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EFFECTS OF SUPERSTRINGS IN $e^+e^- \to \gamma\nu\bar{\nu}$ COLLISIONS

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ABSTRACT

Superstring theory in d = 10 dimensions after Calabi-Yau compactification yields a minimum low-energy gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$. The low-energy theory includes particles with the quantum numbers of 27 representations of E_6, each of which contains an extra neutrino ν^c conventionally called a "right-handed neutrino". We calculate the contributions of ν and ν^c to $\sigma(e^+e^- \to \gamma\nu\bar{\nu})$ through Z^0 and Z_E mixing. We find small contributions of the new right-handed neutrino and of the superstring boson Z_E to $\sigma(e^+e^- \to \gamma + \text{nothing})$.

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Superstring field theories are consistent quantum field theories that unify gravity with the other fundamental forces[1)]. The superstring theory is finite and anomaly-free[2)]. Apparently, there is no consistent quantum theory of gravity based on pointlike particles, but consistent quantum string theory requires gravity. In order to obtain a consistent superstring quantum theory, we require a ten-dimensional space-time. Superstring theory in d = 10 dimensions can be compactified into a vacuum state of the form $M_4 \times K$, where M_4 is a four-dimensional Minkowski space and K is a compact six-dimensional manifold[3)]. In order to have unbroken local N = 1 supersymmetry in M_4 (since supersymmetry may solve the gauge hierarchy problem) K must be Ricci flat with SU(3) holonomy group [i.e., the curvature two-form lies in an SU(3) group]. A large number of such manifolds exist, called Calabi-Yau manifolds. Anomaly cancellation of superstring theory has given two possible Yang-Mills gauge groups, SO(32) or $E_8 \times E_8'$. The heterotic superstring with gauge group $E_8 \times E_8'$ is the most phenomenologically promising theory[4)]. Starting with the $E_8 \times E_8'$ theory in ten dimensions, some Yang-Mills field strengths acquire non-zero expectation values, proportional to the curvature of the internal manifold. Since $E_8 \supset E_6 \times SU(3)$, this causes a breaking of E_8 to E_6, which is the symmetry group for the observable part of the four-dimensional theory. Thus we arrive at a variety of supersymmetric GUT with symmetry group $E_6 \times E_8'$ in which the observed elementary particles are approximately massless relative to the natural mass scale which is the Planck scale of 10^{19} GeV. The spectrum of massless particles, when some of the dimensions are curled up, is determined by specifying which Calabi-Yau space is used as internal manifold. The number of families is determined by the topological structure of the theory and is given by $\frac{1}{2}$ of the Euler characteristic of the space[3)]. The E_8' describes matter which only interacts gravitationally with the charged matter in the E_6 Yang-Mills group. This is a prediction of $E_8 \times E_8'$ theory when it is compactified on any internal manifold with no isometries, so that no new gauge symmetries emerge from the compactification.

The E_6 can break down, by the Wilson loop mechanism, to various subgroups of E_6 containing at least the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$, all having rank five or six. The minimum low-energy gauge group is $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$ with just one extra neutral gauge boson Z_E with mass O(100 GeV to 1 TeV), in addition to the conventional Z^0[5)]. The new neutral gauge boson Z_E mixes with the conventional Z^0 giving two physical states Z and Z' which are the two eigenstates of the (Z^0, Z_E) squared mass matrix. The matter particles occur in generations of 27 fields with the quantum numbers of $\underline{27}$ representations of E_6. These have the following SO(10) and SU(5) decompositions:

$$\left[\underline{27}\right]_{E_6} = \left[\underline{16}+\underline{10}+\underline{1}\right]_{SO(10)} = \left[\left(\underline{10}+\underline{\bar{5}}+\underline{1}\right)+\left(\underline{5}+\underline{\bar{5}}\right)+\underline{1}\right]_{SU(5)}$$

where the $\underline{10}$ and $\underline{\bar{5}}$ of SU(5) are the conventional quarks $Q \equiv \binom{u}{d}$, u^c, d^c and charged leptons $L \equiv \binom{\nu}{e}$, e^c, the first $\underline{1}$ of SU(5) contained in the $\underline{16}$ of SO(10) has the quantum numbers appropriate for a light "right-handed neutrino" ν^c, the $\underline{5}+\underline{\bar{5}}$ of SU(5) in the $\underline{10}$ of SO(10) include two weak doublets with the quantum numbers for the standard model Higgses $H \equiv \binom{H^+}{H^0}$ and $\bar{H} \equiv \binom{\bar{H}^0}{\bar{H}^-}$ as well as a $\underline{3}+\underline{\bar{3}}$ of colour which is an additional charge -1/3 quark D and its conjugate D^c, and the second $\underline{1}$ of SU(5) called N, which is also a $\underline{1}$ of SO(10), can be used to break the gauge symmetry.

In this paper we calculate the cross-section of the process $e^+e^- \to \gamma\nu\bar{\nu}$ through the Z, Z' and W exchange. The Z and Z' couple to the new right-handed neutrinos ν^c, as well as to the left-handed neutrinos. In the $e^+e^- \to \gamma\nu\bar{\nu}$ process, the missing energy-momentum is carried off by the undetectable $\nu\bar{\nu}$ [6]. The unobservable final state is detected by a bremsstrahlung photon which has a characteristic soft spectrum peaked at small angles relative to the $e^\pm$ beams [7]. We present the percentage difference of the $\sigma(e^+e^- \to \gamma\nu\bar{\nu})$ with respect to the standard model cross-section as a function of the vacuum expectation values of the three Higgs fields breaking $SU(2)_L\times U(1)_Y\times U(1)_E$ to $U(1)_{em}$. We find that the percentage change of the cross-section is small because of the previous neutral currents bounds on the Z' mass. Therefore, the previous conclusions [7] on the number of left-handed neutrinos N_ν are still valid.

In the minimal superstring-inspired model $SU(3)_C\times SU(2)_L\times U(1)_Y\times U(1)_E$, we have an extra neutral boson Z_E which mixes with the conventional standard model Z^0 via the mass-squared matrix:

$$(Z^0, Z_E)\; m_{Z^0}\begin{pmatrix} 1 & \alpha \\ \alpha & b \end{pmatrix}\begin{pmatrix} Z^0 \\ Z_E \end{pmatrix} \qquad (1)$$

where

$$m_{Z^0} = \frac{38.65}{\sin\theta_W \cos\theta_W}$$

is the Z^0 mass of the standard model with Higgs doublets H and $\bar{H}$ with expectation values v and $\bar{v}$ respectively, and

$$a = \frac{1}{3} \sin\theta_W \frac{4v^2 - \bar{v}^2}{v^2 + \bar{v}^2}$$
$$b = \frac{1}{9} \sin\theta_W \frac{25x^2 + 16v^2 + \bar{v}^2}{v^2 + \bar{v}^2} \qquad (2)$$

where x is the vev of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet field N. After diagonalization by the unitary rotation $\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$ the mass matrix gives two physical mass eigenstates Z and Z':

$$Z = \cos\phi \; Z^o + \sin\phi \; Z_E$$
$$Z' = -\sin\phi \; Z^o + \cos\phi \; Z_E \qquad (3)$$

with mass

$$m_{Z,Z'} = m_{Z^o} \sqrt{\frac{1}{2}\left[(1+b) \mp \sqrt{(1-b)^2 + 4a^2} \right]} \qquad (4)$$

It is clear that $m_Z \to m_{Z^0}$ and $m_{Z'} \to \infty$ as $x/v \to \infty$ for fixed $\bar{v}/v$. The couplings of the physical neutral gauge bosons Z and Z' to neutrinos ν_L, ν_R and to electrons e_L, e_R are given by:

$$g^Z_{\nu_L} = \cos\phi \; g^{Z^o}_{\nu_L} + \sin\phi \; g^{Z_E}_{\nu_L}$$
$$g^Z_{\nu_R} = \sin\phi \; g^{Z_E}_{\nu_R}$$
$$g^{Z'}_{\nu_L} = -\sin\phi \; g^{Z^o}_{\nu_L} + \cos\phi \; g^{Z_E}_{\nu_L} \qquad (5)$$
$$g^{Z'}_{\nu_R} = \cos\phi \; g^{Z_E}_{\nu_R}$$
$$g^Z_{e_{L,R}} = \cos\phi \; g^{Z^o}_{e_{L,R}} + \sin\phi \; g^{Z_E}_{e_{L,R}}$$
$$g^{Z'}_{e_{L,R}} = -\sin\phi \; g^{Z^o}_{e_{L,R}} + \cos\phi \; g^{Z_E}_{e_{L,R}}$$

where the neutral boson mixing angle ϕ is

$$\tan 2\phi = \frac{2a}{1-b} \tag{6}$$

and the Z^0 and Z_E couplings are listed in the Table. Because of the mixing the ratio m_W^2/m_Z^2 is no longer $\cos^2\theta_W$. We define

$$\cos^2\bar{\theta}_W \equiv \frac{m_W^2}{m_Z^2} \tag{7a}$$

Since

$$\sin^2\bar{\theta}_W = 1 - \frac{m_W^2}{m_Z^2} = 1 - \left(\frac{m_{Z^0}^2}{m_Z^2}\right)\cos^2\theta_W \tag{7b}$$

we get $\sin^2\bar{\theta}_W < \sin^2\theta_W$. The difference of these two values is[5)]

$$\Delta \equiv \sin^2\theta_W - \sin^2\bar{\theta}_W = 0.012 \pm 0.023 \tag{8}$$

Taking the 1-σ limit $\Delta < 0.035$, we find the bounds on x/v for different values of $\bar{v}/v$ which are shown as dashed lines in Figs. 2 and 3.

In the calculations, we consider the Feynman diagrams of Fig. 1. The process $e^+e^- \to \gamma\nu\bar{\nu}$ at the tree level is mediated by the s-channel Z and Z' bosons and the t-channel W boson. In the t-channel diagrams (v), (vi) and (vii), we cannot reach the W pole. For energies of the order of m_W, we have a depression of at most a factor of 2 for these diagrams with respect to the point coupling limit. However, this limit is adequate to discuss the cross-section at energies up to just beyond the Z pole. Then, we will compare the cross-sections of the extended $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$ model to those of the standard model with $\sin^2\theta_W$ adjusted to give $m_Z = m_{Z^0}$[8)].

The differential cross-section for $e^+e^- \to \gamma\nu\bar{\nu}$ via the W and the two massive neutral gauge bosons Z and Z' is:

$$\frac{d^2\sigma}{dx\,dy} = \frac{G_F^2 \alpha}{3\pi^2}\Big\{4M_{Z^0}^2\Big[N_\nu\Big(\big[(g_{\nu_L}^Z)^2+(g_{\nu_R}^Z)^2\big]\big[(g_{e_L}^Z)^2+(g_{e_R}^Z)^2\big]$$

$$\cdot\frac{1}{[s(1-x)-M_Z^2]^2+M_Z^2\Gamma_Z^2} + \big[(g_{\nu_L}^{Z'})^2+(g_{\nu_R}^{Z'})^2\big]\big[(g_{e_L}^{Z'})^2+(g_{e_R}^{Z'})^2\big]$$

$$\cdot\frac{1}{[s(1-x)-M_{Z'}^2]^2+M_{Z'}^2\Gamma_{Z'}^2} + 2\big(g_{\nu_L}^Z g_{\nu_L}^{Z'} + g_{\nu_R}^Z g_{\nu_R}^{Z'}\big)\big(g_{e_L}^Z g_{e_L}^{Z'} + g_{e_R}^Z g_{e_R}^{Z'}\big) \tag{9}$$

$$\cdot\frac{[s(1-x)-M_Z^2]\,[s(1-x)-M_{Z'}^2]-M_Z M_{Z'}\Gamma_Z\Gamma_{Z'}}{\big[(s(1-x)-M_Z^2)^2+M_Z^2\Gamma_Z^2\big]\big[(s(1-x)-M_{Z'}^2)^2+M_{Z'}^2\Gamma_{Z'}^2\big]}\Big) + g_{e_L}^Z g_{\nu_L}^Z$$

$$\cdot\frac{1}{M_{Z^0}^2}\,\frac{[M_Z^2-s(1-x)]}{[s(1-x)-M_Z^2]^2+M_Z^2\Gamma_Z^2} + g_{e_L}^{Z'} g_{\nu_L}^{Z'}\frac{1}{M_{Z^0}^2}\,\frac{[M_{Z'}^2-s(1-x)]}{[s(1-x)-M_{Z'}^2]^2+M_{Z'}^2\Gamma_{Z'}^2}\Big]$$

$$+1\Big\}\ \frac{s}{x(1-y^2)}\Big[(1-x)\Big(1-\frac{1}{2}x\Big)^2+\frac{1}{4}x^2(1-x)y^2\Big]$$

where $x = E_\gamma/E$, $S = 4E^2$, $y = \cos\theta_\gamma$, E_γ is the energy of the photon, θ_γ is the angle of photon with the beam axis, S is the square of the total c.m. energy, $G_F = \sqrt{2}\, g_2^2/8m_W^2$, α is the fine-structure constant, and N_ν is the total number of neutrino types. This expression has infra-red divergence at $E_\gamma = 0$ associated with the radiative process. It also has strong forward-backward peaking at $y = \cos\theta_\gamma = \pm 1$. The total width Γ_z of the Z boson is given in terms of the coupling constants of fermions by

$$\Gamma_Z = \frac{G_F m_Z^3}{3\sqrt{2}\,\pi}\sum_f\big[(g_{f_L}^Z)^2+(g_{f_R}^Z)^2\big] \tag{10}$$

To calculate the cross-section for $e^+e^- \to \gamma\nu\bar{\nu}$, we integrate the differential cross-section over the range $-\frac{1}{2} < y < \frac{1}{2}$, $0.2 < x < 1$, which corresponds to the positions of the photon detection apparatus.

Figure 2a shows the percentage changes $|(\sigma-\sigma_0)/\sigma_0|$ in the total $e^+e^- \to \gamma\nu\bar{\nu}$ cross-section at PEP energies $\sqrt{s}$ = 29 GeV between the superstring inspired model to the standard model with the same value of m_Z. Figure 2b shows the corresponding percentage change in $\sigma(e^+e^- \to \gamma\nu\bar{\nu})$ at the maximum of the cross-section, just above the Z peak $\sqrt{s}$ = 112 GeV. We have taken m_Z = 93.3 GeV. Figures 3a and 3b show the percentage difference of the superstring-inspired model from the standard model in the case where no right-handed neutrinos are considered ($g^Z_{\nu_R} = g^{Z'}_{\nu_R} = 0$). We see that the new right-handed neutrino ν_R has small contributions $[O(1\%)]$ to $\sigma(e^+e^- \to \gamma\nu\bar{\nu})$. The $\sigma(e^+e^- \to \gamma\nu\bar{\nu})$ is proportional to the number of neutrinos apart from W terms. So the determination of the $e^+e^- \to \gamma\nu\bar{\nu}$ cross-section still provides a good method of counting the number of neutrinos N_ν. Since the $\sigma(e^+e^- \to \gamma\nu\bar{\nu})$ in our superstring-inspired model does not change much from the standard model one, this process can still be used to count the number of neutrino types reliably. Conversely, this process is not a good way to look for superstring effects.

For completeness, we give some values of $\sigma_{Z'}(e^+e^- \to \gamma\nu\bar{\nu})$ at energies at the peak of the cross-section just beyond the Z' mass at $\sqrt{s} = 1.1\ m_{Z'}$, for some representative values of $\bar{v}/v$ and x/v. For $\bar{v}/v$ = 0.5 and x/v = 4.0, 6.0, 8.0 corresponding to $m_{Z'}$ = 265 GeV, 394 GeV, 523 GeV, we get $\sigma_{Z'} = 0.32\times10^{-4}$ nb, 0.13×10^{-4} nb, 0.71×10^{-5} nb respectively.

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Table

Left- and right-handed couplings to Z^0 and Z_E

	u(= c = t)	d(= s = b)	e(= μ = τ)	$\nu_e(=\nu_\mu=\nu_\tau)$
$g_L^{Z^0}$	$\frac{1}{2}-\frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W$	$-\frac{1}{2}+\sin^2\theta_W$	$\frac{1}{2}$
$g_R^{Z^0}$	$-\frac{2}{3}\sin^2\theta_W$	$\frac{1}{3}\sin^2\theta_W$	$\sin^2\theta_W$	0
$g_L^{Z_E}$	$\sqrt{\frac{3}{5}}(\frac{1}{3})\sin\theta_W$	$\sqrt{\frac{3}{5}}(\frac{1}{3})\sin\theta_W$	$\sqrt{\frac{3}{5}}(-\frac{1}{6})\sin\theta_W$	$\sqrt{\frac{3}{5}}(-\frac{1}{6})\sin\theta_W$
$g_R^{Z_E}$	$\sqrt{\frac{3}{5}}(-\frac{1}{3})\sin\theta_W$	$\sqrt{\frac{3}{5}}(\frac{1}{6})\sin\theta_W$	$\sqrt{\frac{3}{5}}(-\frac{1}{3})\sin\theta_W$	$\sqrt{\frac{3}{5}}(-\frac{5}{6})\sin\theta_W$

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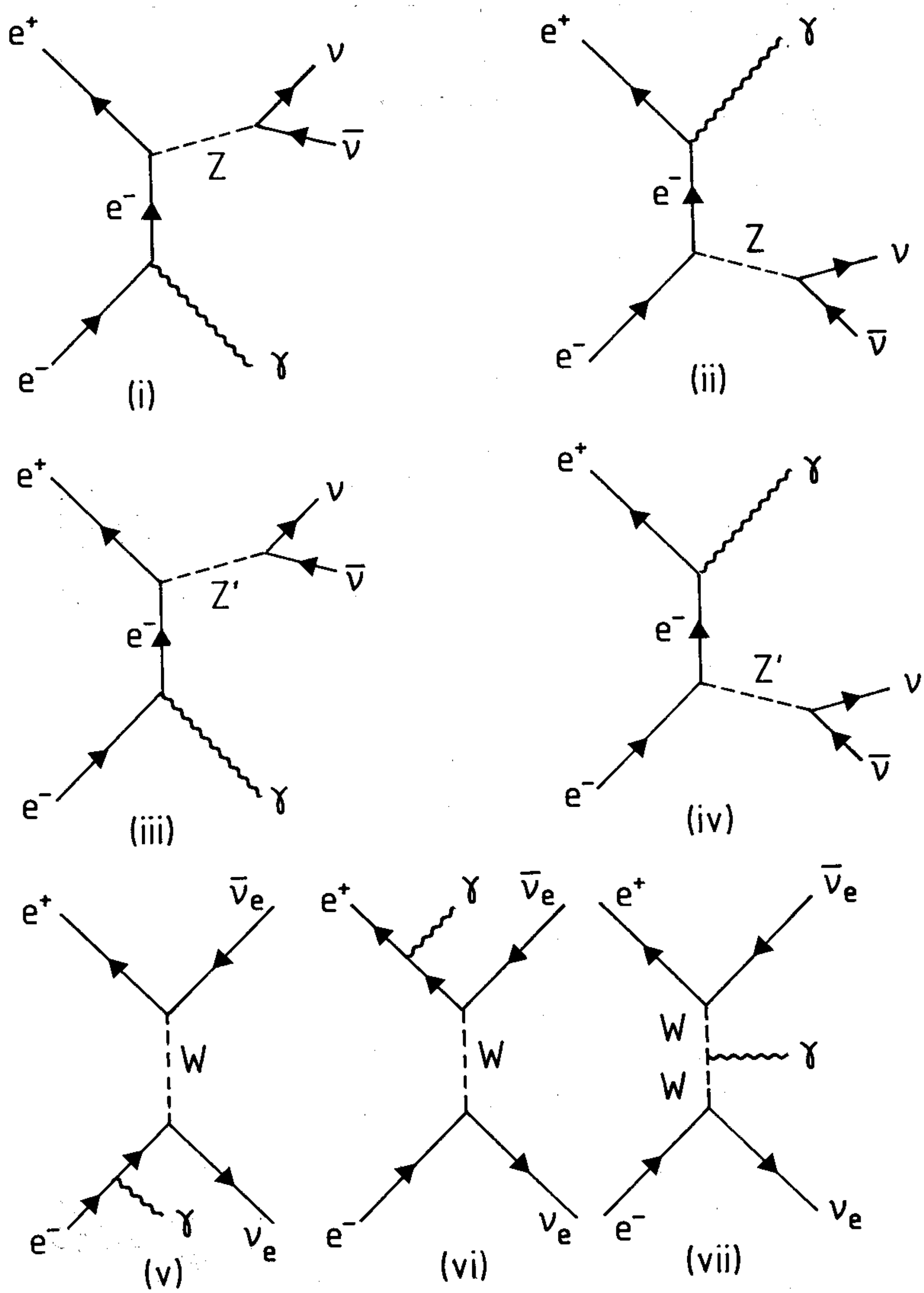
FIGURE CAPTIONS

Fig. 1: The Feynman diagrams contributing to the process $e^+e^- \to \gamma\nu\bar{\nu}$.

Fig. 2: Percentage differences in $\sigma(e^+e^- \to \gamma\nu\bar{\nu})$ from the standard model to the superstring inspired model with the same value of m_Z: (a) for $\sqrt{s}$ = 29 GeV, (b) for $\sqrt{s}$ = 112 GeV. The dashed line corresponds to Δ = 0.035. We have taken m_Z = 93.3 GeV.

Fig. 3: Percentage differences in $\sigma(e^+e^- \to \gamma\nu\bar{\nu})$ from the standard model to the superstring inspired-model with the same value of m_Z in the case where no right-handed neutrinos ν^c are considered: (a) for $\sqrt{s}$ = 29 GeV, (b) for $\sqrt{s}$ = 112 GeV. The dashed line corresponds to Δ = 0.035.


e+
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(i)
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e+
ν
ν̄
Z'
e−
e−
γ
(iii)
e+
γ
e−
Z'
ν
e−
ν̄
(iv)
e+
ν̄e
W
e−
γ
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(v)
e+
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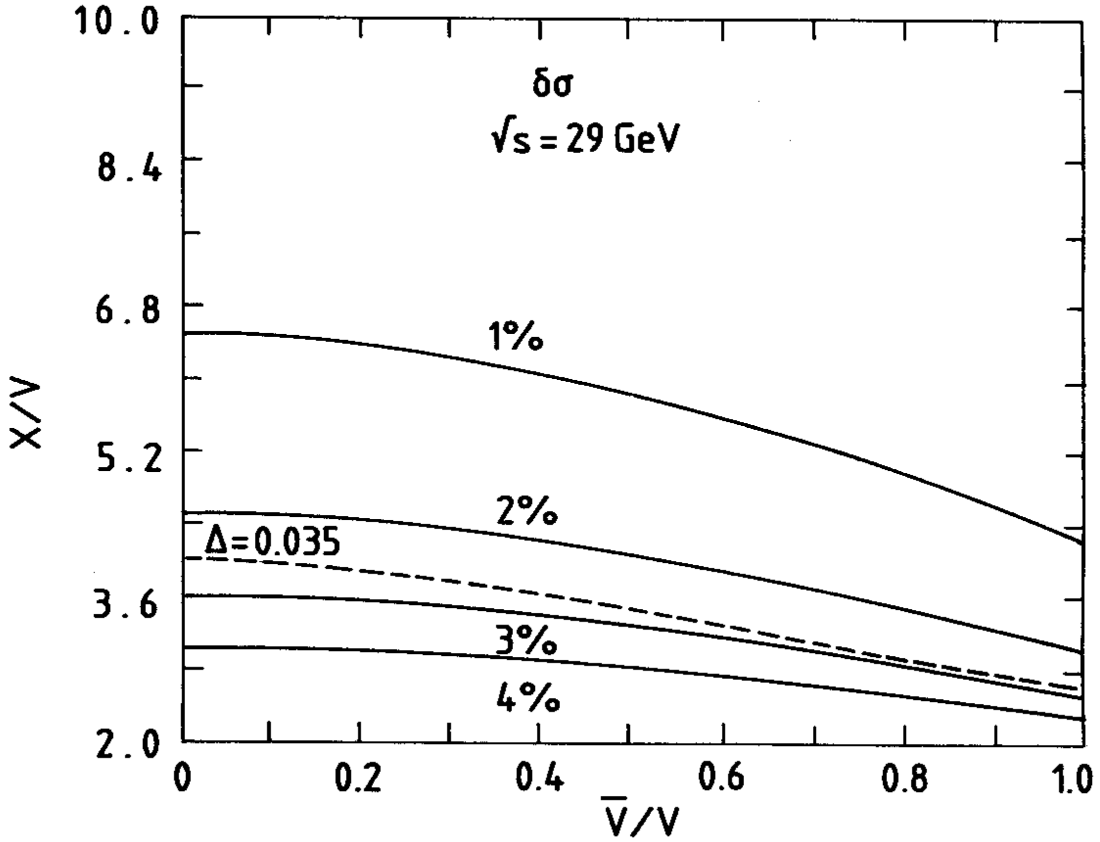
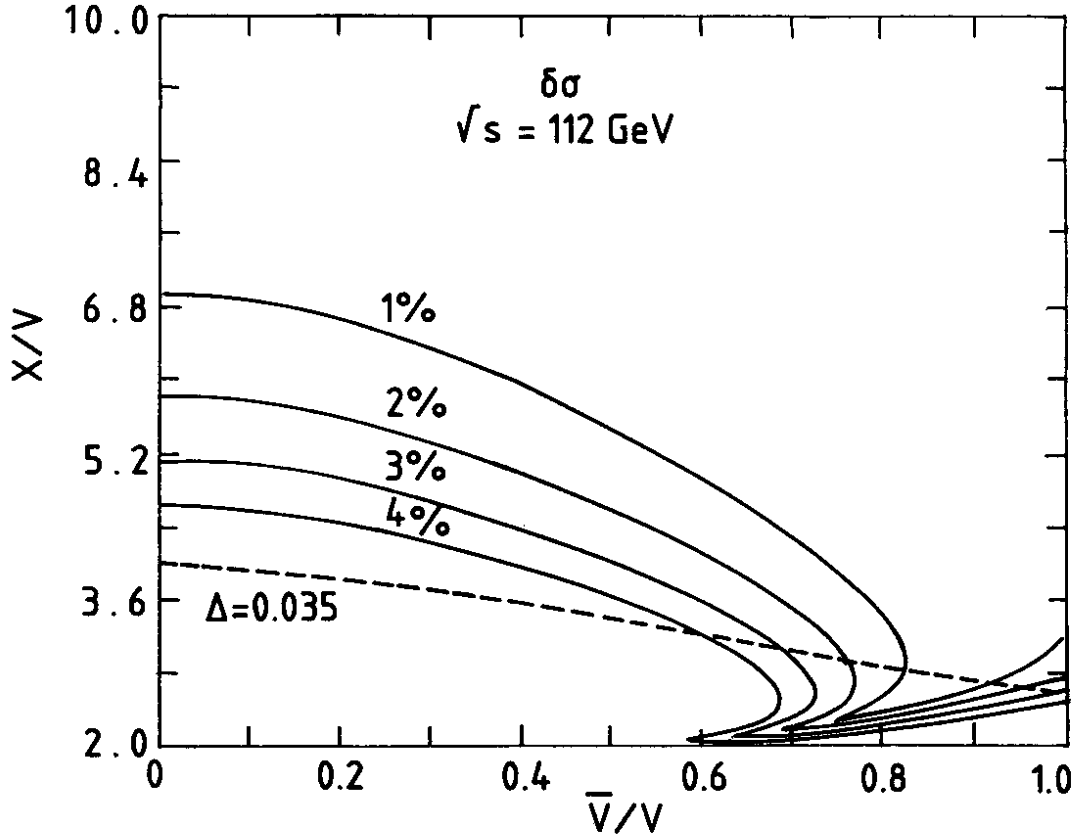
Fig. 1



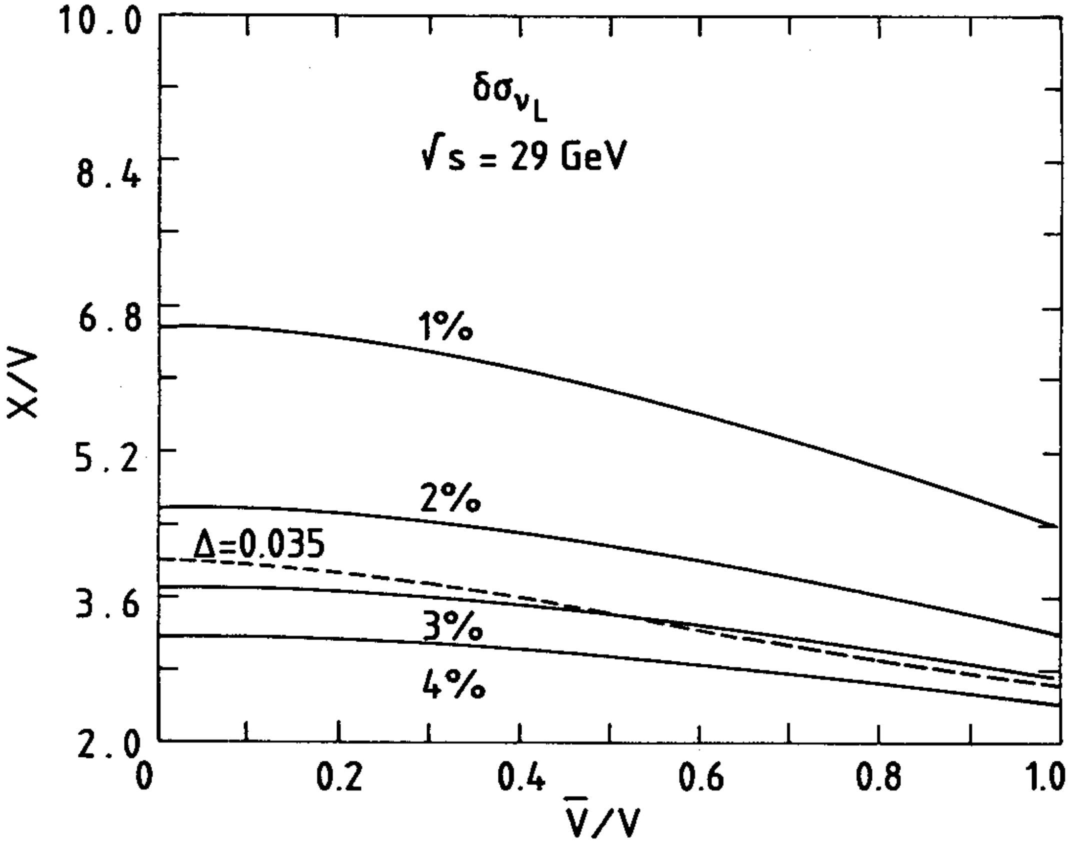
δσ
√s = 29 GeV
1%
2%
Δ=0.035
3%
4%
X/V
V̄/V
10.0
8.4
6.8
5.2
3.6
2.0
0
0.2
0.4
0.6
0.8
1.0

Fig. 2a



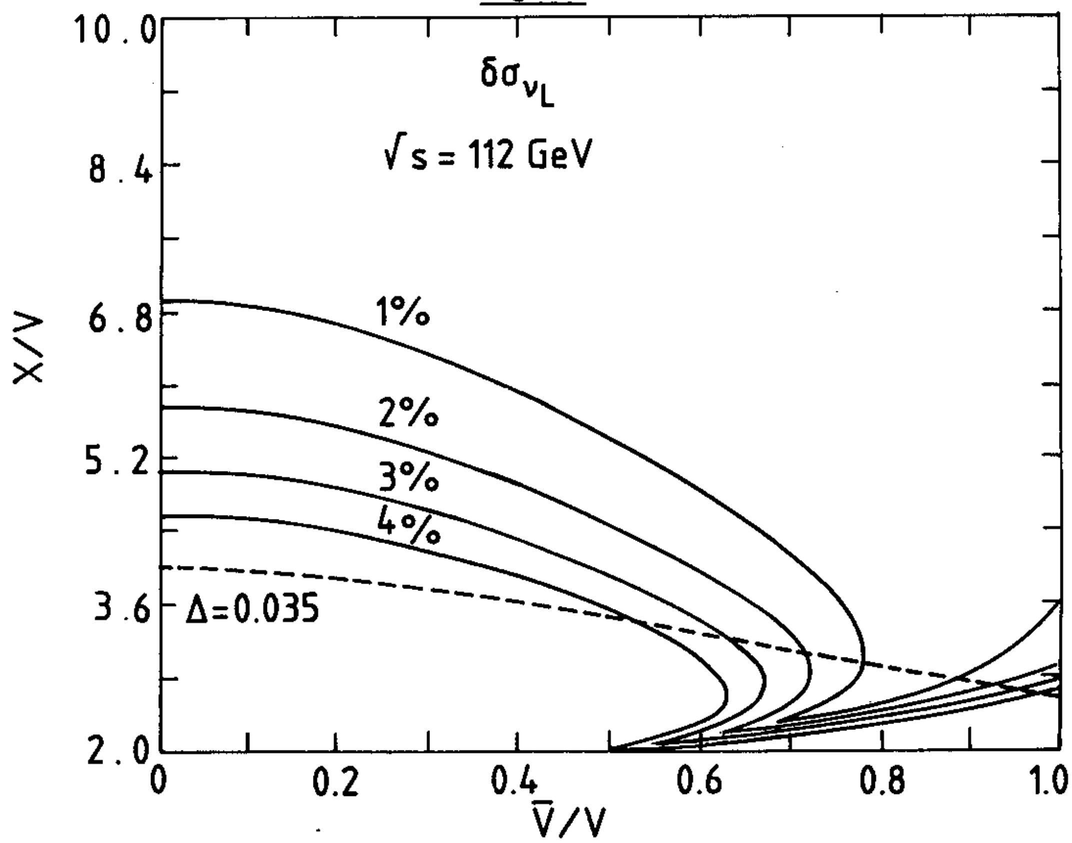
δσ
√s = 112 GeV
1%
2%
3%
4%
Δ=0.035
X/V
V̄/V
10.0
8.4
6.8
5.2
3.6
2.0
0
0.2
0.4
0.6
0.8
1.0

Fig. 2b



$\delta\sigma_{\nu_L}$
$\sqrt{s}$ = 29 GeV
1%
2%
Δ=0.035
3%
4%
X/V
$\bar{V}$/V

Fig. 3a



$\delta\sigma_{\nu_L}$
$\sqrt{s}$ = 112 GeV
1%
2%
3%
4%
Δ=0.035
X/V
$\bar{V}$/V

Fig. 3b