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REDUCTION OF FERMION-GLUON SYSTEMS ON EXTENDED LATTICES

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ABSTRACT

We develop a simple method for dealing with perturbation theory in the presence of twisted boundary conditions. We compare in detail periodic and twisted SU(N) gauge models and stress the importance of twisted boundary conditions to suppress finite size effects. As an application we study staggered fermions in a reduced model for large N_{colour} and $N_{flavour}$. The ratio $N_{colour}/N_{flavour}$ can be made variable by performing only partial colour conversion.

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1. - INTRODUCTION

The original twisted Eguchi-Kawai (TEK) model[1),2)] is a very efficient tool for studying the large N limit on the lattice. In this framework the string tension and some information about the deconfining phase transition at finite temperature have been obtained[2)-5)]. But if one wants to study the deconfining phase transition in the scaling region, the glueball spectrum or staggered fermions and the meson spectrum for large N, the one-point TEK model is insufficient. So one is naturally led to the idea of constructing models with twisted boundary conditions on a lattice which is larger than one hypercube[5)].

Twist can be incorporated in two ways: one is to consider the potentials as periodic and to change the action into a twisted one. This way is the best for Monte Carlo (MC) simulations. The other way is to consider the potentials as periodic up to a gauge transformation which obeys a constraint (the twist), and to leave the action unchanged. The two descriptions are related through a singular gauge transformation. The latter approach has definite advantages for theoretical purposes and we exhibit it in this paper. Sections 2 and 3 treat the pure gauge model and Section 4 gauge models coupled to Kogut-Susskind fermions. Section 5 presents our conclusions.

2. - FEYNMAN RULES IN TWISTED AND PERIODIC BOXES

We study SU(N) lattice models defined by the action

$$S = \sum_{\mu \neq \nu} U_\mu(x)\, U_\nu(x+\mu)\, U^\dagger_\mu(x+\nu)\, U^\dagger_\nu(x) \tag{2.1}$$

and the periodicity condition

$$U_\mu(x + L_{s\nu}\vec{e}_\nu) = \Gamma_\nu U_\mu(x) \Gamma^\dagger_\nu \tag{2.2}$$

Here $\vec{e}_\nu$ is pointing in the ν-direction and $|\vec{e}_\nu| = a$, where a is the lattice spacing. The integers $L_{s\nu}$, $\nu = 0,\ldots,3$ determine the extension of the lattice. The unitary twist matrices obey the commutation relations

$$\Gamma_\mu \Gamma_\nu = \exp i\left(\frac{2\pi}{N}\right) n_{\mu\nu}\ \Gamma_\nu \Gamma_\mu \tag{2.3}$$

We choose a twist $n_{\mu\nu}$ creating a rectangular volume $L_{c0} \times L_{c1} \times L_{c2} \times L_{c3}$. For more details, see Ref. 6). After the transformation $U_\mu(x) \to U_\mu(x)$ or $U_\mu(x) \to U_\mu(x)\Gamma_\mu$ depending on the link (x,μ) as indicated in Fig. 1, the action becomes

$$S = \sum_{x,\mu\neq\nu} t_{\mu\nu}(x)\,\mathrm{Tr}\,U_\mu(x)\,U_\nu(x+\mu)\,U_\mu^+(x+\nu)\,U_\nu^+(x) \tag{2.5}$$

The values of the twist factors are given in Fig. 1. The ground state, which is unique up to gauge transformations, is also shown in Fig. 1. For MC simulations of the model the action (2.5) is most convenient, but because of the irregular distribution of twist factors and ground state matrices on the lattice, the description of the weak coupling limit of the model becomes cumbersome. It is much more convenient to start directly with the action (2.1). Then the ground state is given by

$$U_\mu(x) = \mathbb{1} \tag{2.6}$$

up to gauge transformations. In order to determine the propagator and vertices we write

$$U_\mu(x) = \exp i\,Q_\mu(x) \tag{2.7}$$

We find the propagator after adding the gauge breaking term

$$S_{GB} = -\sum_{x,\mu,\nu} \mathrm{Tr}\,\big(Q_\mu(x) - Q_\mu(x-\mu)\big)\big(Q_\nu(x) - Q_\nu(x-\nu)\big) \tag{2.8}$$

to the bilinear part of the action. Then

$$S + S_{GB} = \sum_{x,\mu,\nu} \mathrm{Tr}\big(Q_\mu(x) - Q_\mu(x-\nu)\big)^2 \tag{2.9}$$

Now we expand the traceless matrices $Q_\mu(x)$ in terms of the unitary, traceless matrices[2)]

$$\tilde{\Gamma}(q) \equiv \exp\Big(i\,\tfrac{1}{2}\,\tfrac{2\pi}{N}\sum_{\mu>\nu} q_\mu n_{\mu\nu} q_\nu\Big)\,\Gamma_0^{q_0}\,\Gamma_1^{q_1}\,\Gamma_2^{q_2}\,\Gamma_3^{q_3}$$

Then

$$Q_\mu(x) = \frac{1}{N^2}\sum_{q\neq(0000)} \tilde{Q}_\mu(q;x)\,\tilde{\Gamma}(q) \tag{2.10}$$
$$0 \leq q_\mu \leq L_{c\mu} - 1$$

From Eq. (2.2) we derive that the functions $\tilde{Q}_\mu(q,x)$ are periodic up to a phase factor

$$\tilde{Q}_\mu(q;x+L_{s\nu}\vec{e}_\nu) = \exp\left(i\,\frac{2\pi}{N}\,n_{\nu\rho}\,q_\rho\right)\tilde{Q}_\mu(q;x)$$

and so their Fourier representation reads

$$\tilde{Q}_\mu(q,x) = \sum_\ell \exp\left(i\,\frac{2\pi}{N}\sum_\rho \frac{x_\rho\,(P_\rho(q)+L_{c\rho}\ell_\rho)}{L_{s\rho}L_{c\rho}}\right)\hat{Q}_\mu(q;\ell) \qquad (2.12)$$

with

$$\sum_\ell \equiv \sum_{\ell_0,\ell_1,\ell_2,\ell_3} \quad ; \quad 0 \le \ell_\mu \le L_{s\mu}-1 \quad ; \mu=0,1,2,3$$

and

$$P_\rho(q) \equiv L_{c\rho}\,n_{\rho\sigma}\,q_\sigma/N$$

We observe that the colour momenta $2\pi(P_\rho/L_{c\rho})$ and the space-time momenta $(2\pi\ell_\rho/L_{s\rho})$ combine to the total momentum

$$P_\mu(q,\ell) = 2\pi\,\frac{(P_\rho(q)+L_{c\rho}\ell_\rho)}{L_{s\rho}L_{c\rho}} \qquad (2.13)$$

Then Eq. (2.12) becomes

$$\tilde{Q}_\mu(q;x) = \sum_\ell \exp(i x\cdot P(q,\ell))\;\hat{Q}_\mu(q,\ell) \qquad (2.14)$$

Later we want to point out the close similarity of the Feynman graphs of the twisted model defined by Eqs. (2.1) and (2.2) on a lattice of volume $\Pi^3_{\mu=0}L_{s\mu}$ and the periodic Wilson model on a lattice of volume $N^2\cdot\Pi^3_{\mu=0}L_{s\mu}$ defined by Eq. (2.1) and

$$U_\mu(x + L_{c\nu} L_{s\nu} \vec{e}_\nu) = U_\mu(x) \qquad (2.15)$$

$\Pi^3_{\mu=0} L_{c\mu} = N^2$ is the twist volume.

For reasons of easy comparability of the models we use also in the periodic case the expansion (2.10). Then the periodic $\tilde{Q}_\mu(q_j x)$ have the Fourier representation (2.14) too with

$$P_\mu(q,\ell) = \frac{2\pi}{L_{c\mu} L_{s\mu}} (q_\mu + L_{c\mu} \ell_\mu) \qquad (2.16)$$

Here P_μ is a pure space-time momentum which, on a lattice of size $\Pi\, L_{s\mu} L_{c\mu}$ can always be decomposed as in Eq. (2.16). The resulting propagators and vertices of the twisted and the periodic models are given in Table 1.

3. - COMPARISON OF THE MODELS

We can now state the differences between the twisted and the periodic models.

a) In the twisted model the colour momenta q and the space-time momenta ℓ form new momenta $P_\mu(q,\ell)$. The propagators and vertices depend on these combined colour-space-time momenta.

b) In the periodic model the colour momenta do not enter propagators and vertices. Every Feynman diagram factorizes into a part depending only on the space-time momenta $P(q,\ell)$ appearing in propagators and vertices and a second part depending only on colour momenta. The sum over the colour momenta can be performed explicitly and gives a polynomial in N.

c) So the combined colour-space-time momenta Eq. (2.13) of the twisted model are related to the pure space-time momenta Eq. (2.16) of the periodic model. Both have the same range if the twisted model has space volume $\Pi^3_{\mu=0} L_{s\mu}$ and colour volume $\Pi^3_{\mu=0} L_{c\mu}$ and the periodic lattice has space volume $\Pi^3_{\mu=0} L_{s\mu} L_{c\mu}$. But in the twisted model all momenta with colour part $q = (0,0,0,0)$ are excluded; see Fig. 2. Therefore, in the twisted case the momentum sums of diagrams cover a slightly smaller set of points. This difference between Feynman graphs is balanced by different polynomials of N^2 multiplying the diagrams.

d) The vertices of the two models differ by a phase factor $\exp\, i\alpha(q_1,\ldots,q_n)$ depending only on the colour momenta. This factor is only present in the vertices of the twisted model.

e) There is an additional difference in the external vertices. Because we consider only gauge invariant operators, the sum of the colour momenta vanishes in every external vertex of the twisted model.

This is clearly not true for the external vertices of the periodic model, as the quantities q, which are equivalent to the colour momenta of the twisted model, are parts of real space momenta and their sum does not vanish. To obtain comparable external operators, we have to guarantee that $\Sigma_{i=1}^{n} q_i = 0$ also in the periodic case. That means that we have to take, instead of $\mathrm{Tr}(Q_{\mu_1}(x_1),\ldots,Q_{\mu_n}(x_n))$, expressions like

$$V_{ext} = \frac{1}{N^2} \sum_k \mathrm{Tr}\; Q_{\mu_1}(x_1+k) \cdots\cdots\cdots Q_{\mu_n}(x_n+k)$$

as observables.
Here

$$k_\mu = \hat{k}_\mu L_{s\mu} \qquad\qquad 0 \leq \hat{k}_\mu \leq L_{c\mu}-1$$

Then

$$V_{ext} = \frac{1}{N^{2n-1}} \bar{\sum_k} \delta(\Sigma k) \exp i\alpha(k_1 \ldots k_n) \times$$

$$\times \sum_{q,\ell} \delta(\Sigma q) \prod_j \hat{Q}_{\mu j}(k_j ; q_j, \ell_j) \prod_j \exp i x_j P_j(q,\ell)$$

This is illustrated in Fig. 3.

Now we can follow exactly the line of reasoning given in Refs. 2) and 7), to show that the planar diagrams of the two models coincide*). The non-planar diagrams differ by phase factors appearing only in the twisted model.

Some consequences:

For any N the Wilson loop in the twisted model on a lattice of volume V and the Wilson loop in the periodic model on a lattice of volume N^2V have equal planar parts*). This is also true for the connected part of correlation functions of two glueballs at rest provided there is no twist in time direction. See Fig. 4.

In general the correlation function in the twisted model has no simple equivalent in the periodic model as depicted in Fig. 5.

4. - REDUCED SYSTEMS WITH FERMIONS

Let us now apply what we learnt in the previous sections to colour gauge fields coupled to quarks in the fundamental representation. This will enable us to study the deconfining and chiral symmetry restoring transitions for a large number N_c of colours, and a large number N_f of flavours if we want to see any effect of dynamical fermions. Let us note that it is impossible to make the reduction for a finite number of flavours. In that case the gluon field and the fermion field would, after colour conversion, have different momentum spaces and no agreement with the unreduced Wilson model could be achieved. But it would also be useless to study a model with finite N_f because the fermions would only show up in the unphysical non-leading terms in $1/N_c$. The case of Wilson fermions has been considered before[8),9)]. It is the Kogut-Susskind type of fermions[10)] that necessitates the panoply we have developed in the previous sections.

a) Twisted boundary conditions for Kogut-Susskind fermions

The fermions of this type have their spin and flavour components scattered over a hypercube. This is why we need our generalized version of reduction. The way the spin (s) and SU(4)(α) degrees of freedom of the spinor $u_{s\alpha}(X)$ are put on the hypercube with centre X and with the 16 corners denoted by x $[x \equiv (x_1, x_2, x_3, x_4); x_\mu$ integer$]$ is familiar from reduction. Take the Dirac

*) Except the small difference in momentum space volume mentioned in 3c.

matrices γ_μ with

$$\{ \gamma_\mu , \gamma_\nu \} = 2 \delta_{\mu\nu} \tag{4.1}$$

Then we denote by $\gamma(x)$ the matrix

$$\gamma_0^{x_0} \gamma_1^{x_1} \gamma_2^{x_2} \gamma_3^{x_3} \tag{4.2}$$

and convert the spin and flavour degrees of freedom of $u(X)$ into space degrees of freedom by putting[11)]

$$u_{\alpha s}(X) = \sum_{\substack{x \in \text{unit cell} \\ \text{around } X}} \gamma(x) \chi(x) \tag{4.3}$$

The anticommuting numbers $\chi(x)$ are promoted to be the basic objects of the theory; we give them a colour index c and some flavour index f. The colour index is of course coupled to colour fields $U_\mu(x)$ living on the lattice links $(x, x+\mu)$. The action S_{fgl} is:

$$S_{fgL} = \sum_{x,\mu} \alpha_\mu(x) \bar{\chi}(x) \left(U_\mu(x) \chi(x+\mu) - U_\mu^+(x-\mu) \chi(x-\mu) \right) \tag{4.4}$$

The phase $\alpha_\mu(x)$ is defined by

$$\alpha_\mu(x) = \gamma^+(x) \gamma_\mu \gamma(x+\mu) \tag{4.5}$$

The point x ranges now through the entire lattice and $\alpha_\mu(x)$ has obviously, from its definition (4.5) and (4.1), periodicity 2.

We now impose the boundary conditions

$$U_\mu(x + L_{s\rho} \vec{e}_\rho) = \Gamma_\rho U_\mu(x) \Gamma_\rho^+ \tag{4.6}$$

$$\chi(x + L_{s\rho} \vec{e}_\rho) = \Gamma_\rho \chi(x) G_\rho^+ \tag{4.7}$$

The temporal-spatial extent of our lattice, L_s, is now necessarily even in all directions, since it is built up not by unit cells, but by cells of size two in all directions. The twist matrix G_ρ couples to the flavour index of the χ field. This matrix will be taken as equal to Γ_ρ for the spatial directions; for the Euclidean time direction we may take it to be such that

$$G_0^{L_{co}} = - \Gamma_0^{L_{co}} \tag{4.8}$$

is satisfied*).

This admits the interpretation of Euclidean time as inverse temperature also for the fermions.

b) Comparison between twisted and periodic models

Like in Section 3, we are going to compare the Feynman rules on the twisted L_s^4 lattice and on the periodic $(L_c L_s)^4$ lattice. For the pure gauge sector nothing changes. For the fermion-gluon sector we have a few peculiarities that we wish to exhibit now. The first peculiarity is the presence of the factor $\alpha_\mu(x)$ in the action (4.4). This factor, since it is periodic modulo 2 lattice spacings, does not spoil the colour momentum conservation in the propagator or in any vertex deriving from (4.4). To see this, look at the propagator term we get from (4.4) by setting $U_\mu(x) = 1$. Then we have to do the Fourier analysis of the previous section on the fermions. We have

$$\chi(x) = \sum_{p_c} \chi(p_c;x)\, \hat{\Gamma}(p_c) \tag{4.9}$$

and from the twisted boundary condition (4.7) and (4.8):

$$\chi(x) = \sum_{p_c, p_s} \chi(p_c;p_s) \exp i\, P(p_c;p_s).x \tag{4.10}$$

The momentum $P(p_c,p_s)$ combines the colour and space momenta for the spacelike components as before in Eq. (2.13). For the timelike components we have, due to the - sign in (4.8):

$$\frac{L_{so}L_{co}}{2\pi} P_o(p_c;p_s) = p_{co} + (p_{so}+\tfrac{1}{2})L_{co} \tag{4.11}$$

Now we have for $\alpha_\mu(x)$ the Fourier representation:

$$\alpha_\mu(x) = \exp(i\, \pi^{(\mu)}.x) \tag{4.12}$$

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*) In fact, one can multiply Γ_μ with any phase from the centre group $Z(N/L_\mu)$.

where $\pi^{(\mu)}$ is a four-vector with

$$\pi^{(\mu)}{}_{\rho} = \pi \ , \rho < \mu \ ; \quad \pi^{(\mu)}{}_{\rho} = 0 \quad \rho \geq \mu \tag{4.13}$$

Thus, the bilinear term becomes, after Fourier transforming:

$$\frac{1}{2i} \sum_{x,\mu} \bar{\chi}(x)\, \alpha_\mu(x) \left(\chi(x+\mu) - \chi(x-\mu) \right) =$$

$$= \sum_{p_c, p_s, \mu} \bar{\chi}(p_c, p_s)\, \delta_{p+\pi^{(\mu)}, p'} \sin p'_\mu(p_c, p_s)\, \chi(p_c, p_s) \tag{4.14}$$

The propagator is only seemingly not diagonal in p_c and p_c'; in fact:

$$\frac{L_s L_c}{2\pi}\left(p + \pi^{(\mu)}\right) = p_c + p_s L_c + \frac{1}{2} L_s L_c \tag{4.15}$$

Since L_s is even, we see that only p_s is affected by adding $\pi^{(\mu)}$, which means only the space-time momentum is non-diagonal. The same holds for any vertex in the twisted model. In the untwisted, periodic model of size $(L_c L_s)^4$ we have exactly the same structure, provided we take antiperiodic boundary conditions in the time direction. The discussion in Section 2 is therefore valid for the fermionic vertices and propagators as well. In the continuum limit, we get in this way an $SU(N_c) \times SU(4N_c)$ theory.

c) Introducing partial colour reduction

In the previous subsection, the ratio of colour over flavour degrees of freedom was 1/4. For reasons of phenomenology[12)], one might like a variable ratio. This can be obtained by only partially reducing the colour degrees of freedom[13),*)]. This will be briefly described now.

The basic observation is that the flavour index on the $\chi(x)$ variables are varying in an SU(L) group, whereas the colour variables are varying in the $SU(N_c = ML)$ group where M is some integer that we choose at our convenience. Now $N_f = 4L$ and $N_c = ML$ and the ratio is variable. Therefore χ will be rectangular

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*) Of course, we can use the SU(2) version of the staggered fermions[11)].

and in index notation:

$$\chi_{b_1+La_1,\,b_2} = \sum_{p_c} \chi_{a_1}(p_c)\,\Lambda_{b_1 b_2}(p_c)$$

The L×L matrices $\Lambda(q)$ are twist matrices in SU(L), with

$$\Lambda_\mu \Lambda_\nu = \exp\left(i\frac{2\pi}{L} n_{\mu\nu}\right)\Lambda_\mu \Lambda_\nu$$

The N×N twist matrices Γ_μ are defined as

$$\Gamma_\mu = \mathbb{1} \otimes \Lambda_\mu$$

where $\mathbb{1}$ is the M×M unit matrix. The boundary conditions on the quark fields are:

$$\chi(x + L_{s\rho}\vec{e}_\rho) = \Gamma_\rho\,\chi(x)\,\Lambda_\rho^{+}$$

and for the gluon fields Q:

$$Q_{b_1+La_1,\,b_2+La_2} = \sum_{p_c} Q_{a_1 a_2}(q)\;\Lambda_{b_1 b_2}(q)$$

In the limit that $L \to \infty$, this theory is equivalent, what regards the planar graphs, to an $SU_c(ML) \times SU_f(4L)$ theory, as can be shown by the same arguments as for the case $M = 1$.

5. - CONCLUSIONS

In this paper we have shown how to do large N simulations of glueball correlations in a box of small spatial size, but where the finite size effects are suppressed by introducing a suitable twisted boundary condition. We did the same for systems containing in addition fermions with a large number of flavours and showed how, by partial reduction, one can achieve a variable ratio of colour to flavour. Our methods can also be used for estimating the reduction of finite size effects[14)] by replacing the now-popular periodic boundary conditions by twisted boundary conditions[15)].

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Table 1

	twisted model	periodic model
Propagator	$\langle\hat{Q}^*_\mu(q',\ell')\hat{Q}_\nu(q,\ell)\rangle = \frac{N^3}{\Pi L_{s\mu}} \frac{\delta_{qq'}\delta_{\ell\ell'}\delta_{\mu\nu}}{\Sigma_\lambda \sin^2(\frac{P_\lambda(q,\ell)}{2})}$	$\langle\hat{Q}^*_\mu(k';q',\ell')\hat{Q}_\nu(k;q,\ell)\rangle = \frac{N}{\Pi L_{s\mu}} \frac{\delta_{kk'}\delta_{qq'}\delta_{\ell\ell'}\delta_{\mu\nu}}{\Sigma_\lambda \sin^2(\frac{P_\lambda(q,\ell)}{2})}$
Internal vertex	$\Sigma_x \mathrm{Tr}(\hat{Q}_{\mu_1}(x_1+\nu_1),\ldots,\hat{Q}_{\mu_n}(x_n+\nu_n) =$	
	$\frac{\Pi L_{s\rho}}{N^{2n-1}} \Sigma_{q,\ell} \exp i\alpha(q_1,\ldots,q_n)\delta(\Sigma q)\delta(\Sigma \ell)$	$\frac{\Pi L_{s\rho}}{N^{2n-3}} \Sigma_{k,q,\ell} \exp i\alpha(k_1,\ldots,k_n)\delta(\Sigma k)\delta(\Sigma q)\delta(\Sigma \ell)$
	$\times \Pi^n_{j=1} \exp iP_{\nu_j} \Pi^n_{j=1}\hat{Q}_\mu(q_j,\ell_j)$	$\times \Pi^n_{j=1} \exp iP_{\nu_j}(q_j,\ell_j) \Pi^n_{i=1} \hat{Q}_{\mu_j}(k_j;P(q_j,\ell_j))$
External vertex	$\mathrm{Tr}(\hat{Q}_{\mu_1}(x_1),\ldots,\hat{Q}_{\mu_n}(x_n)) =$	
	$\frac{1}{N^{2n-1}} \Sigma_{q,\ell} \exp i\alpha(q_1,\ldots,q_n)\delta(\Sigma q)$	$\frac{1}{N^{2n-1}} \Sigma_{k,q,\ell} \exp i\alpha(k_1,\ldots,k_n)\delta(\Sigma k)$
	$\times \Pi^n_{j=1}\hat{Q}_{\mu_j}(q_j,\ell_j) \Pi^n_{j=1}\exp i\, x_j P(q_j,\ell_j)$	$\times \Pi^n_{j=1} \hat{Q}_{\mu_j}(k_j;q_j,\ell_j) \Pi^n_{j=1} \exp ix_j\, P(q_j,\ell_j)$
Notation	q = colour momentum, ℓ = space-time momentum q,ℓ combine to $P(q,\ell)$, see Eq. (2.13)	k = colour momentum; $P(q,\ell)$ = space-time momentum
	$\alpha(q_1,\ldots,q_n) = \Sigma^{n-2}_{\ell=0} \frac{1}{2} \frac{2\pi}{N} (q_1 + \ldots + q_{\ell+1}) n q_{\ell+2}$	

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FIGURE CAPTIONS

Fig. 1: a) $U_\mu \to U_\mu$ on all links of type —— ;
$U_\mu \to U_\mu \Gamma_\mu$ on all links of type $\sim\!\sim\!\sim$.

b) In every plane (μ,ν) the twist is $t_{\mu\nu} = z_{\mu\nu}$ for the upper right plaquette and $t_{\mu\nu} = 1$ for all other plaquettes.

c) ground state: $U_\mu = 1$ on all links of type —— ;
$U_\mu = \Gamma_\mu$ on all links of type $\sim\!\sim\!\sim$.

Fig. 2: Momentum space of a) the twisted and b) the periodic model. o marks an excluded point.

Fig. 3: a) Lattice with twisted boundary conditions and Wilson loop L_0.

b) Lattice of the related periodic model. The Wilson loop $W(L_0)$ of Fig. 3a is equivalent to $(1/N^2)\Sigma W(L_i)$.

Fig. 4: a) Lattice with twisted boundary conditions and no twist in time direction.

b) Equivalent lattice with periodic boundary conditions. The lattices have equal extensions in time direction, but $V_b = N^2 V_a$. Correlation functions $\langle Q(t_1)Q(t_2)\rangle_a^{\text{conn.}}$ and $\langle Q(t_1)Q(t_2)\rangle_b^{\text{conn.}}$ of glueballs $Q(t_i)$ summed over space at $t = t_i$ have equal planar parts.

Fig. 5: a) Lattice with twisted boundary conditions.
b) Lattice with periodic boundary conditions. Then $\langle Q_1(x)Q_2(y)\rangle^{\text{twist.}} \sim \Sigma_{i,j} \langle Q_1(x_i)Q_2(y_i)\rangle^{\text{per.}}$.

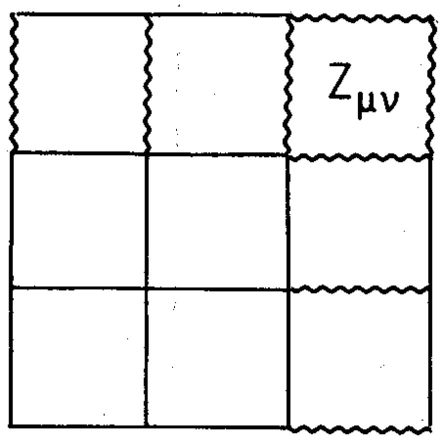

Fig. 1

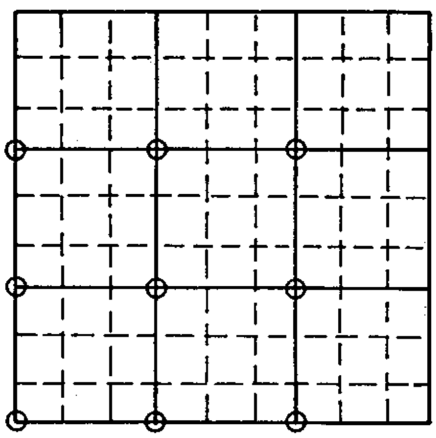

Fig. 2a

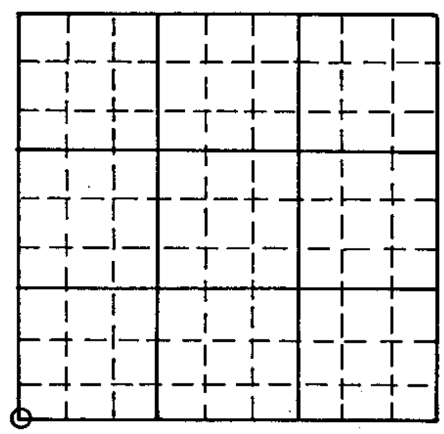

Fig. 2b

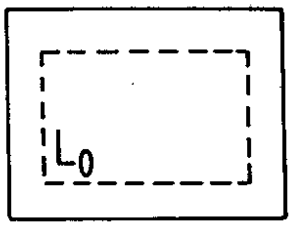

Fig. 3a

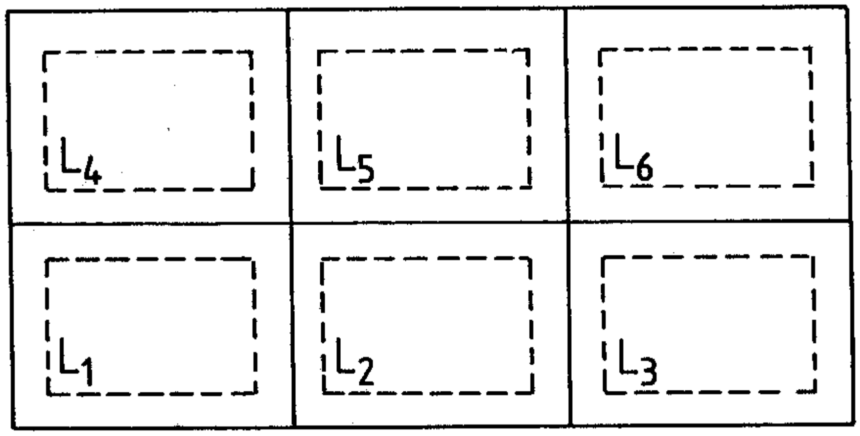

Fig. 3b

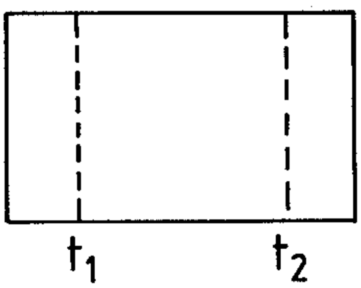

Fig. 4a

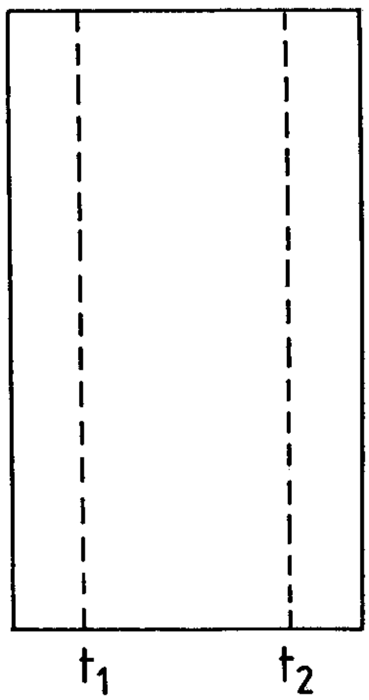

Fig. 4b

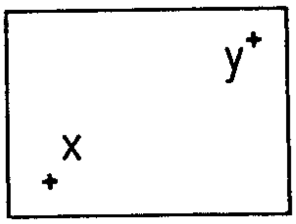

Fig. 5a

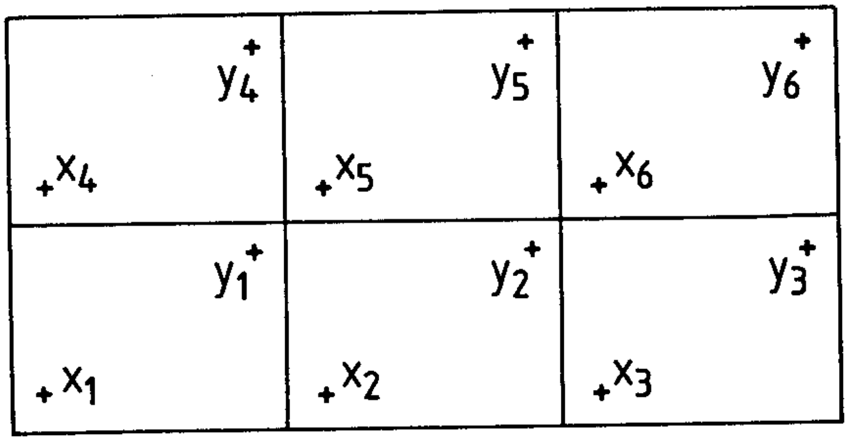

Fig. 5b