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GAUGE AND SUPERSYMMETRY BREAKING IN FOUR-DIMENSIONAL STRINGS

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ABSTRACT

We discuss the problem of gauge and supersymmetry breaking in both the fermionic and bosonic constructions of four-dimensional strings.

(a) We show how to construct consistent string models in the Higgs phase of a spontaneously broken gauge symmetry. The Higgs vacuum expectation value is classically a free parameter, due to the existence of flat directions in the scalar potential. We find a simple rule which selects all such directions and we derive the mass formula for all the string states.

(b) We prove that the existence of a slightly massive gravitino or gaugino implies the existence of an entire tower of such states below the Planck mass, which is a signal of decompactification of some internal dimension.

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1. INTRODUCTION

The problem of gauge and supersymmetry breaking in string theories is of great importance, since it is related on one hand to long outstanding problems in particle physics such as the generation of mass hierarchies and the smallness, if not zero, value of the cosmological constant, and on the other hand to the study of the various string vacua and their possible connection. Here, we study these problems in the context of "Gaussian" four-dimensional models[1-6], constructed with free world-sheet fields but with complicated, in general, boundary conditions.

We start, in Section 2, with a brief presentation of the fermionic construction of the 4d string models[1,3,5,6]. In Section 3, we discuss the Higgs phenomenon in string theories[7,8]; this is related to the existence of a "plethora" of flat directions in the scalar potential. Gauge symmetries are broken by vacuum expectation values of Higgs fields sliding along such flat directions. Their knowledge is essential for the string model building since they lead to the spontaneous breaking of the gauge group in scales which are undetermined at the string tree level and they could thus be hierarchically smaller than the Planck mass, M_P. Furthermore, they determine the number of continuous parameters of the 4d models at the lowest order in the topological expansion. We find a very simple rule which selects out these directions and we derive the general mass formula for all the string states. It is quite remarkable that the mass shifts are given in terms of the charges of the corresponding states with respect to some U(1) currents which in turn correspond to some massless or massive gauge bosons.

In Section 4, we study the supersymmetry breaking. We show that[9] in string perturbation theory, the supersymmetry breaking scale can be small only if it is linked to the size of some internal dimension which is decompactified to a scale, roughly of the same order of magnitude. More precisely, we prove that the existence of a slightly massive gravitino or gaugino ($m_s << M_P$) implies the existence of an entire tower of such states, with masses equal to $(N+1)m_s$, $(2N+1)m_s$, ..., where N is some, not too large, integer. This is a signal of decompactification, meaning that one or more internal radii become large or, by duality, infinitesimal in units of M_P^{-1}, so that the corresponding momenta become quasi-continuous. This phenomenon seems to be a characteristic stringy property and may suggest the existence of some extra dimension at, say, 1-100 TeV. Although there is no direct experimental contradiction, such a scenario has the serious difficulty that[10], by naive dimensional analysis, all couplings become huge very early above the extra dimension scale, invalidating the semi-classical string description and creating a new hierarchy problem. However, a study of the one loop cosmological constant in such a scheme indicates that it might be exponentially suppressed[11] and thus, below the experimental limit even when the size of the internal dimension is close to M_P.

2. 4d MODELS: FERMIONIC CONSTRUCTION

The basic tool to construct consistent closed string vacua is the requirement of (super) reparametrization invariance on a general two-dimensional surface. For a world sheet which has the topology of a sphere, it leads to the condition of conformal invariance, while for topologically non-trivial surfaces, it gives the constraint of multiloop modular invariance. Conformal invariance implies that different string models are characterized as unitary representations of the Virasoro (super-Virasoro) algebra with central charge c = 26 (c = 15) and, thus fixes the 2dim field content. In four dimensions, it can be satisfied if the space-time co-ordinates X_μ (and their fermionic superpartners ψ_μ) are supplemented by internal conformal field theories contributing 22 (9) to the central charge.

In the fermionic construction the internal degrees of freedom are free fermions whose boundary conditions are severely restricted by multiloop modular invariance. In the heterotic case, for instance, one has[1] an extra 18 left-moving and 44 right-moving fermions χ^a and η_A, respectively. World-sheet supersymmetry is non-linearly realized among the χ^a, which must transform in the adjoint representation of a semi-simple Lie group G; the supercurrent is[1]

$$T_F = \psi^\mu \partial X_\mu + \frac{1}{3} f_{abc}\chi^a\chi^b\chi^c \tag{2.1}$$

where the structure constants are normalized such that $f_{acd}f_b^{cd} = \frac{1}{2}\delta_{ab}$. There are just three possible choices for G, namely $SU(2)^6$, $SU(3)\times SO(5)$ and $SU(4)\times SU(2)$. Since modular transformations mix the boundary conditions, one must sum over them in order to obtain an invariant expression. A general boundary condition is defined by a block-diagonal matrix $\mathcal{L}$, according to which the fermions transform

$$\psi^\mu \to -\delta_{\mathcal{L}}\ \psi^\mu \tag{2.2a}$$

$$\chi^a \to \mathcal{L}_G^{ab}\ \chi^b \tag{2.2b}$$

$$\eta^A \to \mathcal{L}_R^{AB}\ \eta^B. \tag{2.2c}$$

when parallel transported around the string. $\delta_{\mathcal{L}}$ is the spin statistics fermion parity, $\mathcal{L}_R$ is an orthogonal matrix, while $(-\delta_{\mathcal{L}}\mathcal{L}_G)$ must, in addition, belong to the group of automorphisms of G, Aut(G), which follows from the fact that the supercurrent T_F (2.1) must have well defined periodicity conditions

$$T_F \to -\delta_{\mathcal{L}}\ T_F, \tag{2.2d}$$

and hence the transformation matrix $(-\delta_{\mathcal{L}}\mathcal{L}_G)$ has to leave invariant the structure constants of G. When $\delta_{\mathcal{L}} = +1(-1)$, T_F is antiperiodic (periodic) and corresponds to the Neveu-Schwarz (Ramond) sector which leads to space-time bosons (fermions). There is a technical simplification when all boundary conditions $\mathcal{L}$ are commuting. Allowing non-commutativity, it is probable that fermionic and bosonic formulations are completely equivalent, in the sense that the fermionic construction can describe any orbifold model and vice versa. When they are commuting, they can be diagonalized simultaneously in a generally complex basis and thus, can be represented by vectors of phases, such that the complex fermion f picks up a phase:

$$f \to -e^{i\pi\alpha(f)}f, \tag{2.3}$$

restricted so that $-1 < \alpha(f) \leq 1$.

In this context, a string model is defined by specifying a set Ξ of vectors of boundary conditions. Its one-loop partition function takes the form[5,6]

$$Z = \Sigma_{\alpha,\beta\epsilon\Xi}\ c\binom{\alpha}{\beta}\ Z_F[{}^{\alpha}_{\beta}] \qquad (2.4)$$

where $Z_F[{}^{\alpha}_{\beta}]$ is the contribution of the spin-structure assignment $[{}^{\alpha}_{\beta}]$ according to which the fermions are twisted with the phases α and β [see (2.3)] along the two non-contractible loops of the torus, i.e., in the σ and τ-directions, respectively. The coefficients $c\binom{\alpha}{\beta}$ are phases which are constrained by multiloop modular invariance:

$$c\binom{\alpha}{\alpha} = -e^{\frac{i\pi}{4}\alpha\cdot\alpha}\ c\binom{\alpha}{1} \qquad (2.5a)$$

$$c\binom{\alpha}{\beta} = e^{\frac{i\pi}{2}\alpha\cdot\beta}\ c^{*}\binom{\beta}{\alpha} \qquad (2.5b)$$

$$c\binom{\alpha}{\beta+\gamma} = \delta_{\alpha}\ c\binom{\alpha}{\beta}\ c\binom{\alpha}{\gamma}. \qquad (2.5c)$$

(2.5a) and (2.5b) are a consequence of the one-loop modular invariance ($\tau \to \tau+1$ and $\tau \to -1/\tau$ respectively), while (2.5c) is derived using a two-loop modular transformation together with the condition of factorization. The "dot" product is Lorentzian (left minus right) and counts each real fermion with a factor ½. Finally, 1 denotes the vector where all fermions are periodic.

It can also be shown that[5,6] the set Ξ forms an Abelian group under addition (mod 2) and can be therefore generated by some "canonical" basis $B = \{b_1 = 1, b_2, \ldots, b_k\}$. To every element α of Ξ there corresponds a sector $\mathcal{H}_{\alpha}$ in the string Hilbert space $\mathcal{H}$ where the 2d fermions are twisted according to α, and to every basis element b_i of B a fermion number GSO-type projection:

$$\mathcal{H} = \bigoplus_{\alpha\epsilon\Xi}\ \Pi^{k}_{i=1}\ \{e^{i\pi b_i F} = \delta_{\alpha} c^{*}\binom{\alpha}{b_i}\}\ \mathcal{H}_{\alpha}, \qquad (2.6)$$

where F is the vector of all two-dimensional fermion numbers. The modular invariance constraints on the basis B and on the coefficients c's can be solved in a systematic way, for generic rational boundary conditions, and provide a well-defined set of rules to construct all consistent four-dimensional models, in the above context[5,6].

The string states of the sector $\mathcal{H}_{\alpha}$ are obtained by acting on the vacuum $|0\rangle_{\alpha}$ with bosonic or fermionic oscillators with frequencies integer or $(1+\alpha(f))/2$ + integer which are allowed by the fermion number projections of Eq. (2.6). The mass formula is

$$\begin{aligned} M^2 &= -\frac{1}{2} + \frac{1}{8}\alpha_L\cdot\alpha_L + \Sigma_L\nu_L \\ &= -1 + \frac{1}{8}\alpha_R\cdot\alpha_R + \Sigma_R\nu_R \end{aligned} \qquad (2.7)$$

where $\alpha_L(\alpha_R)$ is the left (right) part of the vector α and the $\nu_L(\nu_R)$ are frequencies.

A Regge trajectory corresponds to a pair (α,F) of a sector α and a vector F of allowed fermion numbers, consistent with the projections (2.6). One may assign a momentum-winding vector[3,7]

$$p = \tfrac{1}{2}\alpha + F \qquad (2.8)$$

which can be shown to be the momentum of the lowest lying state of the corresponding trajectory in the bosonic lattice formulation. In fact, in the fermionic construction, the lowest lying state can be obtained by acting on the vacuum $|0\rangle_{\alpha}$ with the F(f) lowest-lying oscillators of type f. Hence its "left-moving mass" is

$$\begin{aligned} L_0 &= \Sigma_{\text{left }f}\Big[\frac{1}{8}\alpha^2(f) + \frac{1+\alpha(f)}{2} + \ldots + \\ &+ \frac{1+\alpha(f)}{2} + F(f)-1\Big] - \frac{1}{2} = \frac{1}{2}p_L^2 - \frac{1}{2} \end{aligned} \qquad (2.9a)$$

and likewise its "right-moving mass"

$$\bar{L}_0 = \frac{1}{2} p_R^2 - 1. \qquad (2.9b)$$

Furthermore, acting on this state with the integer-frequency currents $f^* f$, we can build the entire tower of states with the same momentum p. Finally, note that the components of p are at the same time the charges with respect to the U(1) currents

$$\partial\Phi = f^* f \qquad (2.10)$$

where Φ bosonizes the fermionic current $f^* f$.

Defining Γ the set of allowed momenta (2.8) and using the properties of Ξ and the coefficients c's described above, one can show that[7], for instance, in the simplest (bosonic) case, Γ is a Lorentzian even and self-dual lattice[2]. The group structure of Ξ is translated into the lattice character of Γ. Conversely, given a lattice Γ with the above properties we may always rewrite the Hilbert space of the string model in the form (2.6), by reconstructing the set Ξ and the coefficients c's. Ξ turns out to be the set of equivalence classes of lattice momenta, two momenta being equivalent if they differ by an integer vector. Thus, in the case one is able to define a basis of complex fermions the fermionic construction reproduces the bosonic momentum lattices. However, in the presence of real fermions, it is not always possible to define fermionic currents which are periodic under all boundary conditions of Ξ simultaneously, and the fermionic construction reproduces orbifolds, as will become more clear below.

3. HIGGS PHENOMENON IN STRING THEORIES

A first example of continuous deformations of the string vacuum is presented in Ref. 2 and it consists of Lorentz boosting the charge lattice of string states. Considering, for instance, a "Lorentzian" lattice $\Gamma(n_L,n_R)$ any "Lorentz" transformation $\Lambda \in SO(n_L,n_R)$ of the lattice leaves the modular invariance conditions unchanged, since inner products are preserved. Thus, the transformed Γ_Λ is the charge lattice of a new consistent string model, in which the masses of all states are shifted by

$$\delta M^2 = \frac{1}{2} [(\Lambda p)_L^2 - p_L^2] = \frac{1}{2} [(\Lambda p)_R^2 - p_R^2]. \qquad (3.1)$$

Clearly, these mass shifts do not vanish only for Lorentz boosts $\Lambda \in SO(n_L,n_R)/SO(n_L)\times SO(n_R)$. For certain models obtained by compactification of the ten-dimensional superstring, this freedom amounts to changing radii and background gauge and antisymmetric fields[12]. However, from the four-dimensional point of view, it corresponds to a standard Higgs phenomenon and gauge symmetries are broken by vacuum expectation values of scalar fields along flat directions of their potential[7].

An important question is to find and study all possible flat directions of a given fermionic string model constructed with the rules presented above. Obviously, these directions correspond to some massless scalar fields which, in the 2dim underlying conformal field theory, are described with marginal operators. The problem can then be rephrased to find which from those marginal operators are integrable[13]. For pedagogical reasons, we discuss firstly the Z_2-orbifold directions and then we give the general rules[8].

A 4d massless scalar ϕ corresponds to a Z_2-flat direction iff its vertex operator at zero momentum is written in the form

$$\phi \sim \int dz d\bar{z}\; J_L(z) J_R(\bar{z}) \qquad (3.2)$$

where J_L, J_R are two real U(1) currents. When ϕ gets a non-zero vacuum expectation value, the mass-shifts are non-zero only for those string states which correspond to boundary conditions that leave J_L and J_R periodic; they are given by[7,8]

$$\delta M^2 = \frac{1}{2}(Q_L^2+Q_R^2)sh^2\theta + Q_L Q_R sh\theta ch\theta \qquad (3.3a)$$

where Q_L, Q_R are the charges of the states with respect to the two U(1) currents J_L, J_R and

$$\langle\phi\rangle \sim sh\theta \qquad (3.3b)$$

In fact, by introducing Φ_L, Φ_R which bosonize the currents in (3.2) such that $J_{L(R)} = \partial\Phi_{L(R)}$ and completing the basis of the bosonization appropriately, we can describe the same model in terms of a Z_2-orbifold, where Z_2 contains the transformation $\Phi_{L(R)} \to -\Phi_{L(R)}$. Since the product $J_L J_R$ is periodic under all boundary conditions because it corresponds to the vertex operator of the physical 4d scalar ϕ (3.2), in the new bosonized basis the string states form two sectors: the untwisted sector (UT) containing the states which correspond to boundary conditions in (2.6) that leave J_L and J_R (and thus Φ_L, Φ_R) periodic and the twisted sector (T) which correspond to J_L and J_R antiperiodic.

It is known that the partition function of a Z_2-orbifold takes the form[14]

$$Z = \frac{1}{2} Z_{UT}(\Gamma) + Z_T \qquad (3.4)$$

where $Z_{UT}(\Gamma)$ is the partition function of some momentum lattice Γ and the Z_2 transformation is an automorphism of Γ. The momenta in the untwisted sector have the form $(Q_L\cdots;Q_R\cdots)$ where Q_L, Q_R are the charges of the states with respect to the U(1) currents J_L, J_R, the semicolumn separates left from right movers and under Z_2

$$(Q_L\cdots;Q_R\cdots) \to (-Q_L\cdots;-Q_R\cdots); \qquad (3.5)$$

in the twisted sector the momenta become $(0\cdots;0\cdots)$. Performing a Lorentz boost in the hyperplane of (Q_L,Q_R)

$$\begin{pmatrix} Q_L' \\ Q_R' \end{pmatrix} = \begin{pmatrix} ch\theta & sh\theta \\ sh\theta & ch\theta \end{pmatrix} \begin{pmatrix} Q_L \\ Q_R \end{pmatrix} \qquad (3.6)$$

we obtain a new consistent string model for every value of the angle θ, since inner products are preserved and (3.6) commutes with the Z_2 operation (3.5); thus, in the boosted lattice, the modular invariance conditions are satisfied and the Z_2 (3.5) remains an automorphism. The mass-shifts can be calculated from (3.6) using (3.1) and are given by (3.3); they are clearly non-zero only for states in the untwisted sector. A physical understanding of this fact is that in the twisted sector the currents J_L, J_R are antiperiodic and they do not have zero modes, therefore charge operators do not exist. Note that multicritical points correspond to various scalars ϕ_i of the type (3.2) such that $[J^0_{L_i},J^0_{L_j}] = 0$ and/or $[J^0_{R_i},J^0_{R_j}] = 0$, since in this case many inequivalent boostings of the form (3.6) are allowed to be done.

When J_L and J_R are always periodic under all boundary conditions of Ξ (2.6) simultaneously[7], there is no twisted sector and the transformation (3.6) is reduced to an ordinary boosting of a momentum lattice. In this case ϕ transforms in the adjoint representation of the gauge group since it has the same quantum numbers with the gauge bosons obtained by replacing, in the vertex operator (3.2), J_L or J_R by $\partial_z X^\mu$ or $\partial_{\bar z} X^\mu$ respectively; thus Q_L and Q_R correspond to massless gauge bosons and the rank of the gauge group is not reduced. However, when some boundary conditions leave J_L and J_R antiperiodic[8], the scalar ϕ transforms in another non-adjoint representation of the gauge group whose rank is reduced when $\langle\phi\rangle \neq 0$, and Q_L, Q_R do not correspond to massless gauge bosons. One can show[8], though, that these massive gauge bosons are not arbitrary, but they may become massless at some particular points of various scalar expectation values with enhanced gauge symmetry.

It is interesting to observe that without being referred to the massless spectrum, the existence of two real world-sheet currents J_L and J_R with the same periodicity properties (P,P) or (AP,AP) under all boundary conditions of Ξ in (2.6) is sufficient to guarantee the existence of a massless scalar, corresponding to a vertex operator (3.2) which is a flat direction at the string tree level, if it is also supplemented with the condition that the charge operator J_L^0 commutes with the world-sheet supercurrent T_F

$$[J_L^0, T_F^n] = 0 \tag{3.7}$$

where T_F^n are the Fourier coefficients of T_F. The last constraint ensures that world-sheet supersymmetry is respected.

It is finally amusing to try to rewrite the transformed theory in terms of free fermions. In the case of an ordinary boosting of a momentum lattice (J_L and J_R always periodic), it was shown that this is possible at the expense of introducing arbitrary, in general irrational, phases for the 2d fermions[7]. In the case of a Z_2-direction, however, although the initial model is expressed in terms of a Z_2-orbifold at some particular point, after the "boosting" it cannot any more be re-expressed back in terms of free fermions with commuting boundary conditions. It can be fermionized only at the expense of introducing non-commuting boundary conditions between the various sectors. To understand this phenomenon, consider a boson Φ which is fermionized in terms of two real fermions, $\partial\Phi = \psi_1\psi_2$. In the untwisted sector the momentum p of Φ is related to the phase of the complex fermion $\psi_1+i\psi_2$ according to Eq. (2.8) or equivalently:

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \to -\begin{pmatrix} \cos 2\pi p & -\sin 2\pi p \\ \sin 2\pi p & \cos 2\pi p \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{3.8a}$$

In the twisted sector the boson Φ being antiperiodic, ψ_1 is, for instance, periodic and ψ_2 antiperiodic:

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{3.8b}$$

Clearly the matrix in Eq. (3.8b) does not commute with an arbitrary rotation (3.8a).

The generalization of the above rule to include flat directions corresponding to arbitrary Z_N-orbifolds lines is straightforward[8]. A complex massless 4d scalar field ϕ corresponds to such flat direction iff its vertex operator at zero momentum has the form

$$\phi \sim \int dz d\bar{z}\; J_L(z) J_R^*(\bar{z}) \tag{3.9}$$

where J_L and J_R are two complex U(1) currents. Introducing complex 2d scalars Φ_L, Φ_R which bosonize these currents and completing the bosonization basis appropriately, one finds a description of the model in terms of a Z_N-orbifold. The string states form in this case several twisted sectors depending on the periodicity properties of J_L and J_R which are now various phases. The transformation which generalizes the O(1,1) "Lorentz-boost" (3.6) of the Z_2-case, and which at the same time leaves the modular invariance conditions unchanged and commutes with the Z_N-operation, is a two-parameter SU(1,1)/U(1) rotation; it has the same form with (3.6) but the angle θ becomes complex since, in this case, the momenta or charges of the states with respect to J_L and J_R are complex. This transformation leads to the following mass-shifts

$$\delta M^2 = \tfrac{1}{2}\{(|\mathrm{ch}\theta|^2-1)|Q_L|^2+|\mathrm{sh}\theta|^2|Q_R|^2 + \mathrm{ch}\theta\,\mathrm{sh}\theta^* Q_L Q_R^* + \mathrm{sh}\theta\,\mathrm{ch}\theta^* Q_L^* Q_R\} \tag{3.10}$$

for the states corresponding to boundary conditions that leave the currents periodic (untwisted sector); the mass-shifts are zero for all the remaining states. The above rule could formally be generalized to any conformal field theory and in particular to include possible flat directions corresponding to orbifold twisted sectors[15], although we have no direct proof of this statement.

Flat directions may play an important role in the string model building. Among other things, they were recently applied to explain fermion mass hierarchies in a flipped SU(5)×U(1) model constructed using the fermionic formulation of four-dimensional superstrings[16]. Moreover, in the same model, they play an essential role to get rid of all extra gauge factors so that the low-energy group is given by the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$.

4. SUPERSYMMETRY BREAKING

The breaking of space-time supersymmetry in string theories, at a scale hierarchically smaller than the Planck mass, seems to be a very hard, if not impossible, problem in perturbation theory. A first indication is that the method of gauge symmetry breaking described above cannot be applied[7] to break supersymmetry spontaneously because all U(1) charges that do not vanish for the massless spin-3/2 states, and one would like to boost precisely in their direction in order to give them masses according to (3.3), do not commute with the world-sheet supercurrent [see Eq. (3.7)]. Our basic result is that the scale of supersymmetry breaking is necessarily linked to the size of some internal dimension[9].

Similar conclusions are found in the particular case of string solutions with supersymmetry breaking by a Scherk-Schwarz mechanism[17]. Furthermore, recent parallel results[18] based on unitarity properties of N = 2 superconformal theories show that supersymmetry cannot be broken continuously by sliding the vacuum expectation value of a scalar field at an analytic point and in a flat direction of its potential. This does not, however, rule out a number of interesting possibilities: for instance, there could exist vacua with hierarchically suppressed supersymmetry breaking which cannot be continuously connected to supersymmetric ones. Or, the scale of supersymmetry breaking could be proportional to some internal radii, but with a constant of proportionality of the order, say, of 10^{-16}. Or, broken supersymmetry could characterize only the nearly massless, but not all of the massive string modes. Finally, it is not clear that the only continuous string parameters are vacuum expectation values of scalar fields; it is, in fact, noteworthy that in many interesting supergravity models[19] the scale of supersymmetry breaking is not tuned by a Higgs field. In contrast to these, our proof would, if extended to arbitrary string compactifications, exclude all of these possibilities, since we make no assumption about the mechanism producing a slightly massive gravitino or gaugino.

The idea of our proof is that world-sheet supersymmetry severely restricts the form of massless, or infinitesimally massive space-time spinors. Together with modular invariance, this places strong constraints on the allowed superstring spectra. In fact, it follows from the super-Virasoro algebra alone that in a Ramond sector

$$T_F(o)^2 = L_0 - \frac{c}{24} \tag{4.1}$$

where c is the central charge of the Virasoro algebra. One concludes that the conformal dimensions $h = (h_L, h_R)$ of the Ramond primary fields are bounded from below

$$h_L \geq \frac{c}{24} \tag{4.2}$$

and, thus, in any consistent string theory there are no tachyonic fermions. The minimum value of (4.2) corresponds precisely to zero "left-moving" mass [see Eq. (2.7)]:

$$m_L^2 = -\frac{1}{2} + \frac{1}{8} + h_L \tag{4.3}$$

where the factor 1/8 is the contribution of the two real, or one complex, transverse fermions ψ^μ which are periodic, since we are considering a Ramond sector and the "internal" central charge $c = 9$. Thus, a massless gravitino or gaugino corresponds to the vertex operator

$$\psi^\mu_{3/2} \sim \phi_0(z)\, \partial_{\bar z} X^\mu \tag{4.4a}$$

or

$$\psi^\mu_{\frac{1}{2}} \sim \phi_0(z)\, J^a(\bar z) \tag{4.4b}$$

where J^a are gauge currents and ϕ_0 is a Ramond primary field of the underlying 2d superconformal field theory with conformal dimension $h_0 = (c/24, 0)$.

A slightly massive gravitino or gaugino is obtained by replacing ϕ_0 in Eqs. (4.4) with some ϕ with conformal dimension $h = h_0 + \delta h$. The precise statement is then the following: if in a 2d superconformal field theory there exists such a Ramond primary field ϕ, world-sheet supersymmetry and modular invariance imply the existence of a sequence of Ramond primary fields ϕ_k, $k = 1, 2, \ldots$ with conformal dimensions $h_k = h_0 + (Nk+1)\delta h$, where N is some, not too large, integer. These will give rise to an entire tower of gravitinos or gauginos, which is a signal of decompactification of some internal dimension. We have not a general proof of the statement; we can prove it only for "Gaussian" models[9]. As an illustration, in what follows, we will present the proof in the fermionic case. It can easily be extended[9] to the bosonic formulations with generalized 2d supercurrents and to orbifolds.

In the fermionic construction a space-time spinor belongs to a Hilbert-space sector $\mathcal{H}_{\mathcal{A}}$, where $\mathcal{A}$ is some boundary condition matrix defined in Eqs. (2.2) with $\delta_{\mathcal{A}} = -1$. Moreover, the conformal dimension h_L in Eq. (4.3) is replaced by $(1/8)\alpha_G \cdot \alpha_G$ [see Eq. (2.7)], where α_G is the phase vector which corresponds to the diagonalization of the boundary condition matrix $\mathcal{A}_G$ of Eq. (2.2b) according to the convention (2.3). Since $\mathcal{A}_G$ belongs to the group of automorphisms of G, G being the group of realization of world-sheet supersymmetry, one has to examine the phase-vector length $\alpha_G \cdot \alpha_G$ as $\mathcal{A}_G$ ranges over Aut(G). This is minimized for some special automorphisms $\mathcal{A}^0_G$ with the following two crucial properties[6]:

(i) $\alpha^0_G \cdot \alpha^0_G = 1/6 \dim G = 3$, which is precisely the value necessary to make the left-moving vacuum in the sector $\mathcal{H}_{\mathcal{A}}$ massless, and

(ii) $N\alpha^0_G = 0 \pmod 2$ for some even integer N ($N < 12$ for the groups that interest us), which means that $\mathcal{A}^0_G$ is some small root of the identity.

Here we will restrict ourselves to inner automorphisms, the generalization to outer ones being straightforward. An inner automorphism is a group element in the adjoint representation; up to a conjugation it can be written as:

$$\mathcal{A}_G = e^{i\pi\vec\theta\cdot\vec H}, \tag{4.5}$$

with H_ℓ the mutually commuting Cartan generators. The eigenvalues of the matrix (4.5) are one for each of the rank(G) commuting generators, and $e^{i\pi\vec{\theta}\cdot\vec{\rho}}$ for every generator corresponding to the root vector $\vec{\rho}$. Thus, we find:

$$\alpha_G\cdot\alpha_G = \tfrac{1}{2}\,\mathrm{rank}(G) + \Sigma_{\substack{+ve\\ roots}} (\vec{\theta}\cdot\vec{\rho}-1)^2 \qquad (4.6)$$

Normalizing the length of the long roots to two, one has $\Sigma_{+ve}\rho^i\rho^j = c_G\delta^{ij}$ where c_G is the quadratic Casimir of the group G [c_G = n for SU(n) and n-2 for SO(n) with n > 5]. Defining

$$\vec{\theta}_0 = \frac{1}{c_G}\Sigma_{+ve}\,\vec{\rho} \qquad (4.7)$$

and using the Freudenthal-de Vries strange formula:

$$\vec{\theta}_0\cdot\vec{\theta}_0 = \frac{\mathrm{dim}G}{3c_G}, \qquad (4.8)$$

we may rewrite Eq. (4.6) as

$$\alpha_G\cdot\alpha_G = \frac{1}{6}\,\mathrm{dim}G + c_G(\vec{\theta}-\vec{\theta}_0)^2. \qquad (4.9)$$

It follows immediately that the minimum phase vector length is (1/6)dimG and it is obtained for the special automorphism

$$\mathcal{A}^0_G = e^{i\pi\vec{\theta}_0\cdot\vec{H}}. \qquad (4.10)$$

which is called "Coxeter element" and, among inner automorphisms, is unique modulo the choice of the subset of positive roots. Furthermore, $\mathcal{A}^0_G$ is an n^{th} root of the identity with n = c_G for a simply laced group, while n = 6 for SO(5); n is therefore a small integer, and the advertised integer N = 2n.

"Coxeter elements" (4.10) play then an important role in the fermionic construction of string models, since they can yield to massless space-time spinors. For instance, a massless gravitino corresponds to the state:

$$\partial_{\bar{z}}X_1^\mu|0\rangle_S, \qquad (4.11)$$

where the boundary condition vector S = $(1,\alpha^0_G;\alpha_R=0)$, the three positions of the vector being in correspondence with the twists of the three types of 2d fermions [see Eqs. (2.2) and (2.3)], in a self-explanatory notation. Using the mass formula (2.7), and the fact that near S there are no almost periodic fermions with infinitesimal frequency oscillators, it is easy to see that the only candidate of a slightly massive gravitino is of the form:

$$\partial_{\bar{z}}X_1^\mu|0\rangle_{S+\delta S}, \qquad (4.12)$$

where $\delta S = (0,\delta\alpha_G;\delta\alpha_R) \ll 1$, and the mass is

$$m^2_{3/2} = \frac{1}{8}\,(\delta\alpha_G)^2 = \frac{1}{8}\,(\delta\alpha_R)^2 \qquad (4.13)$$

The absence of a linear term in (4.13) is due to the fact that $\alpha^0_G\cdot\delta\alpha_G = 0$ since α^0_G minimized vector length. Suppose now that this state belongs to the Hilbert space of a four-dimensional string model. This means that S+δS is in the group Ξ of allowed boundary conditions, and the state (4.12) survives the fermion-number projections (2.6)

$$e^{i\pi b\cdot F} = -c^*\binom{S+\delta S}{b} = \begin{cases} 1 & \text{if } \delta_b=1 \\ \pm 1 & \text{if } \delta_b=-1 \end{cases} \qquad (4.14)$$

for every element b of the basis B. Equation (4.14) follows from the fact that on the state (4.12), $e^{ib\cdot F}$ acts respectively as the identity or as a chirality operator.

Because Ξ is a group under addition (mod2), and NS = 0 (mod2), we conclude that S+(Nk+1)δS with k = 1,2,... are also in Ξ. We therefore have an entire tower of candidate low-mass gravitinos

$$\partial_{\bar{z}}X_1^\mu|0\rangle_{S+(Nk+1)\delta S};\quad k = 1,2,\ldots \qquad (4.15)$$

with mass differences equal to $Nm_{3/2}$. These states satisfy left-right level matching due to the absence of a linear term in Eq. (4.13).

They also survive all fermion-number projections (2.6). Indeed, using relations (2.5) one has

$$c^*\binom{S+(Nk+1)\delta S}{b} = e^{(i\pi/2)Nkb\cdot S}\, c^*\binom{S+\delta S}{b}^{Nk+1} \qquad (4.16)$$

from which one concludes that the coefficients in the left-hand side of (4.16) satisfy Eq. (4.14) using that

$$\tfrac{1}{2}N\ S\cdot b = \begin{cases} 0(\mathrm{mod}2) & \text{if } \delta_b=1 \\ 0(\mathrm{mod}1) & \text{if } \delta_b=-1 \end{cases} \qquad (4.17)$$

which helps get rid of the extra phase. Equation (4.17) follows from the fact that $\alpha^0_G+b_G+1$, or $\alpha^0_G+b_G$ are the phase vectors of an automorphism, if b_G is an automorphism ($\delta_b=-1$) or antiautomorphism ($\delta_b=1$), respectively, and that $N\alpha^0_G = 0$ (mod2), and finally that α^0_G minimizes vector length over the group of automorphisms. This then completes our proof.

Note that if supersymmetry is not broken at tree level, then non-renormalization theorems[20] guarantee that it will not break through radiative corrections. A possible exception could occur if the gauge group contains an anomalous U(1) factor[21], but one can also show[9] that the chiral charge asymmetry is bounded from below by a not too small number. This makes it unlikely that supersymmetry will break, if at all, at the "weak" scale by such a mechanism, leaving non-perturbative effects as the final alternative, if one excludes the idea of having an extra dimension at relatively low energies. These difficulties in breaking space-time supersymmetry should, among other things, make us rethink about the necessity of having it as an approximate low-energy symmetry in string theories.

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