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Observation of KNO Scaling in the Neutral Energy Spectra from α-α and pp Collisions at ISR Energies

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ABSTRACT

Neutral transverse energy spectra in pp and αα interactions are analyzed in terms of the Wounded Nucleon Model. Analysis of the αα spectrum by application of a multiple nucleon-nucleon collision mechanism is conveniently performed when a Gamma Distribution is used to represent the pp spectral shape. The Wounded Nucleon Model provides a reasonable description of the αα spectrum for the first 3 orders of magnitude, but completely fails to account for the slope of the high energy tail of the distribution. However, the pp and αα spectra can both be fit to the same Gamma Distribution when scaled by their respective mean values and thus exhibit KNO scaling.

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Measurements of the total neutral energy spectrum, E°_{TOT}, for pp dd and αα collisions at $\sqrt{s}_{NN} = 31$ GeV at the CERN ISR have been previously published[1] by this group (Figure 1). The spectrum of total neutral energy emitted in the central region was measured using an electromagnetic shower counter which detected, but did not separately resolve, the photons from the decays of π° and η° particles ($\pi^{\circ} \rightarrow \gamma\gamma$, $\eta^{\circ} \rightarrow \gamma\gamma$). A sum was made of all the neutral energy observed in the detector for each event, and a frequency distribution of this quantity was tabulated, normalized by the integrated luminosity. The center-of-mass acceptance in which the neutral energy was detected covered 90% of 2π in azimuth with an average rapidity acceptance inside this region of $\Delta y = \pm 0.9$ about $y = 0$. Only the statistical errors are plotted. The systematic errors are small for the spectra of E°_{TOT} shown in Figure 1. In this detector, E°_{TOT} and the transverse neutral energy, E°_{T}, are proportional, $E^{\circ}_{T} \sim 0.87\ E^{\circ}_{TOT}$, but the systematic errors for the measurement of E°_{T} are larger.[1] Hence only the E°_{TOT} spectra are used for further analysis. A full description of the experiment[1] and details of the apparatus[2,3] have been given elsewhere.

The quantity E°_{TOT} is a multiparticle measure, like multiplicity, being the sum over many particles in the detector on an event-by-event basis. Thus E°_{TOT} is additive in the case of multiple nucleon-nucleon (N-N) collisions. The E°_{TOT} spectrum in αα interactions is conventionally analyzed as the result of multiple independent N-N collisions, with each N-N collision producing the observed E°_{TOT} spectrum of pp interactions. The observed pp spectrum is treated as the probability function for the collision of two nucleons:

$$f_1(E) = \frac{1}{\sigma_{in}^{pp}} \left(\frac{d\sigma}{dE}\right)_{pp}$$

where $\sigma_{in}^{pp} = \int_0^{\sqrt{s}} (d\sigma/dE)_{pp} dE$ and the $f_1(E)$ is the differential probability for the emission of energy E in dE in our detector for a single nucleon-nucleon collision. Then $f_n(E_0)dE_0$, the probability of observing E_0 in dE_0 for n such collisions overlapped, is given by the n-fold convolution of $f_1(E)$. This is easy to understand by writing $f_n(E_0)$ in the recursive form:

$$f_n(E_0) = \int_0^{E_0} dE\ f_1(E)\ f_{n-1}(E_0-E)$$

where $0 \le E_0 \le n\sqrt{s}_{NN}$. Here, E_0 represents the energy emitted on n collisions. The first term inside the integral is the probability of emitting energy E on one collision and the second term is the probability of emitting a total of E_0-E on n-1 collisions.

It should be noted that all the n-collision spectra are normalized to unity; only the shape is determined. They can be renormalized to the probability or cross section for n simultaneous nucleon-nucleon collisions, if this is known from a model. Alternatively, the data can be used to fit for the n-collision cross sections σ_n, n = 1,2,...m as parameters:

$$\frac{1}{L}\frac{dN^{\alpha\alpha}}{dE^{\circ}_{TOT}} = \sum_{n=1}^{m} \sigma_n\ f_n(E^{\circ}_{TOT}) \ . \qquad (1)$$

The analysis is greatly facilitated by using a Gamma Distribution to represent the N-N probability distribution $f_1(E)$. The pp spectrum in Figure 1 is fit to the function

$$f_1(E) = \frac{\alpha}{\Gamma(p)} (\alpha E)^{p-1}\ e^{-\alpha E} \qquad (2)$$

for the parameters p and α. It should be noted that $p > 0$, $\alpha > 0$, $f_1(E)$ is normalized to unity over the range $0 \leq E \leq \infty$ and that $\Gamma(p)$ is the gamma function of p, = (p-1)! if p is an integer. The function fits the pp data very well with the result:

$\alpha = 1.41 \pm 0.01\ \mathrm{GeV}^{-1}$ $\quad p = 2.50 \pm 0.06$ $\quad \langle E \rangle = p/\alpha = 1.77 \pm 0.03$ GeV

$\sigma_{in}^{pp} = 13.1 \pm 0.3$ mb $\quad \chi^2 = 24.6/15$ d.o.f.

The energy spectrum produced by n simultaneous independent N-N collisions, for n = 1,2,3...., is then simply given by the function:

$$f_n(E) = \frac{\alpha}{\Gamma(np)} (\alpha E)^{np-1} e^{-\alpha E} \tag{3}$$

with the values of α and p determined above. All the $f_n(E)$ remain normalized to unity.

The "Wounded Nucleon Model"[4] has been used successfully by the AFS group[5,6] to relate the central multiplicity distributions ($|y| < 0.8$) of pp, αp, and $\alpha\alpha$ collisions at $\sqrt{s_{NN}} = 31$ GeV. The main distinctive feature of the "Wounded Nucleon Model" is that it counts the number of struck nucleons geometrically, using measured nucleon density distributions; and that a nucleon contributes only once to the production of particles no matter how many times (≥ 1) it is successively struck. The AFS group[5] determined the relative cross sections for n nucleon-nucleon collisions per $\alpha\alpha$ interaction:

$$r_n \equiv \frac{\sigma_n}{\sigma_{in}^{\alpha\alpha}} .$$

The results agree very well with the α-particle geometry. These results can be applied directly in Equation (1), with Equations (2) and (3) used for $f_n(E)$, if n is allowed to take on half-integer as well as integer values. For instance, $n = 2\frac{1}{2}$ corresponds to 5 wounded nucleons (w): $w = 2n$.

The results of the fits are shown in Figure 2. The line on the pp data is the fit to Equation (2) given above. The dotted line on the $\alpha\alpha$ data is the best fit using the n-collision probabilities derived by the AFS collaboration[5] from their charged multiplicity data. This fits the E°_{TOT} spectrum out to 10 GeV, or about $1\frac{1}{2}$ orders of magnitude down in cross section. The dashed line on the $\alpha\alpha$ data allows the σ_n, $n = 1, 1\frac{1}{2}, 2....4$, to take on the values which give the best fit to the data (see Table 1). It is clear from Table 1 that this fit is obtained by saturating the data with the largest number of wounded nucleons allowed, leaving the probability for 1 and 2 pp collisions unchanged from the AFS values. This curve fits the E°_{TOT} spectrum out to 20 GeV, or over 4 1/2 orders of magnitude. Note that both fits have essentially the same shape in the region beyond 15 GeV which is dominated by 8 wounded nucleons, the maximum allowed in this model; whereas the slope of the data is much flatter. In summary, the "Wounded Nucleon Model" works well for the first 90% of the $\alpha\alpha$ cross sections, and can fit the data with unreasonable parameters over half the measured range of E°_{TOT}, but leads to the wrong functional form for the high energy tail of the data over a 6 order-of-magnitude range of cross section.

A surprising result was obtained by fitting the $\alpha\alpha$ spectrum to a single Gamma Distribution (Equation 2) with all parameters free. This is the solid line on the $\alpha\alpha$ data in Figure 2. The fit is quite reasonable over the whole E°_{TOT} range, with parameters:

$\alpha = 0.822 \pm 0.005\ \mathrm{GeV}^{-1}$ $\quad p = 2.48 \pm 0.05$ $\quad \langle E \rangle = p/\alpha = 3.01 \pm 0.05\ \mathrm{GeV}$

$\sigma_{in}^{\alpha\alpha} = 124 \pm 1.5\ \mathrm{mb}$ $\quad \chi^2 = 139.7/30$ d.o.f.

The fact that the parameter p is the same within errors for both the pp and $\alpha\alpha$ data means that the E°_{TOT} spectra for pp and $\alpha\alpha$ interactions measured in this experiment obey KNO scaling.[7] This can be seen mathematically by rewriting equation 2 in terms of the scaled variable $z \equiv E/\langle E \rangle$, where $\langle E \rangle = p/\alpha$. The distribution depends only on the parameter p:

$$\langle E \rangle f_1(E) = \psi(z) = \frac{p}{\Gamma(p)} (pz)^{p-1}\, e^{-pz} \tag{4}$$

A much more graphic illustration of the KNO scaling is obtained by making a KNO-plot of both the pp and $\alpha\alpha$ data (Figure 3). The E°_{TOT} spectra plotted in this way for pp and $\alpha\alpha$ interactions are nearly indistinguishable over 10 orders of magnitude! In particular, the tails of the pp and $\alpha\alpha$ distributions are the same, when measured in units of the average value of the energy $\langle E^{\circ}_{TOT} \rangle$, in each case.

The KNO formulation seems to account beautifully for the shape of the pp and $\alpha\alpha$ spectra but does not provide any guidance for the relationship between $\sigma_{in}^{\alpha\alpha}$ and σ_{in}^{pp} or $\langle E^{\circ}_{TOT} \rangle^{\alpha\alpha}$ and $\langle E^{\circ}_{TOT} \rangle^{pp}$. However, these gross features of data are accounted for very well in the geometrical framework:[4]

$$\langle w \rangle \equiv 2\, \langle E^{\circ}_{TOT} \rangle^{\alpha\alpha} / \langle E^{\circ}_{TOT} \rangle^{pp} = 2 \times (1.70 \pm 0.04)$$

$$\langle n \rangle \equiv 16\, \sigma_{in}^{pp} / \sigma_{in}^{\alpha\alpha} = 1.69 \pm 0.04\ .$$

Also the AFS r_n parameters[5] give directly

$$\langle w\rangle/2 = 1.61$$

These values are also in excellent agreement with the values 1.74 ± 0.06 and 2.0 ± 0.2 for $\langle w\rangle/2$ and $\langle n\rangle$ derived from full solid angle multiplicity measurements in pp and $\alpha\alpha$ interactions at the ISR.[8]

Previous authors[5,9,10] have examined the question of KNO scaling in nuclear collisions, but have always favored the wounded-nucleon, or other similar multiple independent-collision model, to explain the data. In general, these conclusions were based on data spanning only 2 to 3 orders of magnitude in cross section. An expanded plot of the first 3 orders-of-magnitude of Figure 3 is shown in Figure 4, together with the KNO fit, equation 4 with p = 2.48. On this scale, the KNO curve fits the pp data very well, but the $\alpha\alpha$ data systematically miss the curve. This effect is also clear in Figures 2 and 3. The $\alpha\alpha$ data are systematically higher than the KNO fit over the range $12 \leq E^{o}_{TOT} \leq 20$ GeV, and for the lowest data point. Nevertheless, it is clear from Figure 3 that over the 10 orders-of-magnitude for which the E^{o}_{TOT} spectra in the rapidity range $|y| < 0.9$ have been measured for pp and $\alpha\alpha$ interactions, the most reasonable representation of the data is a single Gamma Distribution, Equation 4, exhibiting KNO scaling for the pp and $\alpha\alpha$ distributions at the same value of nucleon-nucleon c.m. energy, $\sqrt{s}_{NN}$ = 31 GeV, with integrated observed $\alpha\alpha$ cross section and $\langle E^{o}_{TOT}\rangle^{\alpha\alpha}$ related to the pp values by the wounded-nucleon or other multiple-collision model.

It is interesting to note that multiplicity distributions in pp and $p\bar{p}$ interactions do not, in general, exhibit KNO scaling.[11,12] However, all the available pp and $p\bar{p}$ charged multiplicity distributions for $\sqrt{s}$ above 10 GeV can be fit by the scaled Negative Binomial Distribution,[11,12] which is the same as Equation 4 in the continuum limit. Thus, a simple representation exists which describes all this "soft" multiparticle physics in terms of systematic variations of the mean value and the parameter p for all measured distributions. However, the underlying explanation for this apparent simplicity remains a mystery[13,14] at the present time.

In conclusion, the high energy tail of the E_{tot} spectrum in $\alpha\alpha$ interactions cannot be obtained from the pp spectrum using the extreme-independent-collision hypothesis fundamental to the wounded nucleon model. In itself this is not surprising since the very simplistic assumptions of the model omit any multiple scattering or coherence between the single particle interactions. The truly surprising result of the data presented is that the KNO scaling of the pp spectra fits the $\alpha\alpha$ data over ten decades including the high energy tail. In a sense KNO scaling implies the opposite of independent particle interactions. Any simple independent particle model for the $\alpha\alpha$ reaction allows the energy release in each N-N interaction to be combined in a random way with that of another N-N interaction. Thus the use of convolution integrals. However, convolutions of a function do not have the same shape as the original function and cannot obey KNO scaling. In terms of the Gamma distribution (Eq. 3), convolutions leave the parameter α the same and let p increase to np. Whereas, the observed scaling leaves p unchanged but varies α by the mean value. The implication, then, of the excellent fit to the data in figure 3 is that the N-N collisions involved in the $\alpha\alpha$ scattering must be, unexpectedly, highly correlated.

Table 1

Parameters of Wounded Nucleon Fit

	AFS r_n Fixed		Best Fit r_n Free
n	w	r_n(percent)	r_n(percent)
1	2	44.7	47.7 ± 8.4
1½	3	19.9	14.1 ± 12
2	4	16.0	28.4 ± 6.4
2½	5	10.2	0 ± 0.8
3	6	5.97	0 ± 0.1
3½	7	2.65	0 ± 0.1
4	8	0.54	9.7 ± 0.3
$\sigma_{in}^{\alpha\alpha}$ (mb)	134 ± 2		129 ± 4
χ^2/dof	1984/32		1044/26

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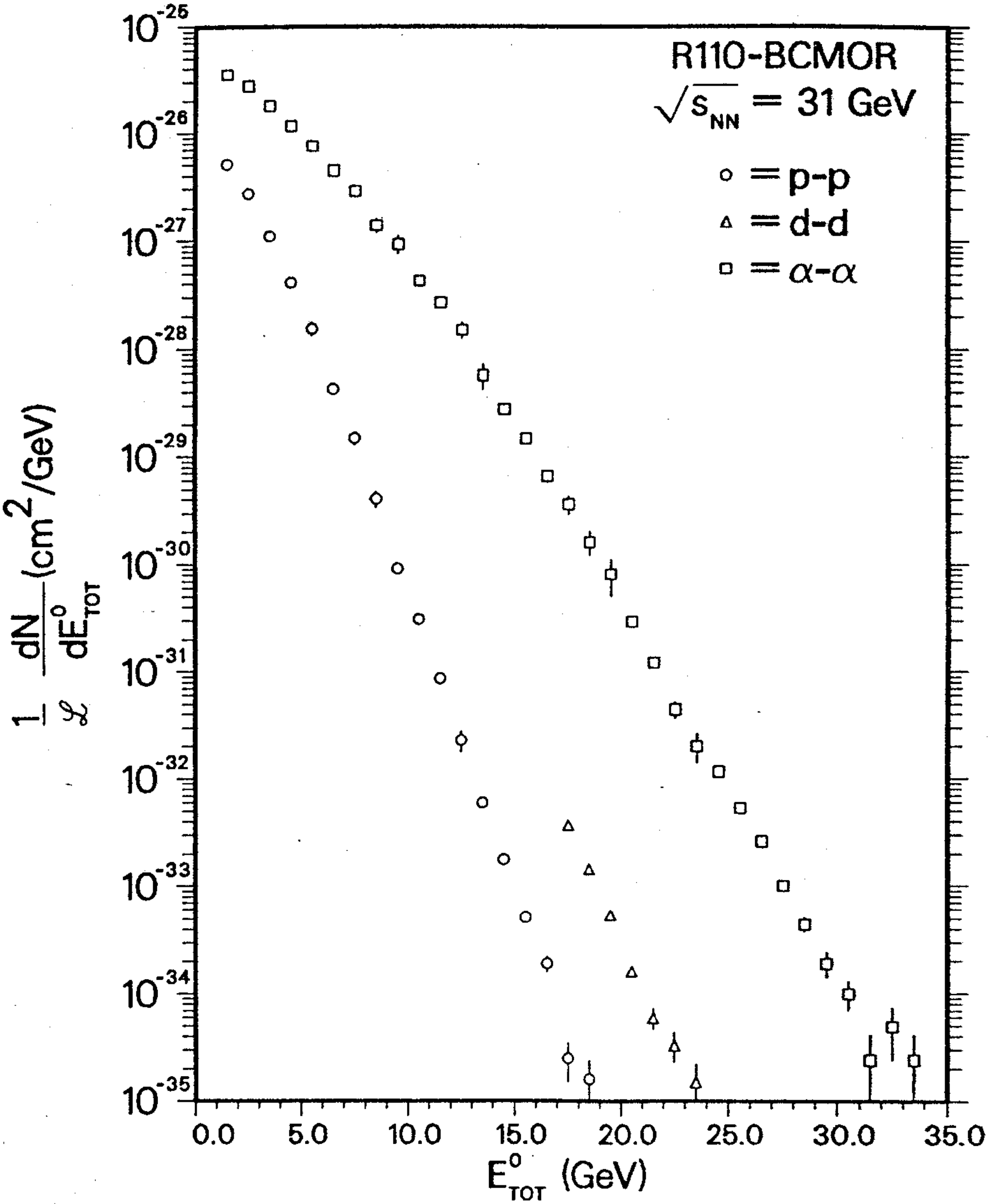
FIGURE CAPTIONS

Figure 1: The total neutral energy spectra for pp, dd and $\alpha\alpha$ interactions at $\sqrt{s}_{NN} = 31$ GeV

Figure 2: The total neutral energy spectra of Figure 1 for pp and $\alpha\alpha$ interactions together with curves representing fits to the data as described in the text.

Figure 3: The pp and $\alpha\alpha$ E°_{TOT} distributions of Figure 1 replotted in the scaled energy variable, $z = E^\circ_{TOT}/\langle E^\circ_{TOT}\rangle$, where $\langle E^\circ_{TOT}\rangle$ is the average value of the energy for each distribution as determined by a fit to Equation 2.

Figure 4: Expanded plot of the highest 3 decades of Figure 2. The solid line is the same as the solid line on the $\alpha\alpha$ data in Figure 2, a fit to Equation 4 with $p = 2.48$.



R110-BCMOR
$\sqrt{s_{NN}}$ = 31 GeV
○ = p-p
△ = d-d
□ = α-α
$\frac{1}{\mathscr{L}} \frac{dN}{dE^0_{TOT}}$ (cm^2/GeV)
E^0_{TOT} (GeV)

Fig. 1

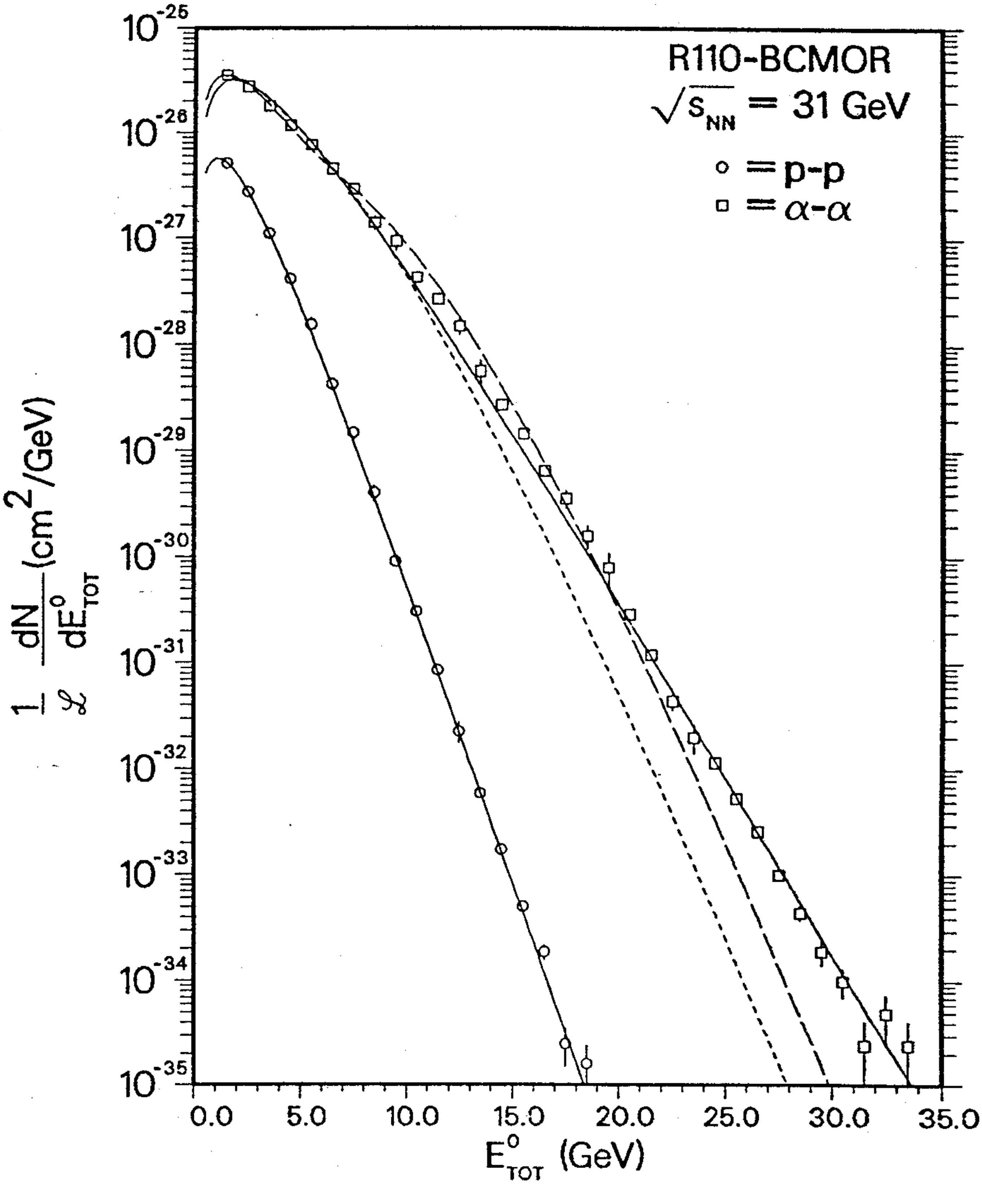

R110-BCMOR
√s_NN = 31 GeV
○ = p-p
□ = α-α
1/ℒ dN/dE⁰_TOT (cm²/GeV)
E⁰_TOT (GeV)
10^-25
10^-26
10^-27
10^-28
10^-29
10^-30
10^-31
10^-32
10^-33
10^-34
10^-35
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0

Fig. 2

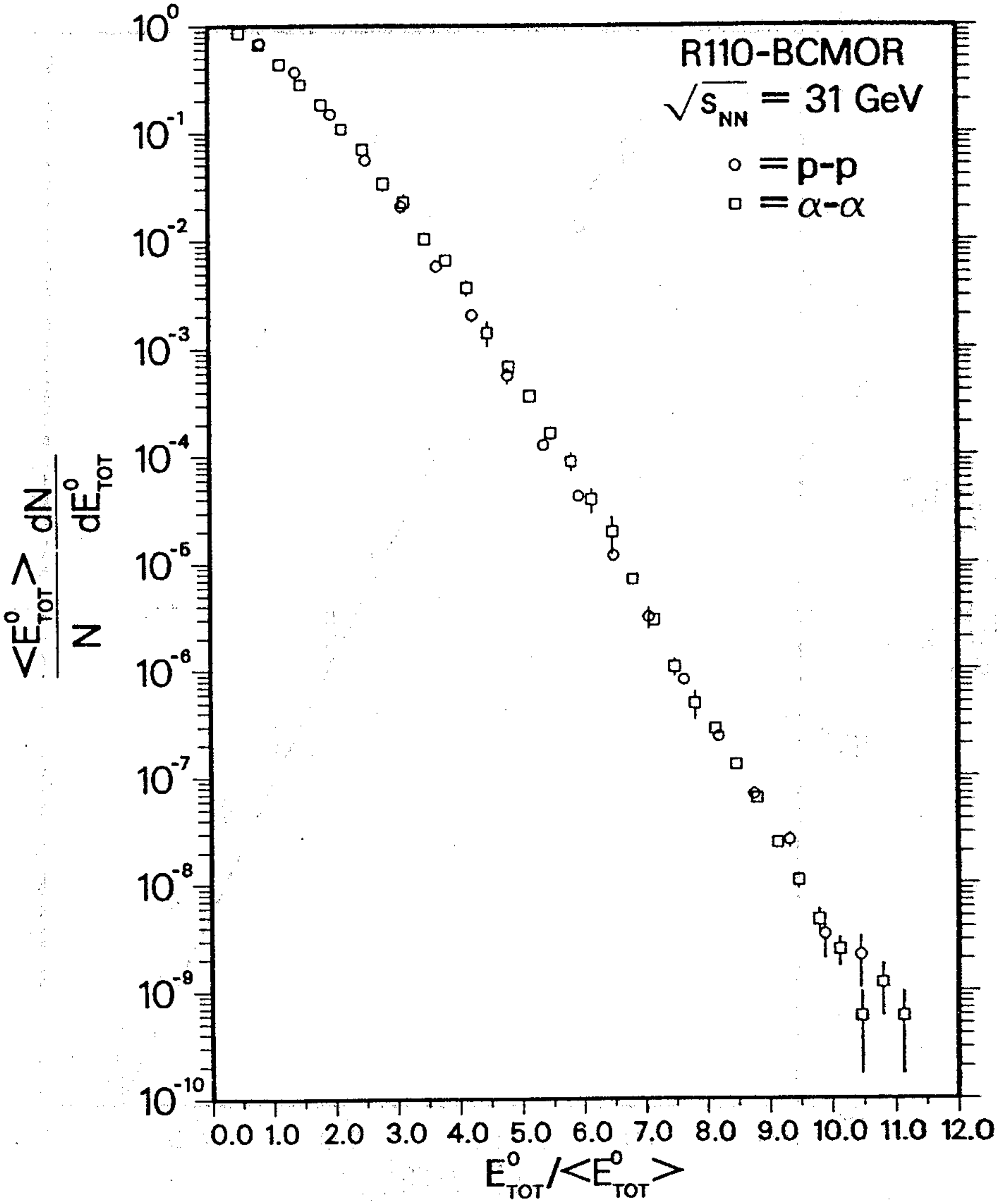

R110-BCMOR
$\sqrt{s_{NN}} = 31$ GeV
○ = p-p
□ = α-α
$\frac{\langle E^0_{TOT} \rangle}{N} \frac{dN}{dE^0_{TOT}}$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 11.0 12.0
$E^0_{TOT}/\langle E^0_{TOT} \rangle$

Fig. 3

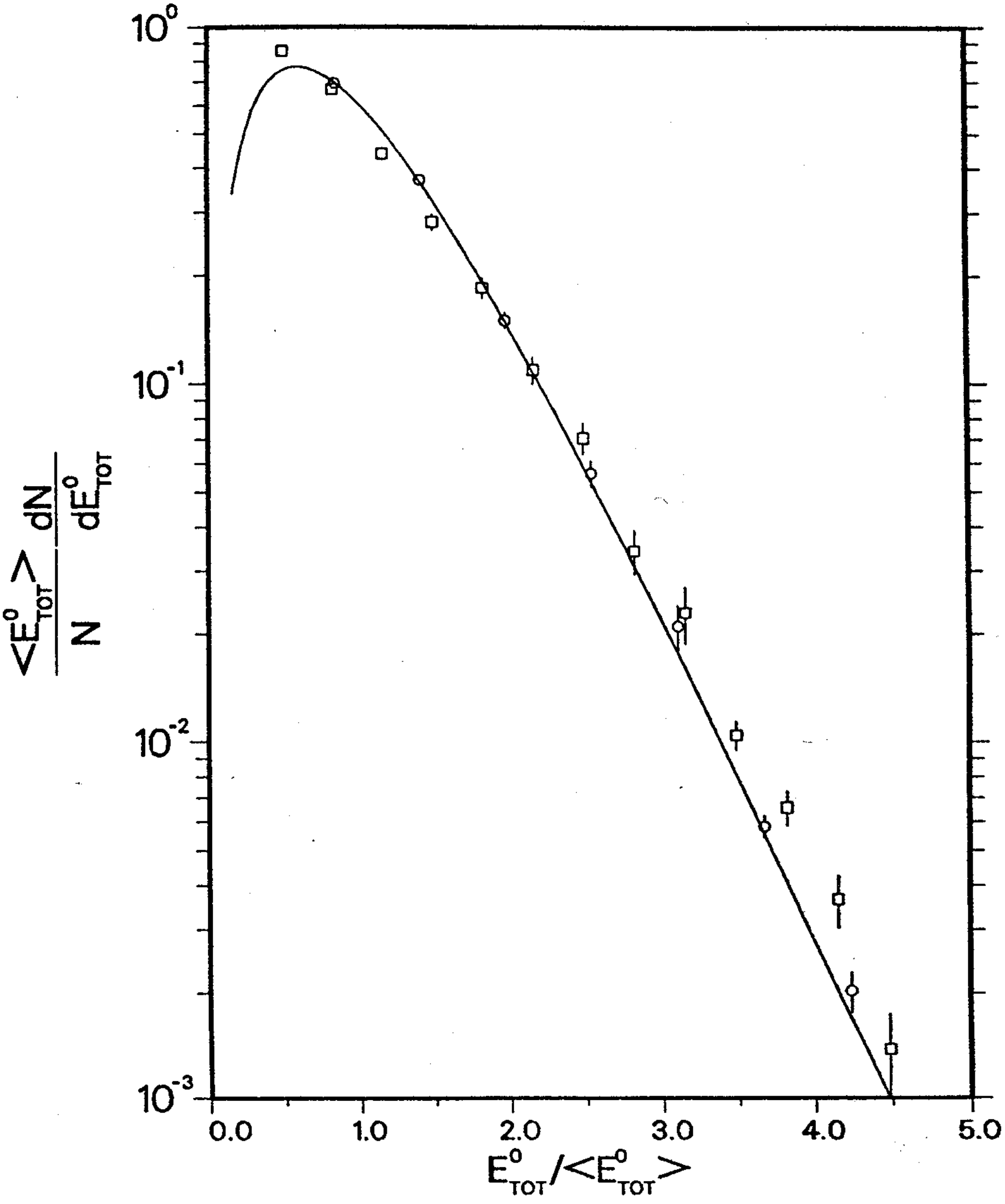


Fig. 4