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A LARGE CLASS OF CALABI-YAU VACUA

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ABSTRACT

A brief review of the construction of all complete intersection Calabi-Yau manifolds is given. These manifolds serve as covering spaces for more realistic vacua with Euler number -6 and -8, and we describe an exhaustive search for such manifolds. Low energy phenomenology is determined by the zero-modes or cohomology on these spaces, about which much is now known.

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1. INTRODUCTION

The first, and perhaps the simplest, viable candidate for a unified model of gravity and all other interactions was the heterotic string. It can be compactified from 10 to 4 dimensions on a smooth background $M^4 \times K^6$ provided K^6 satisfies a simple topological condition; it must have vanishing first Chern class [1]. Although it was historically important that such spaces, which are called Calabi-Yau manifolds, *admit* a unique Ricci-flat metric (in each Kähler class), this is not a solution of the string beyond lowest order in perturbation theory [2] and it seems unlikely that it will be recovered as an exact, non-perturbative solution. Fortunately this is of little practical importance since most, perhaps all, properties of the low energy effective field theory relevant in all forseeable future are determined only by the topology of the hidden space [3].

In the following we describe how to investigate a very large class of Calabi-Yau manifolds [4]. We start by constructing all complete intersection Calabi-Yau (CICY) manifolds. The technique displays all the magic of complex algebraic geometry without any of its complications, and is by far the simplest way to make a manifold. CICY manifolds are in fact so simple that their topology can be thoroughly investigated, even by a machine.

However, all CICY manifolds are probably simply connected, so they cannot by themselves serve as background solutions since we at some point wish to break the GUT group by using the natural Wilson line mechanism. We therefore search for more realistic vacua which are constructed by identifying points on a CICY manifold which are related by a freely acting discrete group G_0. The new manifold $CICY/G_0$ not only has G_0 as fundamental group, but the

Euler number (which is a density integrated over the manifold) of the covering space is also reduced by a factor equal to the order of G_0. This is important because there are no CICY manifolds with Euler number ± 6, which is needed to give three generations of massless particles (we assume here the standard embedding of the spin-connection into the gauge-connection [1]). Thus any CICY with Euler number $\pm 6k$ or $\pm 8k$ ($k = integer \geq 2$) is a viable starting point for constructing a 3 or 4 generation model in this simple and elegant manner.

Although this would seem to be good news for model builders, who often regard Calabi-Yau manifolds as attractive (because they are non-singular) but hopelessly complicated, there are surprisingly few manifolds in this class which admit 3 or 4 generations. In fact, as far as we can see there is only *one* three generation model of this type [5].

This does perhaps increase the predictive power of this approach. But it may also indicate that it is unlikely that a realistic model will be found in the near future unless the string dynamics has chosen precisely *this* manifold as the ground state. This is extremely unlikely since not only are there *at least* thousands (perhaps even an infinite number) of smooth solutions, all of which are of the Calabi-Yau type as far as we know, but millions of orbifolds also seem to be viable background spaces for strings. This 'vacuum degeneracy problem' is one of the most serious facing the string program. We hope that our partial classification, perhaps in conjunction with results from conformal field theory, will help in understanding the vacuum structure of the string.

2. CONSTRUCTION OF ALL CICY MANIFOLDS

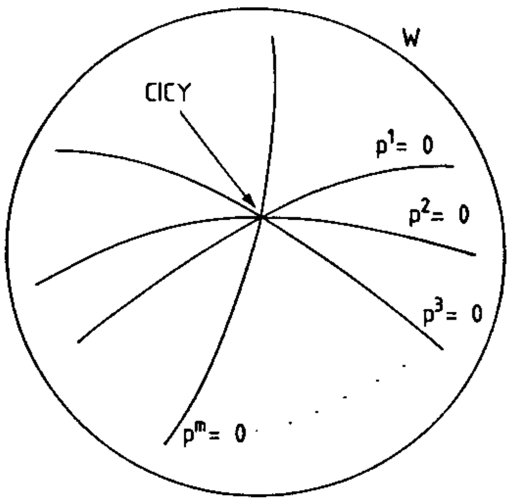
The first thing to realize about complex manifolds [6] is that they despite their name are much simpler than real manifolds, which do not have enough structure to be usefully constrained. While any submanifold of $\mathbf{R}^n$ is a real manifold, the only compact complex submanifold of $\mathbf{C}^n$ is a point. The natural way to avoid this 'no go' theorem is to include the point at infinity and 'compactify' $\mathbf{C}^n$ to the complex projective space P_n. Submanifolds of P_n are not in general trivial, and easily constructed by intersecting hypersurfaces defined by the vanishing set of polynomials in the homogeneous variables of the ambient projective space.

The first remarkable property of complex geometry, which is not true in the real case, is that *any* polynomial constraint has codimension one, i.e. the dimension of the hypersurface is always precisely one less than the ambient dimension.

Any manifold that can be realized by polynomial constraints in P_n (or a product of projective spaces, which is essentially the same thing) is called *algebraic*. From Kodaira's famous embedding theorem and the fact that three dimensional Calabi-Yau manifolds never have any nontrivial holomorphic 2-forms, it follows that all Calabi-Yau manifolds are algebraic.

The common zero-locus of several constraints ($p^1 = 0; p^2 = 0; \ldots ; p^m =$ 0) does not necessarily have codimension m. When it does we have a *complete intersection* (see Fig.1) and all discrete topological properties of the manifold are determined by the *degrees* of the defining polynomials alone! We restrict attention in this section to to this 'best of all worlds', and construct all CICY manifolds [4].

Fig.1. A CICY manifold is the common zero-locus of polynomial constraints in a projective ambient space W.



The number of CICY manifolds is finite because there are only a small number of different ambient spaces $W = P_{n_1} \times P_{n_2} \times \ldots \times P_{n_F}$ into which the CICY manifolds can be embedded [7]. This follows from the fact that the multidegrees d_{ij} of the j'th constraint in the coordinates of the i'th ambient factor must satisfy the condition for vanishing first Chern class:

$$\sum_{j=1}^{m} d_{ij} = n_i + 1, \tag{1}$$

where m is the number of constraints and n_i is the complex dimension of P_{n_i}, and that a bilinear constraint in $P_1 \times P_1$ is simply a P_1, which we write as:

$$\begin{matrix} P_1 \\ P_1 \end{matrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = P_1. \tag{2}$$

Since a linear constraint in P_n is just a P_{n-1} we also require

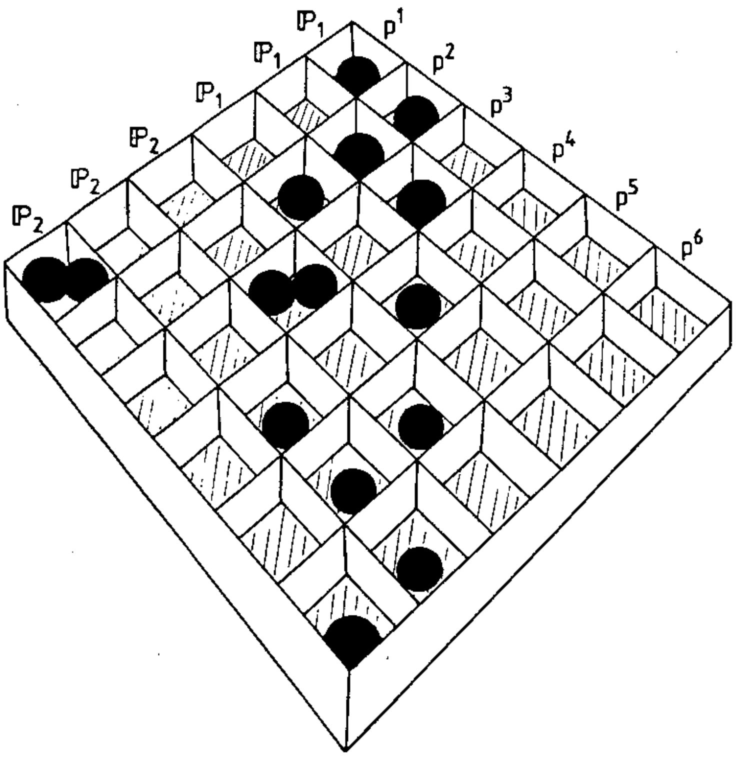
$$\sum_{i=1}^{F} d_{ij} \geq 2 \quad (j = 1, 2, \ldots, m). \tag{3}$$

It is convenient to think of the degree matrix (d_{ij}) as a 'pillbox' like the one shown in Fig.2, where the number of marbles in compartment (i, j) is d_{ij}. The number of columns (constaint equations) must be

$$m = \sum_{i=1}^{F} n_i - D \tag{4}$$

in order to bring the (complex) dimension of the complete intersection manifold down to D.

Fig.2. The construction of all possible CICY manifolds reduces to combinatorics. The space represented by this picture has Euler number −8.



By playing with this picture you can now easily convince yourself that the logistics of distributing marbles over a big pillbox while satisfying all the above restrictions are such that it cannot be done when the size of the box exceeds $(F \times m) = (5D \times 6D)$. The ambient spaces range from P_{D+1} to $(P_1)^{3D} \times (P_2)^{2D}$, with 235 possible choices when $D = 3$. For each of these ambient spaces we now instruct a computer to sprinkle the marbles in all possible (distinct) ways so that each row sums to $n_i + 1$. After also taking into account a number of 'reduction rules' analogous to (2), the result is that there are 7868 distinct degree matrices available to represent 3-dimensional CICY manifolds, with 70 different Euler numbers ranging from 0 to -200. Notice that each manifold may be represented in many different ways in this list, so that we have not *classified* them by this explicit construction.

The degree matrices contain all the information on the discrete topology of these manifolds. In the following sections we shall see how to extract this information.

3. THE SEARCH FOR THREE GENERATIONS

The CICY manifolds are not by themselves viable candidates for superstring vacua, as explained in the introduction, but they are excellent covering spaces (M) for realistic manifolds of the type $\tilde{M} = M/G_0$ [5]. $\tilde{M}$ is obtained by identifying points on M which can be moved into each other by acting with the group G_0, which must be fix-point free in order to avoid sigularities on $\tilde{M}$. (If M is a flat torus and G_0 has fix-points the variety $\tilde{M}$ is called an orbifold.)

To actually exhibit such a group action is usually not easy. First a set of defining polynomials must be chosen for M with the required multidegrees given by the degree matrix and sufficiently general so that the intersection is smooth. This fixes the shape of the manifold which now may posess some discrete (but not continuous) symmetries. If M has Euler number $-6k$ or $-8k$, and we can find a freely acting group G_0 of order k, then the quotient manifold $\tilde{M}$ will have Euler number -6 or -8, which gives 3 or 4 generations of massless matter particles after compactification.

A prototypical example is the Tian-Yau manifold [8], whose covering space is the CICY:

$$\begin{matrix} P_3(x) \\ P_3(y) \end{matrix} \begin{bmatrix} 3 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}^{14}_{-18} \tag{5}$$

which has Euler number -18 and 14 harmonic 2-forms. The 'maximally symmetric' realization

$$\begin{aligned} p^1 &= \sum_{A=0}^{3} x_A^3 = 0 \\ p^2 &= \sum_{A=0}^{3} x_A y_A = 0 \\ p^3 &= \sum_{A=0}^{3} y_A^3 = 0 \end{aligned} \tag{6}$$

admits a freely acting Z_3 symmetry which can be realized by cyclically permuting the first three homogeneous coordinates of each space and multiplying the fourth by cubic roots of unity ($\alpha^3 = 1, \alpha \neq 1$):

$$(x_0, x_1, x_2, x_3) \times (y_0, y_1, y_2, y_3) \rightarrow (x_1, x_2, x_0, \alpha x_3) \times (y_1, y_2, y_0, \alpha^2 y_3) \tag{7}$$

The quotient manifold obtained by identifying points under this group action has Euler number -6, corresponding to three generations.

We know of no topological criteria that are sufficient for deciding whether such a group action exists for some choice of moduli (coefficients in the polynomials), for a given degree matrix. Necessary conditions are however easy to find, and they turn out to be strong enough to eliminate all but three matrices as representing candidate 3-generation vacua. The number of 4-generation survivors is larger, as it should be since several different realizations are known, but there are not many [9].

Our tests are generalizations of the criterion discussed above that the Euler number must be divisible by the order of G_0. This is because by the index theorem any topological index on the manifold (M) can be expressed as an integral over M of a density. The Euler number (χ) is just one of the four classical indices, which count various zero-modes on the tangent bundle T_M of the manifold. On Calabi-Yau manifolds the only non-trivial information is contained in χ, but this changes drastically if we instead compute the indices on $T_M \otimes V$, where the vector bundle V in our case will be built from T_M and the bundle N normal to M inside the ambient space W. These 'twisted' indices are given by

$$I(T_M \otimes V) = \int_M class(M) \wedge ch(V), \tag{8}$$

where $class(M) \equiv class(T_M)$ is either the Chern polynomial $c(M)$, the Hirzebruch polynomial $L(M)$, the Dirac class $\hat{A}(M)$ or the Todd class $td(M)$. These caharacteristic classes are all simple polynomials in the Chern classes [10] which we can compute very easily on complete intersection manifolds.

The Chern character $ch(V)$ (which includes extra factors of 2 if class(M)=L(M)) is extremely well behaved under addition and multiplication of bundles:

$$ch(A \oplus B) = ch(A) + ch(B)$$
$$ch(A \otimes B) = ch(A) \wedge ch(B) \tag{9}$$

and has the simple expansion

$$ch(V) = \sum_k \frac{1}{k!} S_k(V) \ , \tag{10}$$

where $S_k(V)$ is the trace of the k'th power of the curvature 2-form on V. The Chern classes are also simply related to these elementary symmetric functions through the formulæ:

$$S_k - c_1 S_{k-1} + \ldots + (-)^k c_k k = 0 (k \geq 1). \tag{11}$$

Now let x_i denote the 2-form induced on M by the Kähler form on the i'th ambient factor P_{n_i}. Then the symmetric functions on N and M are [4]

$$S_k(N) = \sum_{j=1}^{m} (\sum_{i=1}^{F} d_{ij} x_i)^k \tag{12}$$

and

$$S_k(M) = \sum_{i=1}^{F} (n_i + 1) x_i^k - S_k(N). \tag{13}$$

Setting $k = 1$ in (12) and (13) we find the condition (1) for having vanishing first Chern class.

Combining the above results we see that computing any index twisted with any combination of T_M and N simply boils down to integrating polynomials in the x_i with coefficients that depend only on the degree matrix (d_{ij}). Since the x_i are easily lifted back to the ambient space where they came from, and integrals over P_n are trivial, we can compute a large number of indices in this way. Actually, it turns out that the only independent indices are obtained by restricting attention to $V = T_M^k \otimes N^l$, with $(k,l) = (1,0),(2,0),(3,0)$ or $(0,1)$.

After testing all these indices on the 7868 CICY manifolds for factors appropriate to bring the Euler number down to -6, only 189 survive.

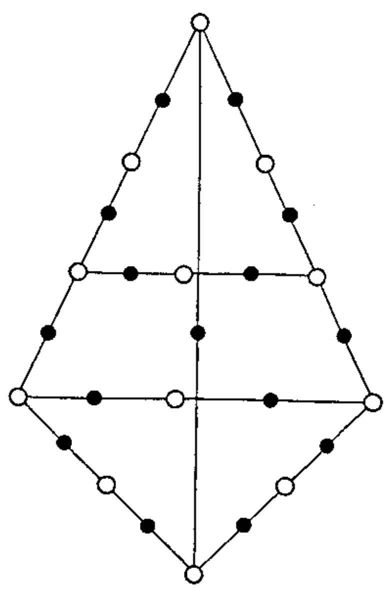
Further tests involving the normal bundles N_i to the individual hypersurfaces $p^i = 0$ can be devised if extreme caution is exercised. We must for example allow for the fact that the group action in some cases may permute factor spaces or polynomials, in which case the $N_i^{\prime}s$ involved cannot be used separately but must be combined into larger bundles. This process can to some extent be automated, and when these tests are implemented on the list of 189 only 21 survive.

A final and extremely laborious weeding must now be performed partly by hand. At this point it is essential to employ a beautiful diagrammatic representation of complete intersection manifolds which was introduced by Green and Hübsch [11]. Let black and white balls represent the constraints and ambient factors respectively, and connect each black ball (j) with each white ball (i) with as many links as the corresponding element d_{ij} of the degree matrix dictates. For example, one of the 21 survivors is:

$$
\begin{matrix} P_1 \\ P_1 \\ P_1 \\ P_1 \\ P_1 \\ P_1 \\ P_2 \\ P_2 \\ P_2 \\ P_2 \\ P_2 \\ P_2 \end{matrix}
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}^{12}_{-12}, \qquad (14)
$$

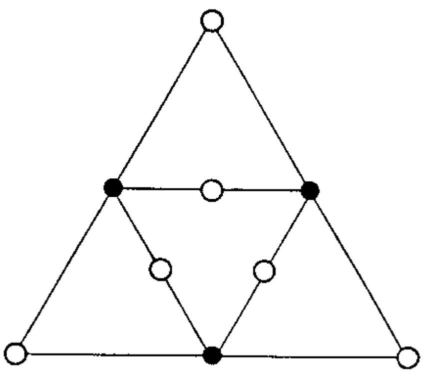
whose diagram may be drawn as shown in Fig.3. Clearly the human brain would prefer the latter representation.

Fig.3. One way to draw the diagram for the manifold with degree matrix (14).

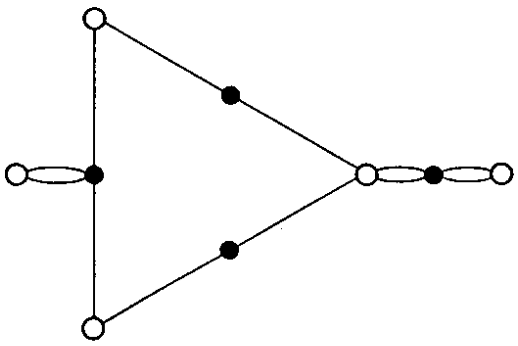


By massaging the diagrams for the 21 survivors into convenient symmetric shapes one may deduce the optimal allowed combinations of N_i-bundles, whose indices may then be tested for divisibility by the order of the required group. The final analysis leaves only the Tian-Yau manifold (5) and the following two manifolds with Euler number -48 as possible candidate three generation vacua:

$$\begin{matrix} P_1 \\ P_1 \\ P_1 \\ P_1 \\ P_1 \\ P_1 \end{matrix} \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}^{6}_{-48} \tag{15}$$



$$\begin{matrix} P_1 \\ P_1 \\ P_1 \\ P_1 \\ P_3 \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix}^{8}_{-48} \tag{16}$$



We have been unable to find freely acting groups of order eight on these manifolds despite vigorous efforts, and do not believe any exist. This leaves the Tian-Yau manifold as the *only* three generation model of this simple type *.

* A similar conclusion was reached in [12] using an erroneous argument.

4. ZERO-MODES AND COHOMOLOGY

Elementary particles as experienced in four dimensions are all massless compared to the compactification scale, and some of the massless fields on Calabi-Yau manifolds may be represented by harmonic forms [1]. These are *unique* representatives of the Dolbeault cohomology classes in $H^{(1,1)}(M)$ and $H^{(2,1)}(M)$. $H^{(1,1)}(M)$ is the group of closed modulo exact $(1,1)$-forms which represent matter superfields transforming as $(\mathbf{27}, \mathbf{1})$ (families) under the gauge group $E_6 \times E_8$, while $H^{(2,1)}(M)$ is the group of $(2,1)$-forms which represent the matter superfields transforming as $(\mathbf{27}^*, \mathbf{1})$ (anti-families).

The number of generations accessible in accelerators is $h^{11} - h^{21}$, where $h^{11} = dim H^{(1,1)}$ and $h^{21} = dim H^{(2,1)}$ are the *only* nontrivial Hodge numbers on a three-dimensional Calabi-Yau manifold. This number is easily obtained from any of several topological indices on the manifold, the simplest being the Euler number $\chi = 2(h^{11} - h^{21})$.

In addition there is in general a large number of scalars labeling the gauge supermultiplets [13], which are parametrized by the group $H^1(End T_M)$, i.e. the 1-forms which take values in the bundle $End T_M \simeq T_M \otimes T^*_M$. In other words 1-forms which transform as mixed rank-2 tensors (octets) under $SU(3)$. Clearly the numbers h^{11}, h^{21} and $h_{ADJ} \equiv dim H^1(End T_M)$ are needed in order to understand the low energy theory, as well as to classify the spaces.

At first sight the problem seems deceptively simple. Start by considering the simplest Calabi-Yau manifolds, originally called 'the Y-series' by physicists [1], which are the only five that can be embedded in a single projective space by complete intersection. They are $P_4[5]^1_{-200}$, $P_5[4,2]^1_{-176}$, $P_5[3,3]^1_{-144}$, $P_6[3,2,2]^1_{-144}$ and $P_7[2,2,2,2]^1_{-128}$, where the sub- and superscripts denote χ and h^{11}, respectively. There are several ways to derive these numbers.

The Euler number is easily computed using the expression for the third Chern class given above, since it is always just the integral of the top Chern class. Also, the naive expectation that the only 2-form on the hypersurface is the restriction of the Kähler-form on the ambient space can be verified in these cases by using the Lefschetz hyperplane theorem.

We can also use Kodaira's deformation theory to compute h^{21} directly. This approach is based on the fact that on Calabi-Yau spaces in D dimensions there is an isomorphism relating the tangent bundle T_M to its dual T_M^*:

$$T_M \simeq (T_M^*)^{D-1}, \tag{17}$$

which together with Serre duality:

$$H^q((T_M^*)^p) \simeq H^{(p,q)}(M) \tag{18}$$

gives the important relation:

$$H^1(T_M) \simeq H^{(D-1,1)}(M). \tag{19}$$

This means that $(D-1,1)$-forms on M can be represented by 1-forms which take values in the tangent bundle (i.e. they carry an extra tangent space index). Furthermore, the group $H^1(T_M)$ parametrizes the space of deformations of the complex structure on M. Since the shape of the manifold, and therefore its complex structure, is determined by the coefficients in the polynomial chosen to define the manifold, the dimension of $H^1(T_M)$ is under certain circumstances just the number of independent ways to choose these coefficients. For a manifold $P_{D+m}[d_1, d_2, \ldots, d_m]$ the count gives [14]:

$$h^{D-1,1} = \sum_{j=1}^{m}[\binom{D+m+d_j}{d_j} - \sum_{d_i \geq d_j} \binom{D+m+d_i-d_j}{d_i-d_j}] - (D+m+1)^2 + 1, \tag{20}$$

which can also be verified by rigorous methods when $D \neq 2$.

In two complex dimensions the only non-toroidal Calabi-Yau is K3, which has $h^{11} = 20$, while the formula (20) gives only 19. This is a warning that deformation theory in general may fail, due to socalled 'obstructions', and a more rigorous technique must be applied.

This is provided by something called 'spectral sequences', which have recently been used to compute all Hodge-numbers on all CICY manifolds [7,15] in three dimensions. We shall not repeat the construction here, but just point out one curious feature. It turns out that the manifolds divide into classes which must be treated separately, depending on whether the associated diagram is 'one-particle reducible' or not. More precisely, if the diagram after cutting a single leg separates into two pieces, one of which represents a one-dimensional manifold, then the spectral sequences are not useful. An example is provided by the Tian-Yau manifold (5), whose diagram

has two links of this type. However, when such *1-legs* are present the manifold may be realized as a hypersurface in the product of two complete intersection 2-folds with positive first Chern class, so that the Lefschetz hyperplane theorem may be applied [16].

The final result is that there are 264 distinct choices of Hodge-numbers (h^{11}, h^{21}) possible in the CICY class, which is therefore a lower bound on the number of topologically distinct manifolds.

The computation of h_{ADJ} has not yet been completed, but again the results for the Y-series are readily available. In the absence of obstructions, a deformation count analogous to the one described above gives [14]:

$$h_{ADJ} = \sum_{j=1}^{m} \Big[(D+m+1) \binom{D+m-1+d_j}{d_j - 1} - \binom{D+m+d_j}{d_j} - \sum_{d_i > d_j} \binom{D+m+d_i-d_j}{d_i - d_j} \Big]. \tag{21}$$

K3 has three representations of the 'Y-type' ($P_3[4]$; $P_4[3,2]$; $P_5[2,2,2]$), all of which have $h_{ADJ} = 45$ according to this formula. That this is the correct answer can be checked by computing the Dirac index on K3 twisted with an SU(2) vector bundle in the adjoint representation, and using the standard embedding of the spin-connection into the gauge-connection [17]. It is unfortunate that this argument does not go through in higher dimensions, so that brute force seems to be required also in this case. On CICY manifolds there will always be many E_6-singlets since h_{ADJ} is larger than h^{21} [18].

5. CONCLUDING REMARKS

As explained in the introduction the motivation for studying Calabi-Yau manifolds is both phenomenological and theoretical. We collect here some remarks relevant to both.

The method which was actually used for compiling the list of all CICY manifolds was somewhat different from what was described above, and much more efficient. The list of all CICY manifolds whose ambient spaces do not contain P_1-factors is rather short (a few hundred) and easy to make. We then generated all other CICY manifolds from these by a technique we called 'splitting' [4], which turns out to be a special case of what mathematicians call 'small resolutions'. By using small resolutions Schoen [19] has recently constructed new Calabi-Yau manifolds with Euler number +6 which may be viable 3-generation vacua, although it is not yet clear if they are non-simply connected.

We still do not know how many of the CICY manifolds are distinct, whether it be as real or complex manifolds. Simply connected real spin-manifolds with torsion-free cohomology can be uniquely labeled [20] by the Euler number, the 'intersection numbers' of 2-forms ω_i:

$$\mu_{ijk} = \int_M \omega_i \wedge \omega_j \wedge \omega_k, \tag{22}$$

and the numbers obtained by 'evaluating' the second Chern class on these 2-forms:

$$\nu_i = \int_M c_2(M) \wedge \omega_i. \tag{23}$$

It is curious that for the CICY manifolds in our list the ν_i seem to only contribute information about the 'connectivity' or 'one-particle reducibility' of the diagram representing the degree-matrix. Namely, let $\#_i^p (p = 0, 1)$ be the number of p-legs in the diagram which are attached to the i'th factor space P_{n_i}. A p-leg is a link which when cut leaves the diagram disconnected with one piece representing a p-dimensional manifold [11]. Then

$$\nu_i = 4\mu_{iii} + 24(n_i + 1 + \#_i^0 - \#_i^1). \tag{24}$$

Work on the classification proper is in progress [21].

The best way to address the vacuum degeneracy problem may be by studying conformal field theories on the world sheet, which is complementary to the space-time approach pursued above. The classification of conformal field theories would presumably include both orbifolds and Calabi-Yau manifolds, and it is perhaps in this context that our classification will prove most useful. For instance, it seems likely that the 'solvable' super-conformal field theories discussed by Gepner [22] elsewhere in these proceedings are closely related to the CICY class.

Whether one is interested in searching for phenomenologically interesting models (bottom-up approach: does the string *admit* our world as a solution?), or in solving the much more ambitious vacuum degeneracy problem on the string's own terms (top-down approach: does the string *predict* our world?), the vacua discussed here seem destined to play a major rôle.

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