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0^{++} TRIGLUONIUM SUM RULES

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ABSTRACT

We estimate the mass of the 0^{++} trigluonium associated to the $g^3 f_{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$ gauge-invariant gluonic operator using QCD-duality sum rules. We also evaluate its mixing with the "standard" digluonium. Then, we conclude that one cannot interpret both the $\sigma(0.5)$ and the G(1.6) as lowest mass gluonia.

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There is increasing evidence that quantum chromodynamics (QCD) is the best candidate for describing the dynamics of hadrons. Due to the self-interactions between the eight gluons which mediate strong interactions, one might expect to have gluonia or glueball bound states of the gluon field strengths. There is a growing interest in the study of the gluonia properties from the theoretical point of view (lattice Monte Carlo simulations[1], effective Lagrangian[2], QCD sum rules[3], bag and potential models[4]). However, various predictions are less accurate than the ones for the quarkonia. The ones of some models based on the notion of constituent quarks suffer from the difficulty in introducing the notion of constituent gluons. Lattice Monte Carlo results are expected to be much affected by various technical uncertainties, like for instance the lattice size effects. The effective Lagrangian method based on the constraints from the U(1) anomalies (the so-called low-energy theorems) should be affected by the chiral symmetry-breaking terms[5] and presumably by the high-dimension gluon operators. Comparatively, QCD sum rules à la SVZ[6], based on the duality between a possibly measured spectral function and the QCD-approximate evaluation of a gluonic two-point correlation, are controlled by high- dimension gluon condensates which are poorly determined either by the use of an instanton-dilute gas model or the factorization hypothesis[7]. In this paper, we study the two-point correlator:

$$\Psi_{3g} = i\int d^4x \, e^{iqx} \langle 0|\mathbb{T}\, J_{3g}(x) \left(J_{3g}(0)\right)^{\dagger}|0\rangle \tag{1a}$$

associated to the local and unique operator of dimension six:

$$J_{3g} = :g^3 f_{abc}\; G^a_{\mu\nu}\, G^b_{\nu\rho}\, G^c_{\rho\mu}: \;, \tag{1b}$$

which has the quantum numbers of the 0^{++} trigluonium. We also study the mixing of the trigluonium with the digluonium associated to the two-gluonic current:

$$J_{2g} = :\alpha_s\; G^a_{\mu\nu}\; G_a^{\mu\nu}: \tag{2a}$$

from the off-diagonal two-point correlator:

$$\Psi_{23} = i\int d^4x \, e^{iqx} \langle 0|\mathbb{T}\, J_{3g}(x) \left(J_{2g}(0)\right)^{\dagger}|0\rangle \;. \tag{2b}$$

1. Trigluonium sum rule

We study the two-point correlator in Eq. (1), using an operator product expansion (OPE) à la SVZ. The evaluation of the Wilson coefficient is standard though very tedious. The contributions of each Feynman diagram are given in Table 1, where the calculation has been done in the convenient Schwinger gauge and by using dimensional regularization. The sum of different contributions gives:

$$\psi_{3g}(q^2) = -\alpha_s^2 \Big\{ \frac{3}{10\pi}\alpha_s (q^2)^4 \log\frac{-q^2}{\nu^2} + 18\pi (q^2)^2 \langle \alpha_s G^2 \rangle - \frac{27}{2}\Big(q^2 \log\frac{-q^2}{\nu^2}\Big) \langle g^3 f_{abc} G^a G^b G^c \rangle + \alpha_s \pi^3\, 36.64\, (\Phi_7 - \Phi_5) \Big\} \tag{3}$$

with the notation given in Table 1. In order to estimate the trigluonium mass which can be introduced via the matrix element:

$$\langle 0 | J_{3g} | G \rangle = \sqrt{2}\, M_{3g}^4\, f_{3g} \;, \tag{4}$$

we use the global duality sum rule:

$$R_n(\tau) = -\frac{d}{d\tau} \log \int_0^\infty dt\, e^{-t\tau}\, t^n \frac{1}{\pi} \mathrm{Im}\, \psi_{3g}(t) \;, \tag{5}$$

which has given interesting predictions for the mass of some other bound states due to its direct sensitivity to the resonance mass squared as well as due to the exponential factor which depresses the continuum and finite width effects into the sum rule. The QCD behaviour of $R_0(\tau)$ is*)

$$R_0^{QCD}(\tau) \simeq 5\tau^{-1} \cdot \left\{ \frac{1 - 27 \cdot \frac{40\pi^2}{5!} g \langle f_{abc} G^a G^b G^c \rangle \tau^3}{1 - 27 \cdot \frac{20\pi^2}{4!} g \langle f_{abc} G^a G^b G^c \rangle \tau^3} \right\} , \tag{6}$$

which is given in Fig. 1. The positivity of the sum rule in the region where the OPE makes sense provides the optimal upper bound of the lowest ground state mass. At the minimum of the sum rule, we obtain (Fig. 1):

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*) We have worked for n = 0. One should notice that for n < 0, the sum rule might be affected by the low-energy behaviour of $\psi_{3g}(q^2)$ which is of non-perturbative origin. For n > 1, the moments will be much more weighted by the high states continuum, so we can lose the optimal information on the lowest ground state.

$$M_{3g} \lesssim 3.7\ \mathrm{GeV}, \tag{7}$$

which is higher than the usual hadronic scale of the order of 1 GeV. The position of the minimum is controlled by the value of the triple gluon condensate which is not known accurately except for its absolute value[7)-8)] which we take to be $gf_{abc}\langle G^a G^b G^c\rangle \approx 1\ \mathrm{GeV}^2 \langle \alpha_s G^2\rangle \approx 0.04\ \mathrm{GeV}^6$. We have used the results of Ref. 7) for our analysis which is consistent in sign with the one coming from an OPE analysis of the mixed quark-gluon condensate[6)]. In the second stage of our analysis, we parametrize the spectral function by using the simple duality ansatz "one resonance" plus a "QCD continuum" where the new parameter is the threshold $\sqrt{t_c}$, in addition to the ones in Eq. (4). Therefore, the phenomenological expression of the moments reads:

$$R_0(\tau) = \frac{2 f_{3g}^2 M_{3g}^{10} e^{-M_{3g}^2 \tau} - 5!\, C_0 \tau^{-6} \rho_5 - 2!\, C_6 \langle O_6\rangle \tau^{-3} \rho_2}{2 f_{3g}^2 M_{3g}^{8} e^{-M_{3g}^2 \tau} - 4!\, C_0 \tau^{-5} \rho_4 - C_6 \langle O_6\rangle \tau^{-2} \rho_1}, \tag{8a}$$

with:

$$C_0 \equiv -\frac{3}{10}\frac{\alpha_s^3}{\pi}\,; \quad C_6\langle O_6\rangle \equiv -\frac{27}{2}\alpha_s^2 \langle g^3 f_{abc} G^a G^b G^c\rangle;$$
$$\rho_i = e^{-t_c \tau}\left\{1 + \frac{t_c \tau}{1!} + \dots \frac{(t_c \tau)^i}{i!}\right\}. \tag{8b}$$

The asymptotic consistency of the two sides of the sum rule gives the finite energy sum rule (FESR) local constraint:

$$2 f_{3g}^2 M_{3g}^8 = -C_0 \frac{t_c^5}{5} - C_6\langle O_6\rangle \frac{t_c^2}{2}, \tag{9}$$

which helps for reducing the number of free parameters in the analysis. One can either confront the two sides of the moments using a least-square fit program or write an equation for M_{3g}^2. Using the latter method here, we show our results Figs. 2-3 versus the changes in t_c and in τ_{MAX}. At the minimum of the curves where we expect to have the optimal information from the moments, we obtain:

$$M_{3g} \simeq 3.1 \ GeV \quad ; \quad \sqrt{t_c} \simeq 3.4 \ GeV \tag{10a}$$

and from Eq. (9):

$$f_{3g} \simeq 62 \ MeV \quad . \tag{10b}$$

Finite width effects might be incorporated by noticing that the lowest intermediate states which can dominate the spectral function are the Goldstone and η-like pair productions. However, there is a large uncertainty for controlling the threshold behaviour of the associated form factor as the so-called low-energy theorem can only tell about its t-behaviour but not its exact normalization. It can become clear that once we have such a control, one can relate the decay amplitude f_{3g} to the hadronic width which behaves as M_{3g}^2/f_{3g}. In this case we would expect a broad state. One should notice that the mass of the trigluonium is much heavier than the one of the digluonium which is known via QCD sum rules to be[5]:

$$M_{2g} \simeq 1. \ GeV \quad . \tag{11}$$

Therefore we might a priori expect that the mixing of the two states will be small as we shall see later on.

2. Trigluonium - digluonium mass mixing

The expected gluonia candidates should be eigenstates of the unmixed tri- and digluonia. We use a two-component mixing formalism in order to get the "physical" gluonia states[*]:

$$\begin{aligned} |\sigma\rangle &= |2g\rangle \cos\theta - |3g\rangle \sin\theta \\ |G\rangle &= |2g\rangle \sin\theta + |3g\rangle \cos\theta \quad . \end{aligned} \tag{12}$$

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*) We ignore, for the moment, the mixing with quark states which will be commented later on.

We follow the procedure used in Ref. 9). The QCD expression of the off-diagonal two-point correlator defined in Eq. (2) (see Table 2) is:

$$\psi_{23}(q^2) = \alpha_s^2 \left\{ \left(\log \frac{-q^2}{\nu^2}\right)\left[\frac{9}{4\pi^3} g^2 (q^2)^3 - \frac{9}{4\pi} g^2 \langle G^2 \rangle q^2\right] - 24\pi g \langle g f^{abc} G_a G_b G_c \rangle \right\} . \tag{13}$$

It is easy to derive the global-duality constraint:

$$\begin{aligned} \int_0^\infty dt\, e^{-t\tau} \frac{1}{\pi} \mathrm{Im}\, \psi_{23}(t) &= \tau^{-4} \alpha_s^2 \left\{ \frac{54\alpha_s}{\pi^2} - 9\alpha_s \langle G^2 \rangle \tau^2 \right\} \\ &\simeq \sin 2\theta\, M_{2g}^2 f_{2g} M_{3g}^4 f_{3g} \left(e^{-M_\sigma^2 \tau} - e^{-M_G^2 \tau} \right) + \\ &\quad \tau^{-4} \alpha_s^2 \left\{ \frac{54\alpha_s}{\pi^2} \rho_3 - 9\alpha_s \langle G^2 \rangle \tau^2 \rho_1 \right\} , \end{aligned} \tag{14}$$

where we have parametrized the "continuum" from the discontinuity of $\psi_{23}(q^2)$. A variation of the sum rule versus τ shows a stability for $\tau \simeq (2\sim3)\ \mathrm{GeV}^{-2}$ which is quite a low value of the Q^2-scale. This is due to the relative weight of the perturbative and gluon condensate effects to Eq. (14). At this value of the optimization scale, the effects of high-dimension condensates might be important. At the approximation, in Eq. (14), we obtain the estimate:

$$\theta \simeq 0.2^\circ , \tag{15}$$

where we have used $f_{2g} \simeq 100$ MeV[5)], $f_{3g} \simeq 62$ MeV; $t_c \simeq 2\ \mathrm{GeV}^2$ $M_\sigma \simeq 500$ MeV and $M_G \simeq 1.6$ GeV[10)].

Equation (15) shows a tiny mixing between the two states as intuitively expected because of the large splitting of the two resonances.

We can conclude from the previous analysis that the existence of both the $\sigma(0.5)$ and $G(1.6)$ as gluonia candidates in the 0^{++} sector cannot be explained from the probable mixing of di- and trigluonia states. The attempt to explain the σ and G from the quarkonium-gluonium mixing also failed. It is likely that the G(1.6) is the "excitation" of the σ.[11)]

ACKNOWLEDGEMENTS

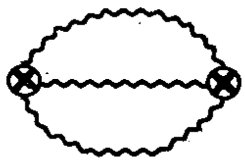
We have have benefitted from discussions with M.S. Chanowitz, K. Johnson and G. Veneziano.

TABLE 1

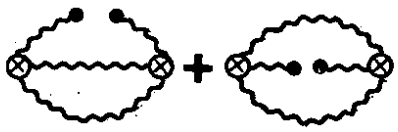
QCD contributions to the two-point correlator:

$$\Psi_{3g}(q^2) = i\int d^4x\, e^{iqx}\langle 0|\mathbb{T}\, J_{3g}(x)\left(J_{3g}(0)\right)^{\dagger}|0\rangle ,$$

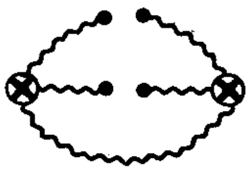
with $J_{3g}(x) = g^3 : f_{abc}\ G^a_{\mu\nu}\ G^b_{\nu\rho}\ G^c_{\rho\nu}:$ in $d = 4+2\varepsilon$ dimensions and for $SU(N)_c \times SU(n)_f$



$$= \frac{N(N^2-1)}{40\cdot 256\pi^4}\, g^6 (q^2)^4 \left\{ \frac{1}{\epsilon} - \frac{87}{20} + 2\log\frac{-q^2}{4\pi\nu^2} \right\}$$



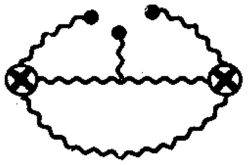
$$= \alpha_s^2 N\, 6\pi (q^2)^2 \langle \alpha_s G^2 \rangle$$



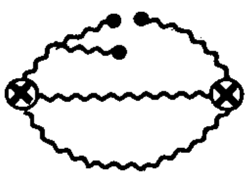
$$= 36\, g^6 (\Phi_7 - \Phi_5) \ :$$

$$\Phi_5 = \frac{1}{16}\, \mathrm{Tr}\langle G_{\nu\mu} G^{\mu\rho} G_{\rho\tau} G^{\tau\nu} \rangle$$

$$\Phi_7 = \frac{1}{16}\, \mathrm{Tr}\langle G_{\nu\mu} G^{\nu\rho} G^{\mu\tau} G_{\rho\tau} \rangle$$



$$= \alpha_s^2 N \frac{9}{4}\, q^2 g^3 \langle f_{abc}\, G^a G^b G^c \rangle$$



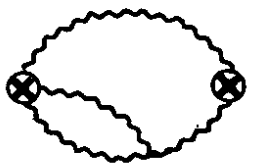
$$\alpha_s^2 N \frac{9}{2}\, q^2 \left(\frac{1}{\epsilon} + \log\frac{-q^2}{4\pi\nu^2}\right) \Big\{ g^3 \langle f_{abc} \cdot G^a G^b G^c \rangle + \frac{32}{3}\, n_f \frac{N^2-1}{N^2}\, \alpha_s^2 \pi^2 \langle \bar{q} q \rangle^2 \Big\}$$

TABLE 2

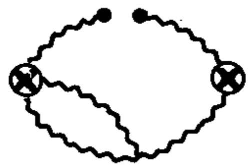
QCD contributions to the off-diagonal two-point correlator:

$$\Psi_{23}(q^2) = i\int d^4x\, e^{iqx}\langle 0|\mathbb{T}\, J_{3g}(x)\left(J_{2g}(0)\right)^{\dagger}|0\rangle ,$$

with: $J_{2g}(x) = :\alpha_s\, G^a_{\mu\nu}\, G^{\mu\nu}_a:$ in $d = 4 + 2\varepsilon$ dimensions and for $SU(N)_c \times SU(n)_f$.

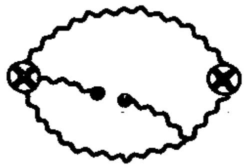


$$-\frac{i}{256\pi^4} N(N^2-1)\, g^6\, \frac{3}{4}(q^2)^3\left\{\frac{1}{\epsilon} + 2\log\frac{-q^2}{4\pi\nu^2}\right\}$$

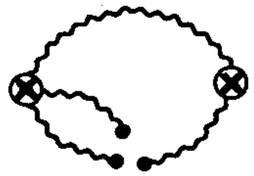


$$\frac{i}{16\pi^2} N g^6 \langle G^2\rangle\, 3q^2\left\{\frac{1}{\epsilon} + \log\frac{-q^2}{4\pi\nu^2}\right\}$$

$$0$$



$$\frac{i}{16\pi^2} N g^6 \langle G^2\rangle\,(-6q^2)\left\{\frac{1}{\epsilon} + \log\frac{-q^2}{4\pi\nu^2}\right\}$$



$$-i\, 6g^5 \langle f_{abc}\, G^a G^b G^c\rangle$$

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FIGURE CAPTIONS

Fig. 1 Behaviour of the QCD moments $R_o(\tau)$ versus the sum rule scale τ for $g\ f_{abc}\langle G^3\rangle = 0.04\ GeV^6$.

Fig. 2 Behaviour of the triglonium mass versus t_c for a given value of the fit interval $[0,\ \tau_{MAX}]$.

Fig. 3 Behaviour of the set $(M_{3g},\ \sqrt{t_c})$ of the optimal values versus the changes of τ_{MAX}.

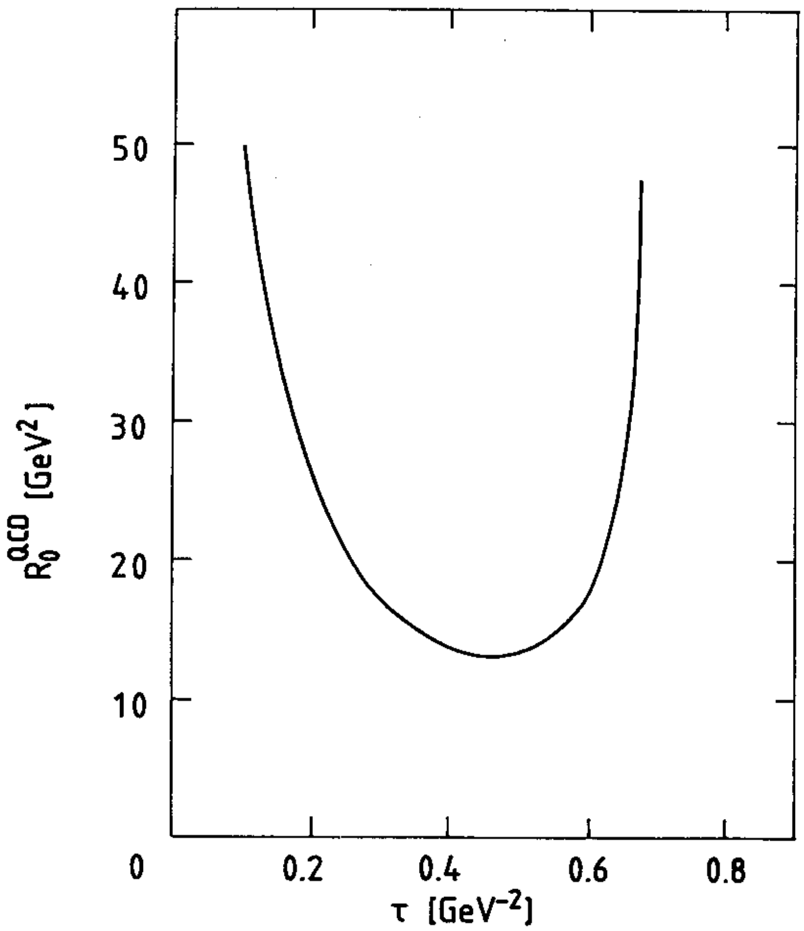
50
40
30
20
10
0
0.2
0.4
0.6
0.8
R_0^{QCD} [GeV^2]
τ [GeV^{-2}]

Fig. 1

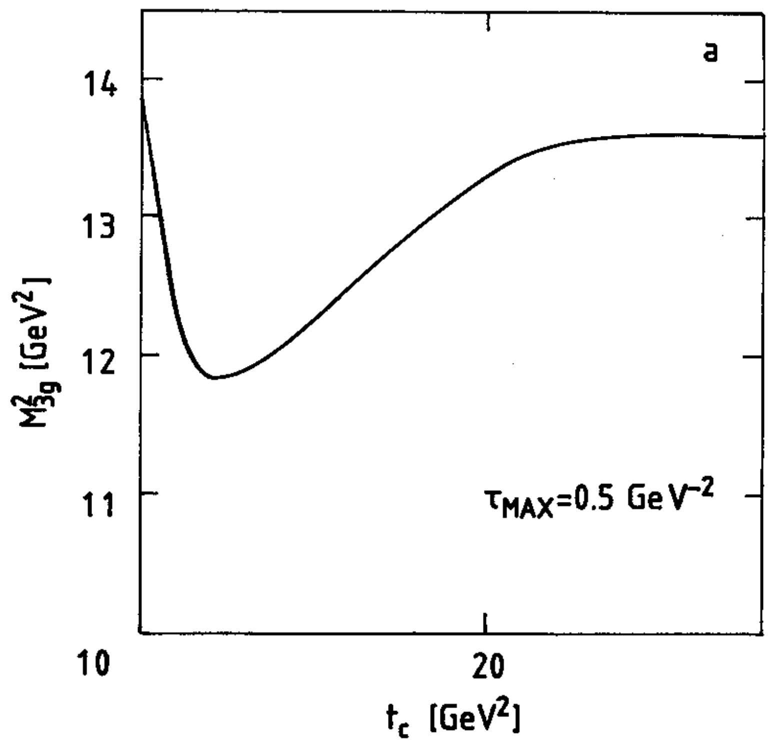
a
14
13
12
11
10
20
M_{3g}^2 [GeV^2]
τ_{MAX}=0.5 GeV^{-2}
t_c [GeV^2]

Fig. 2

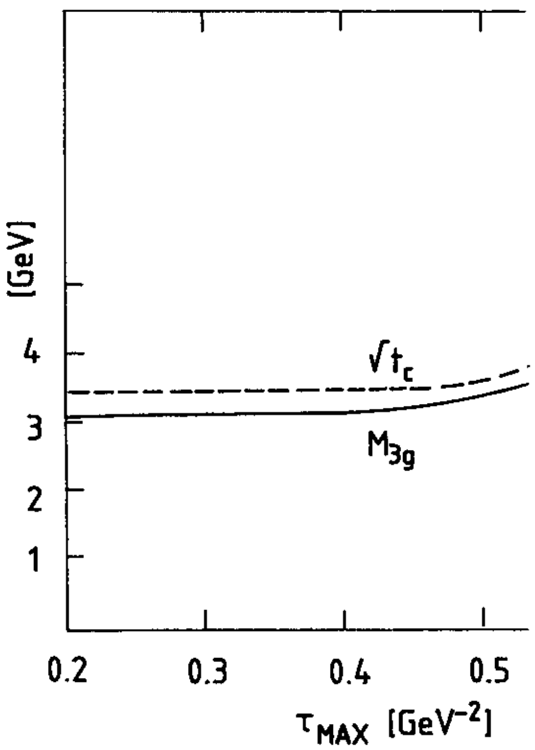

Fig. 3