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ON THE ULTRAVIOLET BEHAVIOUR OF SOFTLY BROKEN N=1-D=2 SUPERSYMMETRIC NON-LINEAR σ-MODELS

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Abstract

Explicit soft breaking terms of N=1-D=2 supersymmetry are added to the N=1 non-linear σ-model action. Supergraph methods are employed to analyse the structure of divergences induced by the breaking terms. Genuine three-loop divergences appear, that are proportional to a dimensionless breaking coupling parameter. They survive the *Ricci*-flatness condition and modify the three-loop β-function of the exact model, which is known to be vanishing.

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Two-dimensional supersymmetric non-linear σ-models consist of the ordinary bosonic non-linear σ-model coupled to fermions in such a way as to guarantee invariance under one, two or four supersymmetries. The restrictions on the nature of the target manifold of the supersymmetric model, arising from the requirement of invariance under extended supersymmetries, provide a very appealing connection between supersymmetry and complex manifold theory[1,2,3].

More recently, the interest for these 2-dimensional models has been stressed since σ-models defined on a *Riemann* surface and taking values in an arbitrary D-dimensional *Riemannian* space keep a close relationship with string theories[4]. Also very fascinating is the remarkable quantum behaviour of the supersymmetric σ-models in the ultraviolet limit. The N=1 and N=2 models are on-shell three-loop finite for *Ricci*-flat manifolds[5], whereas for N=4, finiteness holds through to all orders in perturbation theory[6].

In this letter, it is our purpose to pursue a reassessment of the convergence properties of a general N=1 σ-model when supersymmetry is explicitly broken. The breaking terms we propose to study here are all soft breakings which modify the scalar-fermion couplings of the model. For particular choices of target spaces, there may also occur a shift in the fermionic masses. Performing supergraph computations, suitably modified to account for the explicit breaking terms[7], we shall investigate the divergence structure of the softly broken σ-model and discuss how the breakings may affect the β-function of the exact model.

The following (anti-)commutation relations for N=1 supersymmetry in two dimensions will be of use in the course of our algebraic manipulations:

$$\{D_\alpha, \theta_\beta\} = C_{\alpha\beta}, \tag{1}$$

$$[D^2, \theta_\alpha] = D_\alpha, \tag{2}$$

$$[D_\alpha, \theta^2] = \theta_\alpha, \tag{3}$$

$$[D^2, \theta^2] = -1 + \theta^\alpha D_\alpha, \tag{4}$$

$$D_\alpha \equiv \partial_\alpha + i\theta^\alpha \partial_\alpha, \tag{5}$$

where θ is a Majorana spinor and C is the charge conjugation matrix. We shall adopt here the notation and conventions of ref.[8].

Scalar superfields $\Phi(x;\theta)$ can be defined by the following projections:

$$\begin{aligned} A(x) &= \Phi(x;\theta)|_{\theta=0}, \\ \Psi_\alpha(x) &= D_\alpha\, \Phi(x;\theta)|_{\theta=0}, \\ F(x) &= D^2\, \Phi(x;\theta)|_{\theta=0}, \end{aligned} \tag{6}$$

where $A(x)$ and $F(x)$ are respectively physical and auxiliary scalars and $\Psi(x)$ is the physical fermionic component of $\Phi(x;\theta)$.

Considering now a set of scalar superfields $\Phi^i(x;\theta)$ $(i = 1, \cdots, M)$, we add to the quadratic supersymmetric action,

$$L_0 = -\frac{1}{4}\int d^2\theta\, (D^\alpha \Phi^i)(D_\alpha \Phi^i) + \frac{1}{2}\int d^2\theta M_{ij} \Phi^i \Phi^j \quad , \tag{7}$$

terms which explicitly break N=1 supersymmetry; their net effect is to shift the masses of the physical scalar and spinor fields contained in $\Phi^i(x;\theta)$. They are collected into the breaking Lagrangian, L_B, given by

$$L_B = \frac{1}{2}\int d^2\theta\,\theta^2\,[m^2_{ij}\Phi^i\Phi^j + \mu_{ij}(D^\alpha\Phi^i)(D_\alpha\Phi^j)], \tag{8}$$

where m^2 and μ are real and symmetric $M \times M$ mass matrices.

Concentrating on the terms described in $L_0 + L_B$, the first step will consist in the derivation of the superpropagator $< \mathrm{T}(\Phi^i\Phi^j) >$ with the breaking parameters m^2 and μ taken into account to all orders.

By coupling the superfields $\Phi^i(x;\theta)$ of the Lagrangians (7) and (8) to external sources, it follows that the most general superpropagator has in principle the following form:

$$P(x_1,\theta_1;x_2,\theta_2) = -(1 + \sum_{n=1}^{12} X_n A_n(x_1,\theta_1)) \cdot \tag{9}$$

$$\cdot(D_1^2 + M)^{-1}\,\delta^2(x_1 - x_2)\delta^2(\theta_1 - \theta_2),$$

where the coefficients X_n are c-number valued $M \times M$ matrices to be determined, the operators A_n are defined below, and D_1^2 stands for $\frac{1}{2}D^\alpha(x_1,\theta_1)D_\alpha(x_1,\theta_1)$:

$$\begin{array}{ll}
A_1 \equiv D^2, & A_7 \equiv \partial_{\alpha\beta}D^\alpha\theta^2 D^\beta, \\
A_2 \equiv \theta^\alpha D_\alpha, & A_8 \equiv \partial_{\alpha\beta}\theta^\alpha D^\beta, \\
A_3 \equiv D^\alpha\theta_\alpha, & A_9 \equiv \partial_{\alpha\beta}D^\alpha\theta^\beta, \\
A_4 \equiv \theta^2 D^2, & A_{10} \equiv D^2 D^\alpha\theta^2 D_\alpha, \\
A_5 \equiv D^2\theta^2, & A_{11} \equiv D^\alpha\theta^2 D_\alpha D^2, \\
A_6 \equiv D^\alpha\theta^2 D_\alpha, & A_{12} \equiv \theta^2.
\end{array} \tag{10}$$

Due to the (anti-)commutation relations amongst the D's and θ's given in eqs.(1)–(4), the truly independent operators can be shown to be just A_1, A_2, A_4, A_8 and A_{12} : all the others can be written as linear combinations of them. Moreover, it can be readily seen that A_1, A_2, A_4, A_8 and A_{12} form a closed set under multiplication.

This observation considerably simplifies the task of obtaining the superpropagator of eq.(9), which then reduces to

$$P(x_1,\theta_1;x_2,\theta_2) = -(1 + XA_2 + YA_4 + ZA_8 + WA_{12})\cdot \qquad (11)$$
$$\cdot(D_1^2 + M)^{-1}\,\delta^2(x_1 - x_2)\delta^2(\theta_1 - \theta_2),$$

with the matrices X, Y, Z and W being determined from

$$\begin{aligned}
(1+A)X + \Box CZ &= -A,\\
CX + (1+A)Z &= -C,\\
2(A-B)X - (1+2A-B)Y + 2\Box CZ &= B,\\
2i\Box CX - 2i\Box(A-B)Z + (1+2A-B)W &= D.
\end{aligned} \qquad (12)$$

A, B, C and D are matrices given in terms of the mass matrices of the theory:

$$\begin{aligned}
A &= \frac{1}{\Box - M^2 - m^2}(M\mu + m^2),\\
B &= \frac{1}{\Box - M^2 - m^2}(2M\mu + m^2),\\
C &= i\frac{1}{\Box - M^2 - m^2}\mu,
\end{aligned} \qquad (13)$$

and

$$D = \frac{1}{\Box - M^2 - m^2}(Mm^2 + 2\mu\Box).$$

After manipulating the algebra of the D's and θ's, we find that our superpropagators take the following final form:

$$P(k;\theta_1,\theta_2) \equiv < \mathrm{T}(\Phi(1)\Phi(2)) >= \tag{14}$$

$$= \alpha(k^2)(D_1^2 - M)\,\delta^2(\theta_{12}) - \beta(k^2)\theta_1^\alpha D_{1\alpha}\delta^2(\theta_{12}) +$$

$$-\gamma(k^2)\theta_1^2 D_1^2\,\delta^2(\theta_{12}) + i\eta(k^2)k^{\alpha\beta}\theta_{1\alpha}D_{1\beta}\,\delta^2(\theta_{12}) +$$

$$-\varepsilon(k^2)\theta_1^2\,\delta^2(\theta_{12}),$$

where

$$\begin{aligned}\alpha(k^2) &\equiv \frac{1}{k^2+M^2+m^2},\\ \beta(k^2) &\equiv X\alpha(k^2)M + iZk^2\alpha(k^2),\\ \gamma(k^2) &\equiv Y\alpha(k^2)M - W\alpha(k^2),\\ \eta(k^2) &\equiv iX\alpha(k^2) + Z\alpha(k^2)M\end{aligned} \tag{15}$$

and

$$\varepsilon(k^2) \equiv Yk^2\alpha(k^2) + W\alpha(k^2)M.$$

We next wish to add to the action of the N=1 supersymmetric non-linear σ-model,

$$S = -\frac{1}{4}\int d^2x d^2\theta\, g_{ij}(\Phi)(D^\alpha\Phi^i)(D_\alpha\Phi^j), \tag{16}$$

new coupling terms which explicitly break supersymmetry but, contrary to the mass terms appearing in eq.(8), respect the diffeomorphism invariance of the target manifold, F. The terms we propose to study here are:

$$\frac{1}{2}\int d^2x d^2\theta\, \theta^2 \mu g_{ij}(\Phi)(D^\alpha \Phi^i)(D_\alpha \Phi^j) \tag{17}$$

and

$$\frac{1}{4}\int d^2x d^2\theta\, \theta^2\ \lambda R_{ijkl}(\Phi)(D^\alpha \Phi^i)(D_\alpha \Phi^l)(D^\beta \Phi^j)(D_\beta \Phi^k), \tag{18}$$

where μ and λ are respectively mass dimensional and dimensionless parameters and R_{ijkl} is the *Riemann* tensor of the target space F.

The breaking term (17), besides modifying the coupling between the physical scalars and fermions of the σ-model, may also shift the masses of the fields Ψ^i_α. Indeed, when F is a homogeneous space like the n-sphere, for example, μ is nothing but the mass of the fermionic component fields. As for the breaking term (18), it does not shift masses, but only affects the scalar-spinor couplings of the originally supersymmetric σ-model.

What is said above can be clearly seen if we rewrite the breaking terms (17) and (18) in terms of component fields. They read respectively as below:

$$-\frac{1}{2}\int d^2x\, \mu g_{ij}(A)\Psi^i\Psi^j \tag{19}$$

and

$$-\frac{1}{4}\int d^2x\, \lambda R_{ijkl}(A)(\Psi^i\Psi^l)(\Psi^j\Psi^k). \tag{20}$$

Before turning into our supergraph calculations, we would like to quote below the normal coordinate expansions of the tensors appearing in eqs.(17) and (18). We present their respective expansions only up to the order in the quantum field ξ^i relevant for our loop computations. The results are:

$$g_{ij}(\Phi+\Pi(\xi)) = g_{ij}(\Phi) - \frac{1}{3}R_{ik_1jk_2}(\Phi)\xi^{k_1}\xi^{k_2} - \frac{1}{3!}D_{k_1}R_{ik_2jk_3}(\Phi)\xi^{k_1}\xi^{k_2}\xi^{k_3}+$$
$$+\frac{1}{5!}[\frac{16}{3}R^m_{k_1jk_2}(\Phi)R_{k_3ik_4m}(\Phi) - 6D_{k_1}D_{k_2}R_{ik_3jk_4}(\Phi)]\cdot \qquad (21)$$
$$\cdot\xi^{k_1}\xi^{k_2}\xi^{k_3}\xi^{k_4} + 0(\xi^5)$$

and

$$R_{ijkl}(\Phi+\Pi(\xi)) = R_{ijkl}(\Phi) + D_m R_{ijkl}(\Phi)\xi^m + \frac{1}{2}[D_{m_1}D_{m_2}R_{ijkl}(\Phi)+$$
$$+\frac{1}{3}R^n_{m_1m_2i}(\Phi)R_{njkl}(\Phi) + \frac{1}{3}R^n_{m_1m_2j}(\Phi)R_{inkl}(\Phi)+ \qquad (22)$$
$$+\frac{1}{3}R^n_{m_1m_2k}(\Phi)R_{ijnl}(\Phi) + +\frac{1}{3}R^n_{m_1m_2l}(\Phi)R_{ijkn}(\Phi)]\xi^{m_1}\xi^{m_2} + 0(\xi^3),$$

where Φ is taken as a background field and $\Pi(\xi)$ is the quantum fluctuation expressed in terms of the true quantum field ξ^i [9].

With the results of eq.(14), the mass breaking parameter μ can be summed to all orders in the superpropagators we shall employ in our loop evaluations. Denoting by $\xi^{\mathrm{a}}(x;\theta)$ the quantum superfield with frame index, a, of the target space, our quantum propagator reads:

$$< \mathrm{T}(\xi^{\mathrm{a}}(1)\xi^{\mathrm{b}}(2)) >= \delta^{ab}\frac{1}{k^2}D_1^2\delta^2(\theta_{12})+$$
$$+\delta^{ab}\frac{\mu}{k^2+\mu^2}(\theta_1^\alpha D_{1\alpha} + 2\theta_1^2 D_1^2 + \frac{\mu}{k^2}k_{\alpha\beta}\theta_1^\alpha D_1^\beta+ \qquad (23)$$
$$+2\mu\theta_1^2)\delta^2(\theta_{12}).$$

However, through the quantum-background vertices arising from (17), the parameter μ will still be taken into account when calculating graphs. As for the dimensionless coupling parameter λ, its effect cannot be introduced into the quantum propagators, so it will always appear as a coupling constant governing the quantum-background vertices stemming from (18).

We are now ready to start presenting and discussing the results of our loop corrections.

Besides the well-known metric tensor renormalisation, which at one-loop is the same as in the case of the unbroken model, the only rôle of the one-loop supergraph of fig.1 is to renormalise the supersymmetry-breaking vertex of eq.(18) by means of the term:

$$\frac{1}{32\pi\varepsilon}\int d^2\theta\theta^2\lambda(\frac{1}{2}D_mD^mR_{ijkl}+\frac{4}{3}R_{mi}R^m{}_{jkl})\cdot$$

$$\cdot(D^\alpha\Phi^i)(D_\alpha\Phi^l)(D^\beta\Phi^j)(D_\beta\Phi^k). \qquad (24)$$

Notice however that such a renormalisation is required in the cases of both locally symmetric and Ricci-flat target spaces. The mass-breaking parameter μ does not require an independent renormalisation: the metric tensor renormalisation automatically removes such an infinity. Moreover, it would be worthwhile to remark that, based on power-counting and reparametrisation invariance arguments, one can conclude that only the breaking parameter λ, and not μ, can induce supersymmetric (i.e., non-explicitly θ-dependent) higher-order corrections into the effective action.

Still at the one-loop level, we would like to consider the kind of diagrams depicted in fig.2. They fall into three different categories: order zero, linear and quadratic in the breaking parameter λ. They all give finite one-loop contributions to the effective action, so that no new renormalisations are generated; however, it is interesting to notice that they yield supersymmetric corrections that arise exclusively from the terms which break supersymmetry at the Lagrangian level. These finite one-loop corrections are:

$$-\frac{3\mu}{64}\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2}\frac{1}{k^2+\mu^2}\int d^2\theta\, R_{imnj}R_k{}^{mn}{}_l\cdot$$

$$\cdot\,(D^\alpha\Phi^i)(D_\alpha\Phi^j)(D^\beta\Phi^k)(D_\beta\Phi^l), \tag{25}$$

and

$$\begin{aligned}\frac{\lambda}{128}\int\frac{d^2k}{(2\pi)^2}\frac{1}{(k^2+m^2_{\text{reg.}})^2}\int d^2\theta\, R_i{}^{pq}{}_j(D_pD_qR_{klmn}+\\ +\frac{1}{3}R^h{}_{pqk}R_{hlmn}+\frac{1}{3}R^h{}_{pql}R_{khmn}+\\ +\frac{1}{3}R^h{}_{pqm}R_{klhn}+\frac{1}{3}R^h{}_{pqn}R_{klmh})\cdot\\ \cdot(D^\alpha\Phi^i)(D_\alpha\Phi^j)(D^\beta\Phi^k)(D_\beta\Phi^n)(D^\gamma\Phi^l)(D_\gamma\Phi^m),\end{aligned} \tag{26}$$

where m_{reg} denotes an infra-red cut-off mass.

At order λ^2, no supersymmetric correction arises from the supergraph of fig.2.

At the two-loop approximation, no genuine divergence (i.e., a divergence in $\frac{1}{\varepsilon}$) appears which alters the vanishing of the two-loop β-function of the exact model. Graphs exhibiting the topology drawn in fig.3 do contribute divergent two-loop corrections of order λ to the metric tensor renormalisation. However, such divergences are of the type $\frac{1}{\varepsilon^2}$, and so they do not give any contribution to the β-function.

Finally, going over to the three-loop approximation, the situation changes, as expectated from power-counting arguments. Indeed, by considering the supergraphs whose topology is as shown in fig.4, where we take at the vertex 1 the quantum-background coupling following from the breaking term (18), one can show that a genuine $\frac{1}{\varepsilon}$ three-loop supersymmetric correction is induced which renormalises the metric tensor and is non-vanishing in the *Ricci*-flat case. Up to the numerical coefficient and $\frac{1}{\varepsilon}$-factor coming from the momentum-space loop integrals, the tensorial form of this three-loop divergent contribution is simply:

$$\lambda \int d^2\theta\, R_{ilmn} R_j{}^{kmh} R^n{}_{kh}{}^l (D^\alpha \Phi^i)(D_\alpha \Phi^j), \tag{27}$$

and, as it has been checked, such a divergent correction is not cancelled by any other three-loop contribution. This result clearly shows that the breaking term (18) yields a non-zero three-loop contribution to the metric tensor β-function of the non-linear supersymmetric σ-model which persists even when the target manifold is chosen to be Ricci-flat. This is the lowest non-trivial contribution to β_{ij} induced by the breaking interaction term of eq.(18).

To conclude, we have studied two different ways of explicitly breaking N=1 - D=2 supersymmetry in the framework of an arbitrary non-linear σ-model. The breaking terms we have proposed here are both soft breakings, respect the diffeomorphism invariance of the target space, modify the couplings between the physical scalars and fermions of the σ-model and, eventually, also lead to a fermion mass shift. The net result of our analysis is that the breaking accomplished by the term (18) yields a non-vanishing contribution to the three-loop metric tensor β-function.

There still remains however the investigation and justification for the ad-hoc breaking terms we propose in this letter. Their origin in a superstring compactification context, and also a more detailed analysis of the two-, three- and four-loop supersymmetric corrections generated by the dimensionless coupling parameter λ, is now under investigation [10].

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Figure Captions.

Figure 1: One-loop divergent supergraphs.

Figure 2: Finite one-loop graphs inducing supersymmetric contributions proportional to the breaking parameters.

Figure 3: Two-loop divergent supergraphs.

Figure 4: Three-loop diagram leading to a λ-dependent β-function contribution.

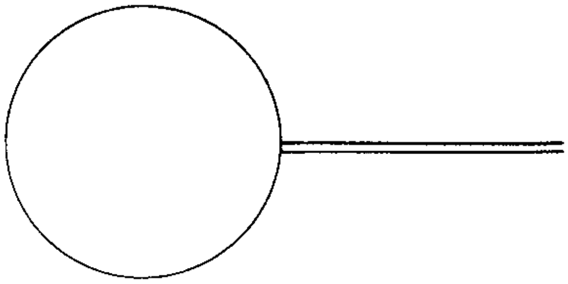

FIG.1

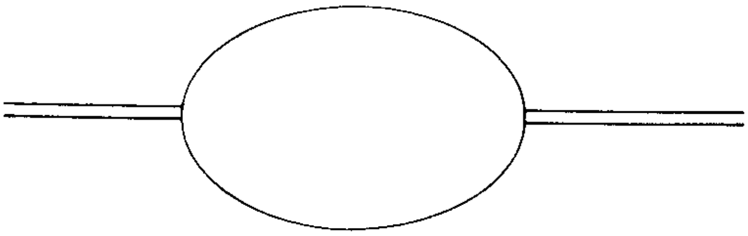

FIG.2

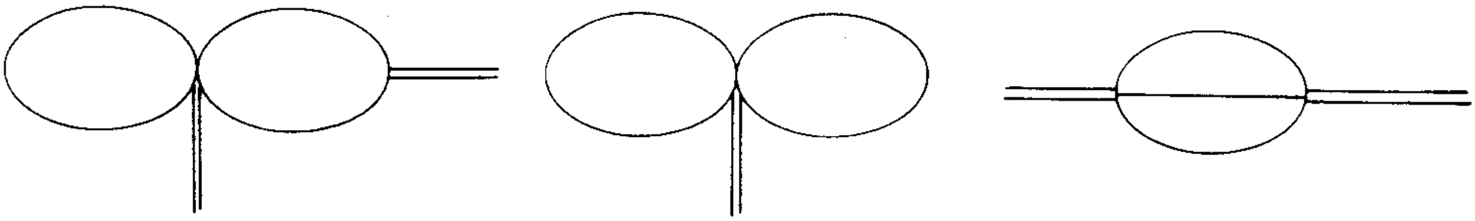

FIG.3

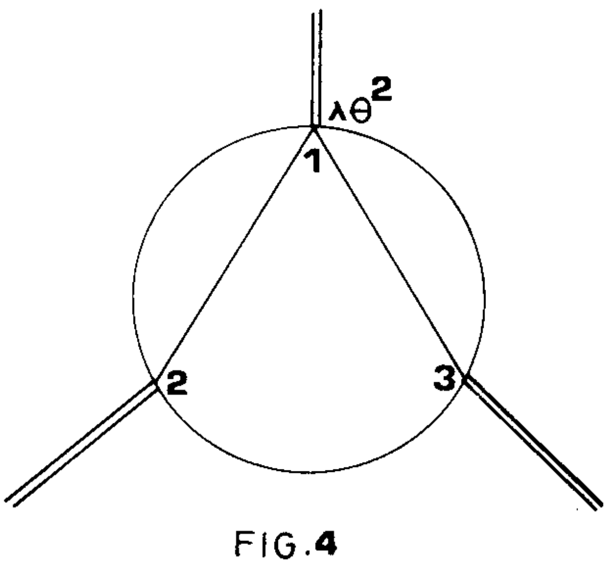

FIG.4