CERN-TH.5391/89

# Supersymmetry of the d=3 Chern-Simons action in the Landau gauge

**F. Delduc** † and **F. Gieres** §
*CERN - Theory Division, CH - 1211 - Geneva 23*

**S. P. Sorella**
*CERN - Theory Division, CH-1211-Geneva 23*
and
*SISSA, INFN, sezione di Trieste, I-34011-Trieste*

**Abstract**

Combining anti-BRS invariance with previously found symmetries [1] [2] of the gauge-fixed Chern-Simons action in three dimensions, we obtain a set of transformations that satisfies an unusual kind of supersymmetry algebra. A superspace formulation of the field equations is given and the Slavnov-Taylor identity associated to these symmetries is presented.

---

† Permanent address: Laboratoire de Physique Théorique et des Hautes Energies, Université Paris VII et CNRS (UA 280), 2 place Jussieu, T24, F-75251-Paris Cédex 05.
§ Supported by "Commission des Communautés Européennes" (DG XII - CCR).

CERN-TH.5391/89
May 1989

# 1 Introduction

The interest in topological field theories has been constantly growing since Witten's pioneering work on this subject [3]. Among these theories, the Chern-Simons action in three dimensions seems to play a quite interesting rôle [4]-[6]. The aim of this letter is to study the very rich symmetry structure of the Chern-Simons theory in a covariant gauge, namely the Landau gauge. Part of these symmetries have been obtained previously by D.Birmingham, M.Rakowski and G.Thompson, who found an abelian superalgebra with Lorentz vector and scalar generators [1] [2]. In the following, we will show that this algebra can be promoted to a non-abelian, supersymmetry-like algebra.

To construct the additional generators of this algebra, use is made of the fact that, in a linear gauge, the action satisfies both BRS and anti-BRS invariance. The basic symmetries are presented and studied in the next section. Then, the transformation laws as well as the equations of motion are cast into a more compact and geometric form by the introduction of an appropriate superspace. Thereafter, the Slavnov-Taylor identity associated to these symmetries is derived. We conclude with some comments on possible further developments and on the relation of the present investigations to recently obtained results.

# 2 The supersymmetry algebra

Using the notation of reference [2], the Chern-Simons action in three dimensions reads:

$$S_{inv}[A] \;=\; -\frac{1}{2}\int d^3x\ \mathrm{Tr}\left[\epsilon^{\mu\nu\rho}\,(A_\mu\,\partial_\nu A_\rho \;+\; \frac{1}{3}\,A_\mu\,[A_\nu\,,\,A_\rho]\,)\right] \quad . \tag{1}$$

Including the gauge-fixing and ghost terms, the full action in the Landau gauge takes the form

$$\Sigma[A,d,b,c] \;=\; S_{inv}[A] \;+\; \int d^3x\ \mathrm{Tr}\,[\;d\,\partial_\mu A^\mu \;+\; b\,\partial^\mu\mathcal{D}_\mu c\;] \quad , \tag{2}$$

where the covariant derivative is defined by $\mathcal{D}_\mu c \;=\; \partial_\mu c + [A_\mu\,,\,c]$. The known symmetries of this action are the BRS invariance [7],

$$\begin{aligned} sA_\mu &= -\mathcal{D}_\mu c & \qquad sc &= cc \\ sb &= d & \qquad sd &= 0 \end{aligned} \tag{3}$$

and a set of anticommuting global symmetries carrying a Lorentz vector index [1] [2]:

$$\begin{aligned} \delta_\rho A_\mu &= \epsilon_{\mu\rho\nu}\,\partial^\nu c & \qquad \delta_\rho c &= 0 \\ \delta_\rho b &= A_\rho & \qquad \delta_\rho d &= \mathcal{D}_\rho c \quad . \end{aligned} \tag{4}$$

Both $s$ and $\delta_\rho$ increase the ghost number by one unit and, altogether, these symmetries form an abelian superalgebra:

$$\{s\,,\,\delta_\rho\} \;=\; 0 \quad , \quad \{\delta_\rho\,,\,\delta_\sigma\} \;=0 \quad .$$

As well-known [8], integration by parts allows to rewrite the action (2) as

$$\Sigma \;=\; S_{inv} \;+\; \int d^3x \;\mathrm{Tr}\,[\,(d \;-\; \{b\,,\,c\})\,\partial_\mu A^\mu \;-\; c\,\partial^\mu \mathcal{D}_\mu b\,] \quad , \tag{5}$$

which expression may be viewed as resulting from (3) by the substitutions

$$\begin{array}{ll} A_\mu \longrightarrow A_\mu & \qquad c \longrightarrow b \\ b \longrightarrow -c & \qquad d \longrightarrow d \;-\; \{b\,,\,c\} \quad . \end{array} \tag{6}$$

Considering the same replacements in eqts. (3) and (4) leads to the anti-BRS transformations

$$\begin{array}{ll} \bar{s}A_\mu \;=\; -\mathcal{D}_\mu b & \qquad \bar{s}c \;=\; -d \;+\; \{b\,,\,c\} \\ \bar{s}b \;=\; bb & \qquad \bar{s}d \;=\; [b\,,\,d] \end{array} \tag{7}$$

and to a new set of anticommuting global symmetries:

$$\begin{array}{ll} \bar{\delta}_\rho A_\mu \;=\; \epsilon_{\mu\rho\nu}\,\partial^\nu b & \qquad \bar{\delta}_\rho c \;=\; -A_\rho \\ \bar{\delta}_\rho b \;=\; 0 & \qquad \bar{\delta}_\rho d \;=\; \partial_\rho b \quad . \end{array} \tag{8}$$

For their part, the transformations (7) and (8) describe an abelian superalgebra:

$$\{\bar{s}\,,\,\bar{\delta}_\rho\} \;=\; 0 \quad , \quad \{\bar{\delta}_\rho\,,\,\bar{\delta}_\sigma\} \;=0 \quad .$$

However, some of the anti-commutation relations with the former symmetries (3) (4) are non-trivial. One finds[1] :

$$\{\,s\,,\,\bar{s}\,\} \;=\; 0 \tag{9}$$

$$\begin{array}{l} \{\bar{s}\,,\,\delta_\rho\} \;\;(\,b,c,d\,) \;=\; -\partial_\rho\,(\,b\,,\,c\,,\,d\,) \\ \{\bar{s}\,,\,\delta_\rho\} \;\;A_\mu \;\;=\; -\partial_\rho A_\mu \;+\; \epsilon_{\mu\rho\nu}\,\dfrac{\delta\Sigma}{\delta A_\nu} \end{array} \tag{10}$$

$$\begin{array}{l} \{\delta_\rho\,,\,\bar{\delta}_\sigma\} \;\;(\,b,c\,) \;=\; \epsilon_{\rho\sigma\nu}\,\partial^\nu\,(\,b\,,\,c\,) \\ \{\delta_\rho\,,\,\bar{\delta}_\sigma\} \;\;d \;\;=\; \epsilon_{\rho\sigma\nu}\,\partial^\nu\,d \;+\; \epsilon_{\rho\sigma\nu}\,\dfrac{\delta\Sigma}{\delta A_\nu} \\ \{\delta_\rho\,,\,\bar{\delta}_\sigma\} \;\;A_\mu \;\;=\; \epsilon_{\rho\sigma\nu}\,\partial^\nu\,A_\mu \;+\; \epsilon_{\rho\mu\sigma}\,\dfrac{\delta\Sigma}{\delta d} \end{array} \tag{11}$$

$$\begin{array}{l} \{s\,,\,\bar{\delta}_\rho\} \;\;(\,b,c,d\,) \;=\; \partial_\rho\,(\,b\,,\,c\,,\,d\,) \\ \{s\,,\,\bar{\delta}_\rho\} \;\;A_\mu \;\;=\; \partial_\rho A_\mu \;+\; \epsilon_{\rho\mu\nu}\,\dfrac{\delta\Sigma}{\delta A_\nu} \end{array} \tag{12}$$

[1]We define the functional derivative by $\frac{\delta}{\delta A_\mu}\int d^3x\,\mathrm{Tr}\,A_\rho B^\rho \;=\; B^\mu$ for $A_\rho$, $B^\rho$ elements of the Lie algebra.

Thus, it is clear that these generators define, up to field equations, a supersymmetry algebra. Using the notation

$$\begin{aligned} \delta^1 &= s & \delta^1_\rho &= \delta_\rho \\ \delta^2 &= \bar{s} & \delta^2_\rho &= \bar{\delta}_\rho \end{aligned} \tag{13}$$

the on-shell algebra can be summarized as follows:

$$\begin{aligned} \{\delta^i_\rho\,,\,\delta^j_\sigma\} &= \epsilon^{ij}\,\epsilon_{\rho\sigma\tau}\,\partial^\tau \qquad (\,\epsilon^{12}\equiv 1\,) \\ \{\delta^i\,,\,\delta^j_\rho\} &= \epsilon^{ij}\,\partial_\rho \quad . \end{aligned} \tag{14}$$

To establish the Hermiticity properties of the transformations in equation (14), we use the assignments of Kubo and Ojima [9], taking into account that our group generators are anti-Hermitian:

$$\begin{aligned} A^\dagger_\mu &= -A_\mu & d^\dagger &= -d \\ c^\dagger &= -c & b^\dagger &= b \quad . \end{aligned}$$

Then, one obtains the following Hermiticity transformations[2] :

$$\begin{aligned} \left(\delta^1_\rho\right)^\dagger &= -\delta^1_\rho & \left(\delta^2_\rho\right)^\dagger &= \delta^1_\rho \\ \left(\delta^1\right)^\dagger &= -\delta^1 & \left(\delta^2\right)^\dagger &= \delta^2_\rho \quad . \end{aligned}$$

The algebra (14) admits an $SL(2,R)$ automorphism group, $(i\delta^1\,,\,\delta^2)$ and $(i\delta^1_\rho\,,\,\delta^2_\rho)$ with $\rho = 0,1,2$ being doublets of this group. This simply reflects the fact that the action (2) admits an $SL(2,R)$ invariance, where $(ic,b)$ represents a doublet and $A_\mu$ , $d-\frac{1}{2}\{b,c\}$ two singlets of this group.

In the Euclidean case, the algebra (14) can be related to a twisted version of the d=3, N=2 supersymmetry algebra. To describe this relation, let us start from generators $Q^i_\alpha$ , $\bar{Q}^\alpha_i$ which are doublets of the internal symmetry group $SU(2)$ (index $i$) and doublets of $SU(2)$, the universal covering of the rotation group $SO(3)$ (index $\alpha$). These quantities satisfy the algebra

$$\{Q^j_\alpha\,,\,\bar{Q}^\beta_k\} = i\,\delta_k{}^j\,(\sigma^\rho)_\alpha{}^\beta\,\partial_\rho \tag{15}$$

where $\sigma^\rho$ denote the Pauli matrices. Let us now redefine the rotation group and choose it as the diagonal subgroup of $SU(2)\times SU(2)$. In eqt. (15), this amounts to an identification of the spinorial and the internal group indices:

$$\{Q^\beta_\alpha\,,\,\bar{Q}^\gamma_\delta\} = i\,\delta_\delta{}^\beta\,(\sigma^\rho)_\alpha{}^\gamma\,\partial_\rho \quad . \tag{16}$$

Finally, we introduce generators belonging to the singlet and triplet representations of the diagonal $SU(2)$,

$$\begin{aligned} Q &= \delta_\alpha{}^\beta\,Q^\alpha_\beta & \bar{Q} &= \delta_\alpha{}^\beta\,\bar{Q}^\alpha_\beta \\ Q_\rho &= (\sigma_\rho)_\alpha{}^\beta\,Q^\alpha_\beta & \bar{Q}_\rho &= (\sigma_\rho)_\alpha{}^\beta\,\bar{Q}^\alpha_\beta \end{aligned} \tag{17}$$

[2] To derive these rules, it is quite convenient to write the transformation of a field as a commutator ($sA_\mu = [s, A_\mu]$) or anti-commutator ($sc = \{s, c\}$).

and derive the commutation relations

$$
\begin{aligned}
\{Q_\rho \,,\, \bar{Q}_\sigma\} &= 2\,\epsilon_{\rho\sigma\tau}\,\partial^\tau \\
\{Q \,,\, \bar{Q}_\rho\} &= 2\,i\,\partial_\rho \\
\{\bar{Q} \,,\, Q_\rho\} &= 2\,i\,\partial_\rho \quad .
\end{aligned}
\tag{18}
$$

This algebra is related to the algebra (14) by the following identifications:

$$
\begin{aligned}
Q_\rho &= \delta^1_\rho + \delta^2_\rho & \qquad \bar{Q}_\rho &= -\delta^1_\rho + \delta^2_\rho \\
Q &= i(\delta^1 + \delta^2) & \qquad \bar{Q} &= i(\delta^1 - \delta^2) \quad .
\end{aligned}
$$

A few remarks concerning the presented invariance algebras are in order. Notice that in the algebra (9)-(12), and contrarily to ordinary supersymmetry, equations of motion of bosonic fields appear. This is not the only place where the usual rôles of commuting and anticommuting fields are interchanged. In fact, this also applies to the order of the equations of motion and to the counting of the degrees of freedom. The free equations of motion for the ghost $c$ and the antighost $b$ are just Klein-Gordon equations and thereby each of these variables corresponds to one degree of freedom. On the other hand, the bosonic fields, i.e. the vector $A_\mu$ and the Lagrange multiplier $d$ satisfy a set of first order differential equations which is analogous to the Dirac equation for free, massless fermions. For the same reason as in the case of Dirac's equation, the number of on-shell degrees of freedom is one-half the number of off-shell degrees of freedom and thus equal to 2. As usual in supersymmetry, the number of commuting and anti-commuting degrees of freedom match. However, from a physical point of view, ghost degrees of freedom are to be counted negatively and therefore the theory has no on-shell degrees of freedom.

# 3 Superspace formulation

We introduce 8 Grassmannian coordinates $\theta_i, \theta_i^\rho$ ($i$ =1,2) and consider the rigid superspace parametrized by $x$ and $\theta = (\theta_i, \theta_i^\rho)$. Then, the "supersymmetry transformations" (13) can be represented on the space of superfields $\Phi(x, \theta)$ by

$$
\delta^i \Phi = Q^i \Phi \qquad , \qquad \delta^i_\rho \Phi = Q^i_\rho \Phi
\tag{19}
$$

where $Q^i, Q^i_\rho$ denote the differential operators

$$
\begin{aligned}
Q^i &= \frac{\partial}{\partial\theta_i} - \frac{1}{2}\,\epsilon^{ij}\,\theta^\rho_j\,\partial_\rho \\
Q^i_\rho &= \frac{\partial}{\partial\theta^\rho_i} - \frac{1}{2}\,\epsilon^{ij}\,\epsilon_{\rho\mu\nu}\,\theta^\mu_j\,\partial^\nu + \frac{1}{2}\,\epsilon^{ij}\,\theta_j\,\partial_\rho \quad .
\end{aligned}
\tag{20}
$$

As in ordinary supersymmetry, we can define another set of derivatives,

$$
\begin{aligned}
D^i &= \frac{\partial}{\partial\theta_i} + \frac{1}{2}\,\epsilon^{ij}\,\theta^\rho_j\,\partial_\rho \\
D^i_\rho &= \frac{\partial}{\partial\theta^\rho_i} + \frac{1}{2}\,\epsilon^{ij}\,\epsilon_{\rho\mu\nu}\,\theta^\mu_j\,\partial^\nu - \frac{1}{2}\,\epsilon^{ij}\,\theta_j\,\partial_\rho \quad .
\end{aligned}
\tag{21}
$$

which quantities anticommute with the generators (20) and satisfy the algebra

$$\begin{aligned} \{ D^i_\rho , D^j_\sigma \} &= \epsilon^{ij} \, \epsilon_{\rho\sigma\tau} \, \partial^\tau \\ \{ D^i , D^j_\rho \} &= \epsilon^{ij} \, \partial_\rho \quad . \end{aligned} \tag{22}$$

The equations of motion associated to the action (2) can be written in our superspace $(x,\theta)$ in terms of anticommuting superfields $\Phi^i(x,\theta)$, $i$=1,2 :

$$\begin{aligned} D^i_\rho \Phi^j + D^j_\rho \Phi^i &= 0 \\ D^i \Phi^j + D^j \Phi^i &= \{ \Phi^i , \Phi^j \} \quad . \end{aligned} \tag{23}$$

To recover the space-time results, one defines component fields by

$$\begin{aligned} \Phi^1 | &= c & \Phi^2 | &= b \\ D^1_\rho \Phi^2 | &= A_\rho & D^1 \Phi^2 | &= d \quad , \end{aligned} \tag{24}$$

the bar denoting the projection to the $\theta = 0$ component of the corresponding superfield. All other on-shell components of the $\Phi^i$ represent functions of $b, c, A_\rho$ and $d$ which may be explicitly determined by using the algebra of the $D$-derivatives and equations (23). The procedure is straightforward and here we only illustrate it by an example. The field equation of $A_\mu$ can be derived by evaluating in two different ways the component

$$\begin{aligned} D^1 D^2_\rho D^2_\sigma \Phi^1 | &= -\partial_\rho A_\sigma + \partial_\sigma A_\rho - [ A_\rho , A_\sigma ] + \epsilon_{\rho\sigma\nu} \{ \partial^\nu b , c \} \\ = -D^1 D^2_\rho D^1_\sigma \Phi^2 | &= \epsilon_{\rho\sigma\nu} \, \partial^\nu d \end{aligned} \tag{25}$$

from which we get

$$F_{\rho\sigma} + \epsilon_{\rho\sigma\nu} \, \partial^\nu d - \epsilon_{\rho\sigma\nu} \{ \partial^\nu b , c \} = 0 \quad . \tag{26}$$

All other equations of motion follow in a similar way. Also, the transformation laws of the space-time fields (24) following from the superfield variations (19) by application of (22) (23) coincide with our initial equations (3) (4) (7) and (8).

It is not clear to us how to deduce the superfield equations (23) from a superspace action. Yet, such a derivation is not necessary for formulating the Slavnov-Taylor identity associated to the basic symmetries (3) (4) (7) (8). This identity which may be used to define the quantum action will be derived in the next section.

# 4 Slavnov-Taylor identity

We introduce global commuting ghosts $\alpha, \alpha^\rho, \bar\alpha, \bar\alpha^\rho$ corresponding to the symmetries $s, \delta_\rho, \bar s, \bar\delta_\rho$ and global anticommuting ghosts $d^\rho$ associated to the translations. (The latter are required for the on-shell closure of the symmetry algebra.) To write down the Slavnov-Taylor identity, we also have to introduce currents $\gamma^\mu, \Omega, \chi, \Lambda$ that are linearly coupled to the variations of the basic fields:

$$\Sigma_2 = \int d^3x \, \mathrm{Tr} \, [ \, \gamma^\mu \, SA_\mu + \Omega \, Sd + \chi \, Sc + \Lambda \, Sb \, ] \quad . \tag{27}$$

Here, the $S$-operation summarizes all invariances of the model:

$$
\begin{array}{rcl}
SA_\mu &=& \epsilon_{\mu\nu\rho}(\,\alpha^\nu\,\partial^\rho c \;+\; \bar{\alpha}^\nu\,\partial^\rho b\,) \;-\; \alpha\,\mathcal{D}_\mu c \;-\; \bar{\alpha}\,\mathcal{D}_\mu b \;+\; d_\rho\,\partial^\rho A_\mu \\
Sd &=& \alpha^\rho\mathcal{D}_\rho c \;+\; \bar{\alpha}^\rho\,\partial_\rho b \;+\; \bar{\alpha}\,[\,b\,,\,d\,] \;+\; d_\rho\,\partial^\rho d \\
Sc &=& -\bar{\alpha}^\rho A_\rho \;+\; \alpha\,cc \;+\; \bar{\alpha}\,(\{\,b\,,\,c\,\} \;-\; d) \;+\; d_\rho\,\partial^\rho c \\
Sb &=& \alpha^\rho A_\rho \;+\; \bar{\alpha}\,bb \;+\; \alpha\,d \;+\; d_\rho\,\partial^\rho b \quad .
\end{array}
\tag{28}
$$

Now, the functionals $\Sigma$ and $\Sigma_2$ are not yet sufficient, since our symmetry algebra only closes on-shell. In the case of supersymmetric theories, this kind of problem can be overcome [10] by including a term which is quadratic in the currents:

$$
\Sigma_3 \;=\; \frac{1}{2}\int d^3x \;\mathrm{Tr}\;[\;\epsilon_{\mu\nu\rho}\,\gamma^\mu\gamma^\nu(-\alpha\,\bar{\alpha}^\rho \;+\; \bar{\alpha}\,\alpha^\rho) \;+\; 2\,\epsilon_{\mu\nu\rho}\,\gamma^\mu\,\Omega\,\alpha^\nu\,\bar{\alpha}^\rho\;] \quad . \tag{29}
$$

Then, the complete action

$$
\Sigma_t \;=\; \Sigma \;+\; \Sigma_2 \;+\; \Sigma_3 \tag{30}
$$

may be shown to be satisfy the identity

$$
\begin{array}{l}
\displaystyle\int d^3x \;\mathrm{Tr}\;\left[\;\frac{\delta\Sigma_t}{\delta\gamma^\mu}\,\frac{\delta\Sigma_t}{\delta A_\mu} \;+\; \frac{\delta\Sigma_t}{\delta\Omega}\,\frac{\delta\Sigma_t}{\delta d} \;+\; \frac{\delta\Sigma_t}{\delta\chi}\,\frac{\delta\Sigma_t}{\delta c} \;+\; \frac{\delta\Sigma_t}{\delta\Lambda}\,\frac{\delta\Sigma_t}{\delta b}\;\right] \\
\qquad\qquad \displaystyle +\;\left[\;\bar{\alpha}\,\alpha^\rho\,\frac{\partial\Sigma_t}{\partial d^\rho} \;-\; \epsilon_{\rho\sigma\tau}\,\alpha^\rho\,\bar{\alpha}^\sigma\,\frac{\partial\Sigma_t}{\partial d_\tau} \;-\; \alpha\,\bar{\alpha}^\rho\,\frac{\partial\Sigma_t}{\partial d^\rho}\;\right] \;=\; 0 \quad .
\end{array}
\tag{31}
$$

# 5 Conclusion

We have shown that, in the Landau gauge, the 3-dimensional Chern-Simons action is characterized by invariances satisfying a supersymmetry-like algebra. In references [1] [6] it was demonstrated that this model is finite up to two loops. It might very well be that these (and possible further) finiteness properties have there origin in the supersymmetric invariances of the gauge-fixed action. To check this in a simple way, it should be quite useful to have an off-shell formulation as well as a superspace action. Due to the relation with the N=2 supersymmetry algebra (section 2), the methods developed by the authors of reference [11] may prove useful for further study of these aspects.


## Acknowledgements

We would like to thank the CERN Theory Division for its hospitality. The work of S.P.S. has been supported by a grant of the Fondazione A. Della Riccia.

# References


[1] D.BIRMINGHAM, M.RAKOWSKI AND G.THOMPSON, "Renormalization of Topological Field Theory", ICTP preprint IC/88/387, Nov. 1988, submitted to Nucl.Phys.B.

[2] D.BIRMINGHAM AND M.RAKOWSKI, "Superfield formulation of Chern-Simons Supersymmetry", ICTP preprint IC/89/38, March 1988.

[3] E.WITTEN, *Comm.Math.Phys.* **118** (1988) 601 .

[4] E.WITTEN, "Quantum Field Theory and the Jones Polynomial", IAS preprint IASSNS-HEP-88/33, September 1988.

[5] S.SEN AND P.MURPHY, "Constraints on two dimensional conformal field theories from knot invariants", preprint Trinity College Dublin ;

M.BOS AND V.P.NAIR, "U(1) Chern-Simons theory and c=1 conformal blocks", preprint Columbia University, Jan. 1989 ;

T.P.KILLINGBACK, "Quantum Chern-Simons theory", preprint DAMTP/88-31 ;

H.MURAYAMA, "Explicit quantization of the Chern-Simons action", preprint University of Tokyo, UT-542, March 1989 .

[6] E.GUADAGNINI, M.MARTELLINI AND M.MINTCHEV, "Perturbative aspects of the Chern-Simons field theory", preprint CERN-TH.5324/89, March 1989 .

[7] C.BECCHI, A.ROUET AND R.STORA, *Comm.Math.Phys.* **42** (1975) 127 ;

C.BECCHI, A.ROUET AND R.STORA, *Ann.Phys.* **98** (1976) 287 .

[8] G.CURCI AND R.FERRARI, *Phys.Lett.* **63B** (1976) 91 ;

I.OJIMA, *Progr.Theor.Phys.* **64** (1980) 625 ;

L.BAULIEU AND J.THIERRY-MIEG, *Nucl.Phys.* **B197** (1982) 477 .

[9] T.KUGO AND I.OJIMA, *Progr.Theor.Phys.* **60** (1978) 1869 .

[10] L.BAULIEU, M.BELLON, S.OUVRY AND J.C.WALLET, in preparation .

[11] A.GALPERIN, E.IVANOV, S.KALITZIN, V.I.OGIEVETSKY AND E.SOKATCHEV, *Class.Quant.Grav* **1** (1984) 469 .