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KÄHLER-CURVATURE EFFECTS CAN SOLVE THE INITIAL CONDITION PROBLEM IN INFLATION

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ABSTRACT

In a large class of non-linear σ-models there arise at tree level important thermal effects due to the curvature of the field manifold. We show how these can provide naturally the initial conditions necessary for inflation.

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Non-linear sigma models may often arise as the low-energy effective theory of a more fundamental theory. In sigma models the scalar fields z^A can be thought as co-ordinates on some manifold G/H with a field-dependent induced metric g. A well-known example of such a situation is encountered in N = 1 supergravity[1], where the bosonic sector is simply a sigma model coupled to Einstein gravity. In N = 1 supergravity G/H is a Kähler manifold which means that locally one can express the metric as $g_{A\bar{B}} = \partial_A \partial_{\bar{B}} G$, where $G(z^A, \bar{z}^{\bar{A}})$ is a real function [we use the notation $\bar{z}^{\bar{A}} = (z^A)^*$]. In N = 1 supergravity G/H is not fixed, whereas in extended supergravities one often finds non-compact symmetry groups. The phenomenologically successful no-scale supergravity models[2),3)] are based on SU(n,1)/SU(n)×U(1) Kähler manifolds, and they may also be[4] the field-theory limit of superstrings[5].

The cosmological consequences of sigma models can be expected to be interesting because of the non-trivial modifications caused by the curvature of the scalar field manifold[6]. In the present paper we will address the question of initial conditions for inflation in the framework of sigma models.

It has been argued by Linde[7] that in inflationary models based on supercooling the field responsible for inflation (the inflaton) in fact cannot relax into the local minimum created by high temperature effects before the inflationary era is supposed to start. The argument is shortly the following. The T = 0 potential has an overall scale H_0^2 fixed by the energy density perturbations to be roughly $O(10^{-12})$ (in natural Planck units $M_P/\sqrt{8\pi} = 1$). The high temperature thermal corrections are therefore of the order of $H_0^2 T^2$. Therefore, in a radiation dominated Universe the typical time scale for an O(1) change in the field value is $\Delta\phi \sim \omega t \sim H_0 T^{-1}$. As inflation starts when $\rho_{rad} \sim T^4 < H_0^2$, one finds that $\omega t \sim H_0^{\frac{1}{2}} << 1$. As a natural initial value for the inflaton is presumably O(1), one then concludes[7] that it has not had time to relax to the local minimum.

A proposal for a solution that evades this problem is chaotic inflation[8] which is not based on supercooling. There one assumes that at $t = M_P^{-1}$ the initial field value can take any random value compatible with $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \lesssim 1$. If fluctuations in the kinetic energy E_k are much smaller than V, the Universe may then inflate provided the initial value of the inflaton was large enough. However, as a natural initial value for the fluctuation of the kinetic energy density is $E_k = \frac{1}{2}\dot{\phi}^2 \simeq 1$ (in natural units), this would imply that the initial field value $\phi_0 \gtrsim 10^3$ if the potential is given by $V \approx H_0^2\phi^4$ (the size of the quartic self-coupling λ is determined by the requirement of correct energy-density fluctuations). The scalar potential should be smooth in order not to produce unwanted perturbations in energy density, and moreover, there should not exist global minima where the inflaton may be trapped. It is difficult to see how these difficulties could be avoided if $\phi \gg 1$ because one expects the potential to be subject to gravitational corrections which are of the general form $\delta V = \Sigma\, a_n(\phi)(\phi/M_P)^n$. For example, if $\phi \gg 1$ one expects $\lambda \sim O(1)$ rather than $\lambda \lesssim 10^{-12}$ as required by energy density perturbations.

Gravitational corrections can presumably be neglected if $\phi \lesssim 1$. If this situation is to be realized in chaotic inflation, one needs $\dot\phi \ll 1$ to satisfy $\dot\phi \ll V$. We do not consider such a fine-tuning satisfactory: the problem of why initially $\phi_0 \approx 0$ (the value of ϕ at the global metastable minimum) has merely been traded with the problem of why initially $\dot\phi_0 \approx 0$.

In the present paper we will study inflation with supercooling in the context of non-linear sigma models. We will show that in these models the dominant thermal effects are present already at the tree level because of the non-trivial curvature of the field manifold. These effects can easily confine the fields to the local minimum and thus avoid the above problem. In what follows we will assume the field manifold to be Kähler. Generalizations to non-Kählerian situations are obvious.

Consider the following action

$$S = \int d^4x \sqrt{-g}\left[-\tfrac{1}{2}R - g_{A\bar B}\, g^{\mu\nu}\, \partial_{\{\mu} z^A\, \partial_{\nu\}} \bar z^{\bar B} + V(z,\bar z)\right] \qquad (1)$$

In the Friedman-Robertson-Walker background the field equation for homogeneous fields $z^A(t)$ reads

$$\ddot z^A + 3H\dot z^A + \Gamma^A_{BC}\,\dot z^B \dot z^C + g^{A\bar B}\,\partial_{\bar B} V = 0 \qquad (2)$$

where $\Gamma^A_{BC} = g^{A\bar D}\partial_B g_{C\bar D}$ is the connection and the Hubble parameter $H = \dot R/R$, where R is the cosmic scale factor as usual. Of special interest are cases where one has flat directions in the potential energy. Such situations arise, e.g., in no-scale models[2),3)], where they may also be responsible for cancelling the cosmological constant[9)]. In the following we will concentrate on the case of a single field z having a potential with a flat direction, which we choose to be Im z (our main arguments do not depend crucially on the assumption of a precisely flat direction). Therefore we take

$$g_{z\bar{z}} \equiv g = g(z+\bar{z}) \;;\; V = V(z+\bar{z}) \tag{3}$$

The potential V we envisage to contain all radiative corrections and it may or may not be of the usual supergravity form[1]. Denoting $u \equiv \mathrm{Re}\, z$ and $v \equiv \mathrm{Im}\, z$, one then finds from (2)

$$\ddot{u} + 3H\dot{u} + \Gamma(u)(\dot{u}^2 - \dot{v}^2) + \tfrac{1}{2} g^{-1} \partial_u V = 0 \tag{4a}$$

$$\ddot{v} + 2\Gamma(u)\dot{u}\dot{v} + 3H\dot{v} = 0 \tag{4b}$$

Equation (4b) immediately yields

$$\dot{v} = Q g^{-1} R^{-3} \tag{5}$$

where Q is a constant. We expect $Q \neq 0$ because of quantum fluctuations of the field z. The value of Q can differ in causally disconnected regions. Defining a properly normalized field, which has canonical kinetic energy, by

$$\dot{\phi} = \dot{u} g^{1/2} / \sqrt{2} \tag{6}$$

one obtains from (4a)

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi U_Q = 0 \tag{7}$$

where

$$U_Q = Q^2 g^{-1} R^{-6} + V(\phi) \tag{8}$$

is the effective potential depending on R. The equation of motion for the flat direction v has therefore influenced the field equation in the non-flat direction in a drastic way. Note that we have not yet specified the metric g in any way.

Let us now assume that at high temperatures $T \gg V^{1/4}$ we are in a radiation dominated Universe where $R = R_0 t^{\frac{1}{2}}$. This assumption is consistent because $\rho_{rad} =$

$\pi^2 NT^4/30$, where N is the number of light degrees of freedom, whereas $U_Q \sim T^6$. Then the equation motion (7) can be integrated to yield

$$\varepsilon = \left(\frac{d\phi}{d\xi}\right)^2 + 8Q^2 g^{-1} R^{-6} \tag{9}$$

where $\xi = t^{-\frac{1}{2}}$ and ε is a positive constant (this is because g must be positive in order not to have ghosts). For any reasonably smooth metric g it then follows that $\dot{\phi} \sim t^{-3/2}$, and thus one expects that ϕ will tend to a constant as $t \to \infty$. Let us call this constant $\bar{\phi}$ and expand the potential as

$$V(\phi) = 3H_0^2\left[1 - a(\phi-\bar{\phi}) + \tfrac{1}{2}b(\phi-\bar{\phi})^2 + \cdots\right] \tag{10}$$

where $H_0 \sim 10^{-6}$. The equation of motion (7) is dominated by Q-dependent part until $T < T_0$, where

$$T_0^{\,2} \simeq \left(\frac{aH_0^2}{Q^2}\right)^{1/3} R_0^2\, g(\bar{\phi}) / (N g'(\bar{\phi}))^{1/2} \tag{11}$$

The transition to a de Sitter era takes place when $T < T_c$ where

$$T_c^{\,2} \simeq 3N^{-1/2} H_0 \tag{12}$$

If $g(\phi)$ is smooth enough and we can take it to be O(1), then also $QR_0^{-3} \sim$ O(1) at most. Otherwise the energy density of v would be too large so that the homogeneous field z would collapse to form a black hole. Then from (11) and (12) we see that the Q-dependent part of the effective potential (8) will have died out before inflation. However, because one-loop thermal corrections[10)] will give terms of the order of $H_0^2 T^2$, the Q-dependent part dominates over these until $T^2 < H_0$, i.e., until the onset of inflation [see(12)]. Therefore, in sigma models these one-loop thermal corrections will never be important for inflation, in contrast to the theories with minimal kinetic terms. In fact, if the potential has a flat direction, as we have assumed, the high-temperature behaviour of the theory is not related to the zero temperature potential at all, and considerations of one-loop thermal corrections are irrelevant.

To implement inflation there should be a local minimum in the neighbourhood of $\bar{\phi}$. Q-dependent corrections to the potential V will then carry the field there. When inflation starts, these Q-dependent terms will become completely negligible because of their R^{-6}-dependence, and inflation can proceed in the conventional way.

Let us now address the crucial question of how long does it take the field ϕ to move from an initial value $\phi_P \sim O(1)$ to the neighbourhood of $\bar{\phi}$. Let us take $\dot{\phi}(M_P) = 0$. Then $\varepsilon = (8Q^2/R_0^6)g^{-1}(\phi_P)$ and when $T \sim T_c$, or $t \sim 10^6$, one finds that

$$\phi(T_c) \simeq \bar{\phi} + 8Q^2R_0^{-6}\left[g^{-1}(\phi_P) - g^{-1}(\bar{\phi})\right] \times 10^{-3} \tag{13}$$

Therefore, denoting by $\rho_v = g(\phi_P)\dot{v}(M_P)^2$ the energy density of the initial fluctuation (which we assume to be constant in a causally connected region), we can write

$$|\phi(T_c) - \bar{\phi}| \sim \mathcal{O}(1)\,\rho_v \times 10^{-3} \tag{14}$$

with $\rho_v < 1$. This is our main result. It is general and assumes no particular form of the metric g. It now depends on the details of a particular model whether all initial fluctuations of ρ_v will produce inflation or only some.

In conclusion, we have considered the initial conditions for inflation in models where the inflaton spans a curved Kähler manifold. Such models appear, e.g., in non-minimal supergravities. We assumed that the inflaton moves in a non-trivial Im z background as is the case in a large class of curved Kähler manifolds[6]. Instead of an empty classical vacuum, the inflaton sees the heat bath of its imaginary part. In that case we found an effective temperature-dependent potential for the inflaton field, which at high energies dominates over all other (one-loop) thermal corrections and can naturally give rise to proper initial conditions needed for inflation. The origin of such a term is due to the non-trivial coupling between the real and imaginary parts of the inflaton caused by the curvature. We expect that the mechanism described in this paper will work also in the existing models of primordial inflation[11] which make use of non-minimal kinetic terms for the fields.

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