary Internal Note, Nov. (1985).
- [4] T. J. Orzechowski et al., Phys. Rev. Lett., <u>57</u> (1986) 2172.
- [5] J. W. Watson, Nucl. Inst. and Method in Phys. Research <u>A250</u> (1986) 1;
 J. A. Edighoffer et al., Phys Rev. Lett. <u>52</u>, (1984) 344.
- [6] E. T. Scharlemann, Journ. Applied Physics <u>58</u> (1985) 2154.
- [7] J. M. Slater and D. D. Lowenthal, Journ. Appl. Physics <u>52</u>, (1981) 44.
- [8] P. Sprangle and C.-M. Tang, Appl. Phys. Lett. 39, (1981) 677.
- [9] E. T. Scharlemann, A. M. Sessler, and J. S. Wurtele, Phys. Rev. Lett. <u>54</u>, (1985) 1925; and Nucl. Instr. and Methods in Physics Research A239, (1985) 29.
- [10] G. T. Moore, Optics Communic <u>52</u> (1984) 46.
- [11] V. Neil and D. H. Whittum, "Transverse Resistive Wall Instability in a High-power, Short-Wavelength FEL", Proc. of the Ninth FEL" Conference, Williamsburg (1988), and Lawrence Livermore National Laboratory Report UCRL-96712.
- [12] S. S. Yu et al., Nucl. Instr. and Methods in Physics Research A259, (1987) 219.
- [13] R. A. Jong, E. T. Scharlemann, and W. M. Fawley, Nucl. Instr. and Methods in Physics Research A272, (1988) 99.
- [14] D. Prosnitz, Bull. of the Am. Phys. Soc. 33, (1988) 1014.
- [15] N. Bloembergen et al., Rev. of Mod Phys.No 3 Part II 59, (1987).
- [16] D. B. Hopkins et al, "An FEL Power Source for a TeV Linear Collider", Proceedings of the 1988 Linear Accelerator Conference, Williamsburg, Oct. (1988). Lawrence Berkeley Laboratory Report LBL-25937.
- [17] E. T. Scharlemann, "Selected Topics in FELs", Proc. of the Int. School on Electromagnetic Radiation and Particle Beams", Varena, June 1988, Nucl. Instr. and Methods in Phys. Res. (to be published); Lawrence Livermore National Laboratory Report UCRL-99922 (1988).
- [18] "Free Electron Laser Handbook", North Holland Press (to be published).