

# GOLDSTONE FERMION COUPLINGS AND SUPERSYMMETRY BREAKING

## IN SUPERSTRING MODELS

John Ellis, A.B. Lahanas\*) and D.V. Nanopoulos+)

CERN -- Geneva

#### ABSTRACT

We re-examine supersymmetry breaking in the observable sectors of superstring-inspired supergravity models by computing Goldstone fermion couplings at the one-loop level. We find that a single global U(1) phase invariance is sufficient to forbid masses for gauge non-singlet chiral scalar bosons, and that Heisenberg symmetry is not necessary.

<sup>\*)</sup>Permanent address: Nuclear and Particle Physics Section, Physics Department, University of Athens, Athens 157-71, Greece.

<sup>+)</sup>Permanent address: Physics Department, University of Wisconsin, Madison, WI 53706, U.S.A.

The most interesting supergravity models are those whose effective scalar potential has one or more flat directions at the tree level. These so-called noscale models [1] have no holes in the potential with cosmological constant of  $0(m_p^4)$ , which would be cosmologically catastrophic, and offer the possibility of dynamical generation of low mass scales by radiative corrections creating a nonvanishing effective potential along the flat direction. Moreover, no-scale models appear naturally in the low-energy effective field theories obtained from string Supersymmetry must be broken in such theories, and a likely mechanism appears to be gaugino condensation in a hidden sector of the low-energy The question then arises how this supersymmetry effective field theory [3,4]. breaking feeds through to the observable sector of the theory. It is well known that no supersymmetry breaking squared scalar masses appear at the one-loop level [5], whereas gaugino masses may appear at the one-loop level [6,7], and there are two-loop contributions to squared scalar masses [8]. These results were initially derived in the minimal superstring-inspired no-scale supergravity model whose Kähler potential is [2]

$$G = \ln(S+S^{\dagger}) + 3\ln(T+T^{\dagger}-g(C_{i},C_{j}^{\dagger})) - \ln|W(C)+W(S)|_{(1a)}^{2}$$

with

$$g(c^i, c^t_i) = c^i c^t_i$$
(1b)

where S and T are gauge-singlet fields, and the C<sup>i</sup> are non-singlet fields. The vanishing of the one-loop squared scalar masses was obtained by looking directly at the effective scalar potential [5]. Both this result and the non-vanishing one-loop gaugino mass were found in vacua which did not have a vanishing cosmological constant in general. Moreover, the reason for the vanishing one-loop squared scalar masses is not clear. A deeper understanding of this result would indicate whether it is more general than the simple model (1) in which it was initially derived, and so might be true in a more general class of superstring-inspired supergravity models. It has been suggested that the reason for the vanishing of the squared scalar masses at the one-loop level is a Heisenberg symmetry [9]

of the Kähler potential (1).

In this paper we argue that the Heisenberg symmetry (2) is not necessary for the squared scalar masses to vanish at the one-loop level. Instead, we show that the one-parameter global U(1) transformation  $C^i \rightarrow e^{i\alpha}C^i$ 

$$C^{i} \rightarrow e^{i\kappa}C^{i}$$
 (3)

is sufficient to guarantee their absence. This means that the specific form (1b) of the function  $g(C^i,C^+_j)$  in the Kähler potential (1a) is not necessary for the scalar masses to vanish. Any function  $g(C^iC^+_i)$  would do equally well, and such functions do not in general have the Heisenberg symmetry (2).

These results are derived by computing the one-loop corrections to the Goldstone fermion couplings, and using the results that (i) the one-loop effective action for a supergravity theory may be characterized by a Kähler potential with one-loop corrections [10], and (ii) all supersymmetry-breaking mass splittings within supermultiplets  $(X,\tilde{X})$  are proportional to the  $X-\tilde{X}$ -goldstino couplings. Although the sufficiency of our custodial U(1) symmetry could perhaps be derived in other ways, our technique of calculating the Goldstone fermion couplings may be useful in a broader context.

We start by reviewing the argument linking Goldstone fermion couplings to supersymmetry-breaking mass splittings within supermultiplets. We consider the matrix element of the supercurrent  $J^{\mu}$  between a boson B and a fermion F in a chiral supermultiplet:  $\langle B, p_B | J^{\mu} | F, p_F \rangle$ . Assuming parity conservation, and using the onshell condition  $p_F u = m_F u$  for the fermion spinor, the matrix element may be written as [11]

where  $q^{\mu} \equiv p_B^{\mu} - p_F^{\mu}$ . Current conservation  $q^{\mu} \langle B, p_B | J_{\mu} | F, p_F \rangle = 0$  imposes the following on-shell condition

$$\left[A(m_{B}^{2}-m_{F}^{2})+Bq^{2}-Cm_{F}+F(q^{2}-m_{F}^{2})\right]+\left[C+D(m_{B}^{2}-m_{F}^{2})+Eq^{2}+m_{F}F\right]_{B}^{V}=0$$
(5)

We now study the Goldstone fermion contribution to Eq. (4), which is

$$\langle B, p_{B} | J_{n} | F, p_{F} \rangle = \int \int_{\mathbb{R}^{2}} \int_{\mathbb{R}^{2}} (a + b / k_{B}) u$$

$$= \int \int_{\mathbb{R}^{2}} \int_{\mathbb{R}^{2}} (a + m_{F} b) (k_{B} - m_{F}) + q^{2} b J u$$
(6)

where f is the Goldstone fermion decay constant:  $\langle 0|J_{\mu}|\widetilde{G}\rangle = f\gamma_{\mu}$ . This tells us about the singular pieces of the form factors A, B, C, D, E, F in expression (4):

$$F = \frac{f}{q^2} \left( a + m_F b^2 \right) + \frac{f}{q^2} \left( a + m_F b^2 \right) +$$

Retaining only the leading terms at small  $q^2$  in Eq. (5), we find

$$A(0)(m_{\rm g}^2 - m_{\rm F}^2) + \lim_{q^2 \to 0} (q^2 F) - m_{\rm F} \lim_{q^2 \to 0} (C + m_{\rm F} F) = 0$$
 (8)

Substituting the Goldstone fermion expressions (7), we finish up with

$$A(0)(m_g^2 - m_F^2) = -fa(0)$$
 (9)

Therefore, since  $A(0) \neq 0$ ,  $\Delta m^2 = m_B^2 - m_F^2 \neq 0$  if and only if a(0) = 0. In perturbation theory, we have

$$A(b) = P_0 + K A_1 + ..., a(0) = K a_1 + ..., \Delta m^2 = K \Delta m_1^2 + ...$$
 (10)

and so

$$A_{o} \cdot \Delta m_{i}^{2} = -f \cdot \alpha_{i}$$
(11)

Hence, to see if  $\Delta m_1^2 = 0$ , we need only to see if  $a_1$  vanishes on mass-shell, i.e., if  $a_1((p_B-p_F)^2=0)|_{p_B^2=0, p_F^2=0}=0$ . Moreover, it is consistent to use tree-level masses in evaluating this one-loop quantity.

Our next step is to extract from the Kähler potential (la) with arbitrary  $g(C,C^+)$  the propagators needed to evaluate  $a_1$ . We start with the fermionic kinetic terms which are

$$\mathcal{L}_{E}^{kin} = G_{j}^{i} \bar{\chi}_{R_{i}} \chi_{R_{j}}^{j} + (hem. conj.)$$
 (12a)

where  $G_{i}^{j} \equiv \partial^{2}G/\partial\phi^{i}\partial\phi^{*}_{j}$  as usual is given by

$$-G_{i}^{j} = \begin{pmatrix} (5+5^{\dagger})^{-2} & 0 & 0 \\ 0 & 3e^{-2G/3} & -3e^{-2G/3}c \\ 0 & 3e^{-2G/3} & 3e^{-2G/3}c & 3e^{-2G/3}c \\ 0 & 3e^{-2G/3}g_{e} & 3e^{-2G/3}g_{e} + 3e^{-2G/3}g_{e}g_{e} \end{pmatrix}_{(12b)}^{(12b)}$$

and we denote  $\partial g/\partial C \equiv g^C$ ,  $\partial g/\partial C \equiv g_C$ , etc. The following matrix

$$A = \begin{pmatrix} S+S+ & O & O \\ O & \frac{1}{13}e^{9/3} & \frac{g^{c}}{\sqrt{3}g^{c}}e^{9/6} \\ O & O & \frac{1}{\sqrt{3}g^{c}}e^{9/6} \end{pmatrix}$$
(13)

has the property that  $AA^+ = -G^{-1}$ , and so can be used to diagonalize the kinetic terms (12): the transformed fermions  $f_R^i = (A^*)_j^i \chi_R^j$  have canonical kinetic terms

$$\mathcal{L}_{F}^{kin} = -\left(\overline{f}_{R_{i}} \mathcal{X}_{R}^{i} + \text{herm. conj.}\right) + \left(\overline{f}_{R_{i}} \left(A^{T} G^{T} \mathcal{X}_{A}^{*}\right)^{i} f_{R}^{i} + \text{herm. conj.}\right)$$
+ herm. conj.

In this new fermionic basis, the fermion mass terms are

$$\mathcal{J}_{F}^{\text{mass}} = -M_{F}^{ij} \mathcal{J}_{R_{i}} \mathcal{J}_{L_{j}} + \text{herm. conj.}$$
 (15a)

where

$$M_{F}^{ij} = e^{-4/2} \begin{pmatrix} (S+S^{\dagger})^{2} \frac{W}{W} & \sqrt{3}(S+S^{\dagger})(S+S^{\dagger})^{-} \frac{W}{W} \end{pmatrix} - \frac{e^{4/6}}{\sqrt{3}g_{c}^{c}} \frac{W^{c}}{W} \\ \text{sym} & 2 & -\frac{z}{3} \frac{e^{4/6}}{\sqrt{3}g_{c}^{c}} \frac{W^{c}}{W} \\ \text{met} & \text{ric} & \frac{e^{4/3}}{3g_{c}^{c}} \left( \frac{W^{c}}{W} - \frac{g_{c}^{c}}{g_{c}^{c}} \frac{W^{c}}{W} \right) \end{pmatrix}$$
(15b)

and in the tree-level vacuum

If we denote  $f_L \equiv (\tilde{S}_L, \tilde{T}_L, \tilde{C}_L)$ , the matrix (16) tells us that the  $\tilde{C}_L$  fermion is massless, whilst the  $\tilde{T}_L$  fermion has a mass  $2m_{3/2}$  and can be identified as the Goldstone fermion. It is indeed the only fermion with a bilinear coupling to the gravitino:

$$\mathcal{L}^{Y-\widetilde{T}} = -\sqrt{3} \, M_{3/2} \, \overline{\mathcal{I}}_{\mu_{L}} \, \mathcal{J}^{\mu} \, \widetilde{T}_{L} + \text{herm. conj.}$$
 (17)

Hence, after redefining

we recover the expected feature that the orthogonal  $\widetilde{T}_L^{\dagger}$  field is massless.

One-loop contributions to all can be divided into two categories: those containing gravitinos and those without. To evaluate the latter, we need the interactions contained in (14) and (15), and others which do not involve the gravitino, and come from the following Lagrangian terms:

$$-\overline{\chi}_{R_{i}} \underset{=}{\mathbb{Z}_{j}} \chi_{R}^{k} \left( G_{R}^{ij} + \frac{1}{2} G_{R}^{i} G^{j} \right) + \left( -\frac{1}{2} G_{Rl}^{ij} + \cdots \right) \overline{\chi}_{R_{i}} \chi_{L_{j}} \overline{\chi}_{L}^{k} \chi_{R}^{l} + \text{hem. conj.}$$

When extracting vertices from (14), (15) and (18), we assume that  $g(C,C^+)$  is invariant under the global U(1) transformation  $C \to e^{i\alpha}C$ , so that  $g = g(C,C^+)$  and that

$$\langle g^c \rangle = \langle g_c \rangle = \langle g^c \rangle = \langle g_{cc} \rangle = 0$$
 (19)

in the tree-level vacuum. These latter conditions are automatic if  $g(CC^+)$  is analytic and the tree-level  $\langle C \rangle = 0$  as we expect. Under these assumptions, the

<sup>\*)</sup> The tree-level potential for such a model [4] is positive semidefinite if S+S+,  $g_C^C > 0$  as required for the positivity of the kinetic terms, leading naturally to  $\langle C \rangle = 0$  at the tree level.

interaction terms relevant for the calculation of non-gravitino loop contributions to all are (in two-component notation and using the Minkowski metric):

$$J^{int} = 3J_{2} M_{3/2} e^{-\zeta G}_{3} T_{R}(\tilde{T}_{1}^{2} + h.c.) + e^{-\zeta G}_{3} J_{R}T_{1}^{2} \tilde{T}_{0}^{2} h_{1}^{2}$$

$$-3M_{3/2} e^{-\zeta G}_{3} \zeta_{G}' > CC^{\dagger}(\tilde{T}_{1}^{2} + h.c.) + \frac{1}{2} \zeta_{G}' > e^{-\zeta G}_{3} (C^{\dagger}_{0}^{2} C) \tilde{T}_{0}^{2} h_{1}^{2}$$

$$- e^{-\zeta G}_{3} J_{R}T_{1}^{2} \tilde{C}_{0}^{2} h_{C}^{2} - 2\zeta_{G}' \zeta_{e}^{2} e^{-\zeta G}_{6}(i J_{R}C \tilde{C}_{0}^{2} h_{1}^{2} C) \tilde{T}_{0}^{2} h_{1}^{2}$$

$$- \frac{3}{57} e^{-2\zeta G}_{3} \zeta_{G}' > [\delta^{4}T_{R}(C^{\dagger}_{0}C + h.c.) - i \delta^{4}T_{1}(C^{\dagger}_{0}C - h.c.)]$$

$$+ \frac{1}{3} (\tilde{T}_{R}\tilde{C}_{L})(\tilde{T}_{L}\tilde{C}_{R})$$

They give rise to the Feynman diagrams in Fig. 1: the first six terms give graphs (a) to (e), the seventh term comes from the scalar kinetic energy term  $G_{1}^{j}\partial_{\mu}z_{j}^{*}\partial^{\mu}z_{j}^{i}$  and is needed for graphs (f) and (g), and the eighth term gives graph (h). We have denoted  $g'(x) \equiv \partial/\partial xg(x)$  and  $T_{R,I}$  are the real and imaginary parts of the scalar component of the T superfield:  $T = (1/\sqrt{2})(T_{R}^{+i}T_{I}^{-})$ . Explicit evaluation of the graphs gives couplings

$$-4m_{32} \langle g' \rangle^{1/2} e^{-\langle G \rangle 6} (i \partial_{M} (\stackrel{+}{\tau} \sigma \overset{-}{\sigma} M \chi) \int d^{4}k \frac{k}{(k^{2} + m_{32}^{2})(q + k)^{2}} \rightarrow (21a)$$

$$-4m_{32} \langle g' \rangle^{1/2} e^{-\langle G \rangle 6} (i \partial_{M} (\stackrel{+}{\tau} \sigma \overset{-}{\sigma} M \chi) \int d^{4}k k_{\lambda} (k - k)_{\lambda} \\ (k - k)^{2} (k^{2} - 4m_{32}^{2}) \rightarrow -4m_{\lambda} e^{-\langle G \rangle 6} (21b) \\ \times (c^{4}k^{2} C_{\lambda}) \int d^{4}k \\ \times (c^{4}k^{2} C_{\lambda}) \int d^{4}k k_{\lambda} (k - k)_{\lambda} \\ \times (c^{4}k^{2} C_{\lambda}) \int d^{4}k k_{\lambda} (k - k)_{\lambda} \\ \times (c^{4}k^{2} C_{\lambda}) \int d^{4}k k_{\lambda} (k - k)_{\lambda} \\ \times (c^{4}k^{2} C_{\lambda}) \int d^{4}k k_{\lambda} (k - k)_{\lambda} \\ \times (c^{4}k^{2} C_{\lambda}) \int d^{4}k k_{\lambda} (k - k)_{\lambda} \\ \times (c^{4}k^{2} C_{\lambda}) \int d^{4}k k_{\lambda} (k - k)_{\lambda} (k - k)_{\lambda} \\ \times (c^{4}k^{2} C_{\lambda}) \int d^{4}k k_{\lambda} \\ \times (c^{4$$

$$-\frac{8}{3}m_{\frac{3}{2}}\langle 9'\rangle^{\frac{1}{2}}e^{-\langle G'\rangle_{6}}\left(i\,2C^{\dagger}\tilde{\tau}\tilde{c}\right)\int_{\mathbf{k}^{2}(\left(\mathbf{k}-\mathbf{k}\right)^{2}-4m_{\frac{3}{2}}^{2})}^{d^{4}k}\frac{\mathbf{k}^{n}}{\mathbf{k}^{2}(\left(\mathbf{k}-\mathbf{k}\right)^{2}-4m_{\frac{3}{2}}^{2})}\longrightarrow 0 \tag{21h}$$

where we have indicated in each case the result of taking the limit  $p_B^2 = p_F^2 = (p_B^- p_F^-)^2 = 0$  and using the fact that  $m_B^- = m_F^- = 0$  at the tree level. It is transparent from adding together the mass-shell limits of (21a) to (21g) that

$$\left. \begin{array}{c} a_1(0) \Big|_{(2i)} = 0 \quad \Rightarrow \quad \left. \triangle m_1^2 \right|_{(2i)} = 0 \end{array} \right. \tag{22}$$

The vertices needed for the calculation of the one-loop gravitino diagrams shown in Fig. 2 are

where  $\widetilde{C}$  and  $\widetilde{\psi}_{\mu}$  are two-component spinors, and we again use the Minkowski metric. The graphs in Figs. 2a through 2h give couplings

$$-\sqrt{2} \langle g' \rangle^{\frac{1}{2}} e^{-\langle G \rangle_{R}} \left[ i \lambda C^{\dagger} \widetilde{T}_{\alpha} \left( \sigma^{2} \overline{\sigma^{\nu}} \widetilde{C} \right)_{\beta} \right] \left[ d^{4} k \frac{\left( k - \frac{1}{2} \right)_{\alpha} k_{\beta} S^{\lambda \lambda_{\beta}} d^{\beta} \left( k - \frac{1}{8} \right)}{\left( k - \frac{1}{2} \right)^{2} k^{2}} \rightarrow 0$$
 (24a)

$$-\sqrt{2} \langle g' \rangle^{2} e^{-\langle G \rangle / 6} i \partial_{\lambda} C^{\dagger} \left( d^{4} k \frac{(k-p)_{a}(k-p)_{a}k_{\tau}}{(k-p)^{2}k^{2}} (f S^{a})(k-p)_{a} \sigma^{\tau} \tilde{C} \right) \rightarrow 0 \quad (24b)$$

$$2 \langle g \rangle^{\frac{1}{2}} e^{-44} \frac{1}{6} i \partial_{k} C^{\dagger} \int_{0}^{4} d^{4}k \frac{(q-k)}{(q-k)^{2}} \left( \widehat{C} S^{\dagger + k}(k) \widehat{T} \right) \longrightarrow 0$$
(24e)

$$-4\langle g'\rangle^{\frac{1}{2}}e^{-\langle G\rangle_{k}}i\partial_{k}C^{\dagger}\int d^{q}k\frac{(k-k)_{m}}{(k-k)^{2}}(\mathcal{E}S^{m}(+k)\widehat{\tau})\rightarrow 0 \tag{24f}$$

$$-24 m_{3/2}^2 \langle g' \rangle^{1/2} e^{-\langle G \rangle / 6} i \partial_{\lambda} C^{\dagger} (\widetilde{T} S^{M}) (-q) \widehat{C}) \frac{b}{b^2} \int_{\overline{Q} - k)^2} (k^2 - 4 m_{3/2}^2) \longrightarrow 0$$
 (24g)

$$-2 m_{32}^{2} < 9' > 2 e^{-(4)} k_{1} a_{1} c_{2} c_{3} c_{4} c_{5} c_{4} c_{5} c_{4} c_{5} c_{5} c_{4} c_{5} c_{5} c_{4} c_{5} c_$$

where  $S^{\mu\lambda}(k)$  is the gravitino propagator. In every case the coupling is proportional to  $\partial_{\mu}C$  and hence vanishes in the mass-shell limit. We therefore conclude that

$$\left. \left. \left\langle \alpha_{1}(0) \right|_{(24)} \right|_{(24)} = \bigcirc \qquad \Longrightarrow \quad \left. \left\langle \Delta w_{1}^{2} \right|_{(24)} \right|_{(25)}$$

It follows that there are no supersymmetry-breaking squared scalar masses at the one-loop level in the effective potential derived from the tree-level Kähler potential (la), under the assumption that g has a global U(1) symmetry:  $C \rightarrow e^{i\alpha}C$  and is an analytic function of  $CC^+$ . The multi-parameter Heisenberg symmetry (2) is sufficient but not necessary to prove that  $\Delta m_1^2 = 0$ . Our U(1) symmetry does seem to be the minimal necessary: it is easy to write down additional interaction vertices and one-loop diagrams which appear if the U(1) symmetry is relaxed, and in general contribute to  $a_1(0)$  and hence  $\Delta m_1^2$ . These could of course be cancelled by some other symmetry, but it is unlikely to be more minimal than the global U(1) symmetry that we have identified.

The approach to evaluating supersymmetry-breaking mass splittings that we have adopted in this paper may be useful for other applications. We have in mind the exploration of other superstring-inspired no-scale supergravity models, and the extension to two-loop effects. The advantage of this approach, combined with the results of Ref. [10], are that one can in principle proceed to construct systematically higher-loop approximations to the full effective action for any N = 1 supergravity theory, including those inspired by the superstring.

## REFERENCES

- [1] E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, Phys. Lett. 133B (1983) 61; J. Ellis, A.B. Lahanas, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 134B (1984) 429; J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. B241 (1984) 406.
- [2] E. Witten, Phys. Lett. 155B (1985) 151; S. Ferrara, C. Kounnas and M. Porrati, Phys. Lett. 181B (1986) 263; I. Antoniadis, J. Ellis, E. Floratos, D.V. Nanopoulos and T. Tomaras, Phys. Lett. 191B (1987) 96; S. Ferrara, L. Girardello, C. Kounnas and M. Porrati, Phys. Lett. 192B (1987) 368 and 194B (1987) 358.
- [4] M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. 156B (1985) 55.
- [5] J.D. Breit, B.A. Ovrut and G. Segré, Phys. Lett. 162B (1986) 347;
  P. Binétruy and M.K. Gaillard, Phys. Lett. 168B (1986) 347.
- [6] P. Binétruy, S. Dawson and I. Hinchliffe, Phys. Lett. B179 (1986) 262; and Berkeley preprint LBL-22322 (1986).
- [7] J. Ellis, D.V. Nanopoulos, M. Quiros and F. Zwirner, Phys. Lett. 180B (1986) 83.
- [8] J. Ellis, A.B. Lahanas, D.V. Nanopoulos, M. Quiros and F. Zwirner, Phys. Lett. B188 (1987) 408.
- [9] P. Binétruy and M.K. Gaillard, LAPP-Berkeley preprint LAPP-TH-188/87 (1987),
   LBL-23429, UCB-PTH-87/14 (1987), to appear in Phys. Lett.;
   U. Ellwanger and M. Schmidt, Heidelberg preprint HD-THEP-86-21 (1986).
- [10] E. Floratos, A.B. Lahanas, C. Chiou-Lahanas and X. Maintas, CERN preprint TH.4826/87 (1987).
- [11] See, for instance: C.H. Llewellyn Smith in "Fundamental Forces", edited by D. Frame and K.J. Peach, Proceedings of the 27th Scottish Universities Summer School in Physics, St Andrews, August 1984.

### FIGURE CAPTIONS

- Fig. 1 : One-loop contributions to the Goldstone fermion vertex which do not involve gravitinos.
- Fig. 2 : One-loop contributions to the Goldstone fermion vertex which do involve gravitinos.

