TRI-PP-87-40
Mar 1987

Anomalous Magnetic and Quadrupole Moments of the W-boson in the Two-Higgs-Doublet Model

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ABSTRACT

We present limits on the anomalous magnetic and quadrupole moments of the W-boson in SU(2)⊗U(1) models with two Higgs doublets. We give the contributions to these moments from the charged and neutral Higgs bosons beyond the Standard Model. The main result is that these extra components increase the moments by 0.1% and 0.03% respectively.

(Submitted to Physical Review D)

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The two-Higgs-doublet extension[1] of the Standard Model (SM) has been studied for various reasons. These include spontaneous CP violation[2], and supersymmetric extensions of the SM[3]. It is also possible that the two-Higgs-doublets are among the debris left over from the breaking of some grand unified physics. Whatever the deeper reason for its existence may be, we deem that the effects of the two-Higgs-doublet extension are worth examining. In this paper, we focus on its contributions to the anomalous magnetic moment, κ, and the quadrupole moment,Q , of the W-boson. The important reasons for studying these quantities are well emphasized in the literature[4] and we shall not repeat them here. It suffices to mention that κ will be measured in experiments at LEP II[5],at the SSC, possibly at the Tevatron, at SLC[6], and also at high energy photon-electron colliders[7].

In the SM, κ is one at the tree level, and Q is zero. It is well known that a useful quantity to probe the physics beyond the SM is the deviation of κ from unity.We denote this deviation by $\Delta\kappa$. In particular , $\Delta\kappa \geq 1$ is not ruled out by current data[8].Theoretically, in some models of composite W-bosons a value as high as 3 can be obtained[9]. Moreover, in most weakly coupled, renormalizable theories, $\Delta\kappa$ is expected to be of order α/π; in marked contrast to composite models. An upper bound of $\Delta\kappa = 1.5$ % and $\Delta Q = 0.25$ % has been obtained in ref.10 for the SM, for favorable values of t-quark and Higgs boson masses.

The modest aim of this paper is to obtain similar bounds on $\Delta\kappa$ and ΔQ for the two-Higgs-doublet model. This is done with the assumption that perturbation theory holds for the model, which in turn implies that the masses of the three neutral and two charged Higgs bosons are less than a few Tev$/c^2$.[11] Otherwise, the theory becomes strongly interacting and invalidates our calculation. Furthermore, for technical simplicity, we will assume that all neutral Higgs bosons have the same mass, M_0. As we shall see later, the masses of the neutral Higgs bosons enter as

M_0^2/M_W^2, where M_W is the mass of the W boson, in logarithmic functions. Hence, using an average mass will not introduce large inaccuracies and reduces the number of free parameters. This enables us to see more clearly the physics that enters into the calculation.

As we shall see, an upper limit on $\Delta\kappa$ of the order of a percent is obtained and is less than the contribution of the t-quark. This is in agreement with expectation from a weakly coupled theory. It also means that $\Delta\kappa$ cannot be used experimentally to distinguish between different weakly coupled theories such as the SM with extra fermion families or a two-Higgs-doublet model; unless a measurement better than 5% is achieved. This will probably be very difficult with the above mentioned machines. On the other hand, if $\Delta\kappa \geq 20\%$ is measured, it would likely imply that non-perturbative physics is at work. This would be very interesting indeed!

In section II, we give a brief description of the two-Higgs-doublet model. We shall only present the contributions beyond the SM . Section III contains numerical results for $\Delta\kappa$ and ΔQ over the mass range 50 GeV/c^2 to 1 TeV/c^2 both for the neutral and charged Higgs bosons. Our conclusions and the relevance of the calculation when the model is embedded into the supersymmetric version are discussed in section IV.

II Calculation of $\Delta\kappa$ and ΔQ

We begin by writing down the terms we need in the two-Higgs-doublet model. The Lagrangian involving the scalar fields Φ_1 and Φ_2 is given by

$$\mathcal{L} = \sum_{i=1}^{2} (D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) + V(\Phi_1, \Phi_2), \qquad (2.1)$$

where

$$D_\mu \equiv \partial_\mu - i\frac{g'}{2}B_\mu - i\frac{g}{2}\vec{\tau}\cdot\vec{A_\mu}, \qquad (2.2)$$

with A_μ and B_μ denoting the SU(2) and U(1) gauge fields, respectively.The Yukawa terms are omitted and the specific form of the scalar potential is not relevant to us. After spontaneous symmetry breaking (SSB) we get

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ a_1/\sqrt{2} \end{pmatrix}$$

and

$$\langle\Phi_2\rangle = \begin{pmatrix} 0 \\ a_2/\sqrt{2} \end{pmatrix}, \qquad (2.3)$$

We shall assume CP invariance and set a_1 and a_2 to be real. Usually, one proceeds to translate the fields and derive the Feynman rules. We find it convenient to first rotate the fields via

$$H_1 = \Phi_1 \cos\alpha + \Phi_2 \sin\alpha \ , \qquad (2.4a)$$

and

$$H_2 = -\Phi_1 \sin\alpha + \Phi_2 \cos\alpha \ , \qquad (2.4b)$$

such that $\langle H_1\rangle = v/\sqrt{2}$ and $\langle H_2\rangle = 0$,where $v^2 = a_1^2 + a_2^2$ and $\tan\alpha = a_1/a_2$. H_1 can be interpreted as the real Higgs field. The advantage of rotating the fields first can be seen when H_1 and H_2 are written in terms of the physical and unphysical would-be-Goldstone fields. Explicitly, we write

$$H_1 = \begin{pmatrix} G^+ \\ v/\sqrt{2} + iG^0 + H_1^0 \end{pmatrix}, \qquad (2.5a)$$

$$H_2 = \begin{pmatrix} H^+ \\ H_2^0 + iH_3^0 \end{pmatrix} , \tag{2.5b}$$

where H_1^0 and H_2^0 are physical scalar fields, H_3^0 is a physical pseudoscalar field. The unphysical fields G^+ and G^0 are just the SM ones. The usual gauge fixing conditions hold. Substituting eqs.(2.5a) and (2.5b) into 2.1, it can be shown that G^0 does not mix with H_3^0 and similarly for G^+ and H^+ . Then, it is clear that new contributions come entirely from H^+ H_2^0 and H_3^0. Due to the assumed CP invariance H_3^0 will not mix with either H_1^0 or H_2^0. However, H_1^0 and H_2^0 can in general mix. We can parametrize this mixing with yet another angle θ. Denoting the mass eigenstates by h_1^0 and h_2^0 we get

$$h_1^0 = H_1^0 \cos\theta + H_2^0 \sin\theta , \tag{2.6a}$$

$$h_2^0 = -H_1^0 \sin\theta + H_2^0 \cos\theta . \tag{2.6b}$$

Henceforth, we shall work only with these physical states.

The vertices can be easily derived from the Lagrangian when expressed in the fields defined in eqs.(2.5) and (2.6).The useful ones are listed in fig.1 . Using these, we obtain the one loop contributions to the γW^+W^- vertex, which are depicted in fig. 2, where the kinematics is also defined.

Following previous notation[10,12] we can write the most general CP and electromagnetic gauge invariant vertex as:

$$\begin{aligned}\Gamma_{\mu\nu\lambda} =& ie\Big\{ A[2p^\lambda g^{\mu\nu} + 4(Q^\nu g^{\lambda\mu} - Q^\mu g^{\lambda\nu})] \\ &+ 2(\kappa - 1)(Q^\nu g^{\lambda\mu} - Q^\mu g^{\lambda\nu}) + 4\frac{\Delta Q}{M_W^2} p^\lambda Q^\mu Q^\nu \Big\}\end{aligned} \tag{2.7}$$

It can be shown that the diagrams of fig.2(b-d) contribute to gauge terms and are not relevant to $\Delta\kappa$ or ΔQ. The sole contribution comes from fig.2a. After some calculation and using standard techniques of dimensional regularization in

the t'Hooft-Feynman gauge we obtained:

$$\Delta\kappa = -3a \int_0^1 \frac{(-2t^4 + (2+F)t^3 - Ft^2)dt}{t^2 - tF + \delta} , \tag{2.8}$$

$$\Delta Q = 2a \int_0^1 \frac{t^3(t-1)dt}{t^2 - tF + \delta} , \tag{2.9}$$

where $a = g^2/96\pi^2$ and $F = 1 + \delta - \epsilon$ and where $\delta = \frac{M_0^2}{M_W^2}$ and $\epsilon = \frac{M_+^2}{M_W^2}$.

From these, we derive the upper bound on $\Delta\kappa$ and ΔQ for the two-Higgs-doublet model. The above equations were given for one neutral Higgs boson. One sums over h_1^0,h_2^0 and H_3^0 to obtain the total contribution.

III Results

The main results of this paper are contained in the previous equations. For completeness, the contributions in the SM are also given below[12]:

$$\Delta\kappa = 3a\int_0^1 \frac{(2t^4 - 2t^3 + 4t^2 - t^2\delta^2(t-1))dt}{t^2 + \delta^2(1-t)} \,, \qquad (3.1)$$

and

$$\Delta Q = 2a\int_0^1 \frac{(t^3 - t^4)dt}{t^2 + \delta^2(1-t)} \,, \qquad (3.2)$$

where δ^2 is now M^2_{Higgs}/M^2_W. From eqns. 2.8 and 2.9, we see that the main uncertainty comes from the unknown masses of the Higgs bosons; the dependance is logarithmic. In the limit of an infinite M_0 and finite M_+ we get

$$\Delta\kappa \to 2a \qquad (3.3)$$

$$\Delta Q \to 0 \qquad (3.4)$$

Similarly, when reversing those limits one gets

$$\Delta\kappa \to -a \qquad (3.5)$$

$$\Delta Q \to 0 \qquad (3.6)$$

In the case where both masses are large and equal, $\Delta\kappa$ and ΔQ are 0. The maximum value of ΔQ is 0.6a in our mass range and occurs when all masses take the minimum value allowed by current data. This peak decreases very quickly and by the time the masses are 300 GeV/c^2 the contribution is almost 0. These limits also serve as a useful check on the numerical integrations. In figs.3a and 3b, we show the dependence of $\Delta\kappa$ and ΔQ as a function of M_0 for fixed values of M_+, while figs.4a and 4b show the behaviour of $\Delta\kappa$ and ΔQ as a function of M_+ and M_0. Clearly, one sees that large contributions can come from small values of M_0 and

M_+. We have loosely chosen 50 GeV/c^2 as the lower limit for both masses.This is indicated by considerations[13] involving muon decays, asymmetries in e^+e^- annihilations and other reactions. Direct searches of charged Higgs set a much lower limit at $25GeV/c^2$ from e^+e^- annihilations. From figs. 4a and 4b, we see that the maximum contribution to $\Delta\kappa$ and ΔQ from the extra Higgs bosons are 0.1% and 0.03% respectively. These values allow us to conclude that the bound on $\Delta\kappa$ and ΔQ from the two-Higgs-doublet model is

$$(\Delta\kappa)_{max} = 32a = 1.6\% \qquad (3.7)$$

$$(\Delta Q)_{max} = 5.6a = 0.28\% \qquad (3.8)$$

This is to be compared with the SM values of

$$(\Delta\kappa)_{max} = 30a = 1.5\% \qquad (3.9)$$

$$(\Delta Q)_{max} = 5a = 0.25\% \qquad (3.10)$$

These are absolute maximum since we simply added the maximum value from the extra Higgs bosons of the THD model to the maximum value of the SM. We see that the extra Higgs bosons add insignificantly to $\Delta\kappa$ and ΔQ, from an experimental point of view.

IV Conclusions

We have calculated one loop corrections to the magnetic and quadrupole moments of the W boson in the two-Higgs-doublet extension of the SM. It is found that the maximum contributions to $\Delta\kappa$ and ΔQ in this extension are respectively 1.6% and 0.28%.This is in accordance with the expectation of perturbation theory if the Higgs boson is not strongly interacting. It was found previously that each possible heavy fermion in an SU(2) doublet gives a contribution to $\Delta\kappa$ less than 0.4%. It is interesting that heavy fermion doublets give a larger contributions to $\Delta\kappa$ than Higgs bosons. We can conclude that extending the SM by adding one extra fermion family and/or one extra Higgs doublet cannot add to $\Delta\kappa$ by more than 0.03. To get a large correction to the anomalous magnetic moment this way would require a ridiculously large number of Higgs bosons or fermion families. Enlarging the scalar sector by adding SU(2) singlets will not affect $\Delta\kappa$ or ΔQ significantly at the one loop level. Therefore, if a $\Delta\kappa$ of 10% or more were measured, it would be a very strong indication that non-perturbative physics is at work; such as compositeness of the W-boson or strongly interacting Higgs boson. We have certainly not exhausted all possible extensions of the Higgs sector in $SU(2) \otimes U(1)$ theories and determined their contribution to $\Delta\kappa$ and ΔQ. Our work does indicate that $\Delta\kappa$ and ΔQ are not sensitive to variations in this sector as long as it remains a weakly coupling one. It would be interesting to determine at least semi-quantitatively $\Delta\kappa$ and ΔQ for other, more elaborate embeddings of the Standard Model. In particular, the supersymmetric standard model with broken supersymmetry and a plethora of super-particles may add to $\Delta\kappa$ and ΔQ beyond the contributions we calculated. Exact calculations will be interesting and our work will be of use in such an attempt.

Acknowledgements: this work was supported in part by the U.S. Department of Energy, contract no.W-7405-Eng-82, Office of Basic Science (KA-01-01), Division of High Energy Physics and Nuclear Physics, and by Center of Particle Theory, DOE grant number DE-FG05-85ER40200, and by the Natural Sciences and Engineering Research Council of Canada.

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Figure Captions.

Figure 1

Vertices involved in the one-loop correction of the γW^+W^- vertex.

Figure 2

Feynman diagrams to the one-loop correction to the γW^+W^- vertex.

Figure 3a

$\Delta\kappa$ as a function of M_+ for different values of M_0: 100 GeV/c^2 (continuous line), 500 GeV/c^2 (dash-dotted line), 1000 GeV/c^2 (dotted line). The vertical scale is in units of a $(g^2/96\pi^2)$.

Figure 3b

ΔQ as a function of M_+ for different values of M_0: 100 GeV/c^2 (continuous line), 200 GeV/c^2 (dash-dotted line), 500 GeV/c^2 (dotted line). The vertical scale is the same as in fig. 3a.

Figure 4a

$\Delta\kappa$ as a function of M_+ ansd M_0. The vertical scale is the same as in fig. 3. Both Horizontal axes are 50-1000 GeV/c^2, the point (50,50) being the origin.

Figure 4b

ΔQ as a function of M_+ and M_0. The vertical scale is the same as in fig. 3. Both horizontal axes are 50-500 GeV/c^2.

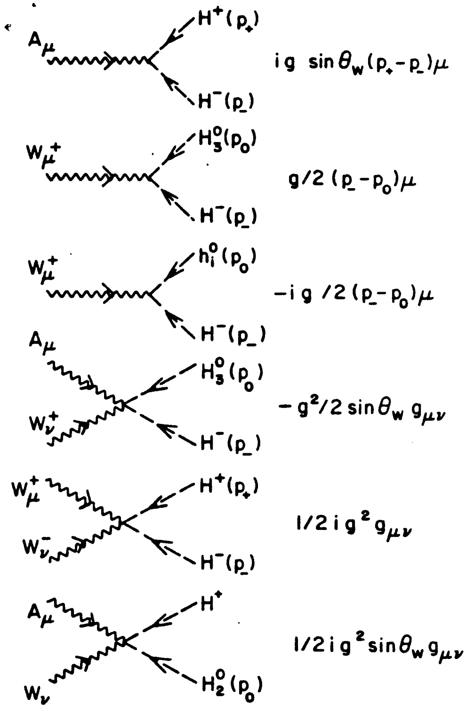

Fig. 1

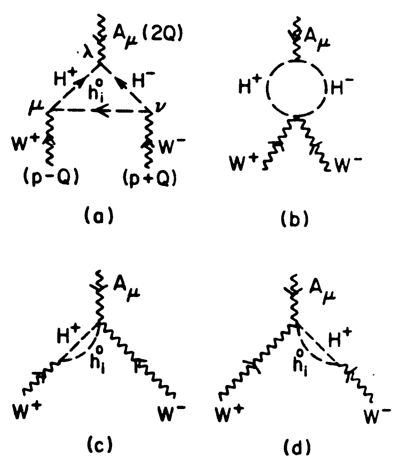

Fig. 2

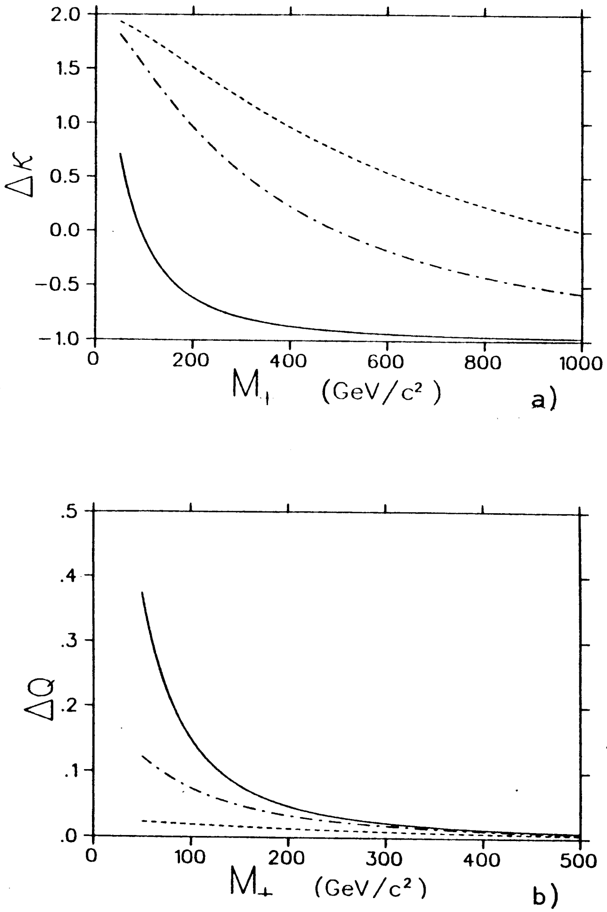

Fig. 3

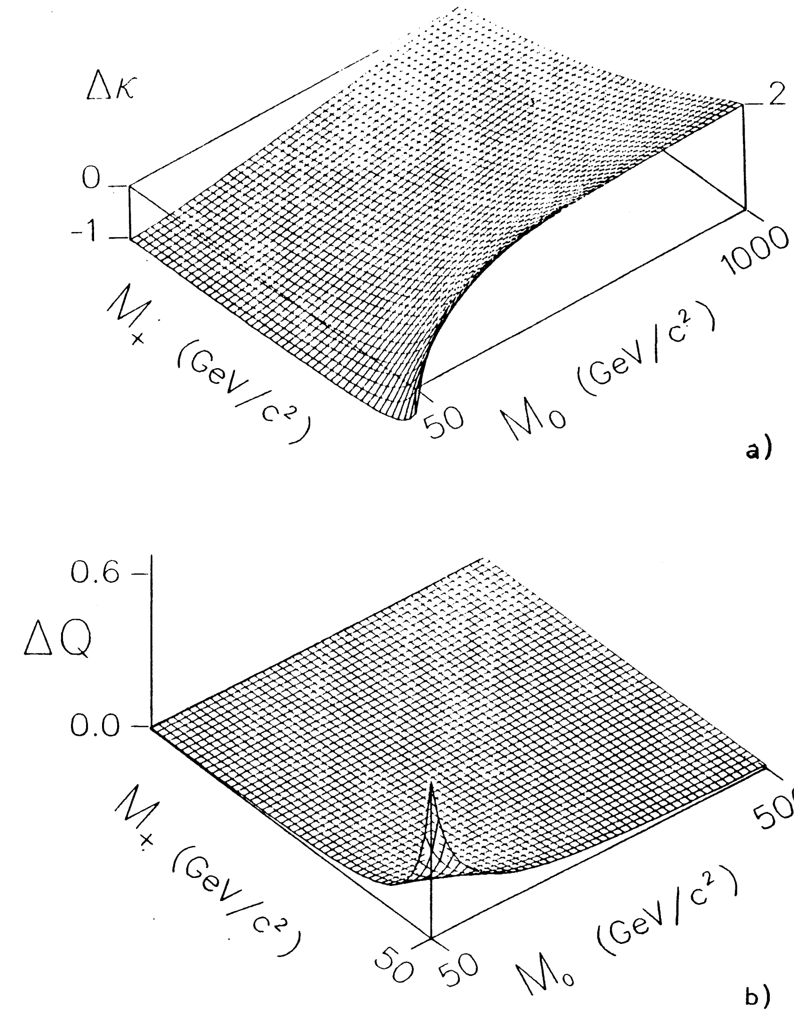

Fig. 4