

MONOPOLES AND DIRAC SHEETS IN COMPACT U(1) LATTICE GAUGE THEORY^{*)}

V. Grösch⁺⁾, K. Jansen⁺⁾, J. Jersák⁺⁾
C.B. Lang^{#)}, T. Neuhaus⁺⁾ and C. Rebbi^{#)}

CERN - Geneva

ABSTRACT

We point out that in compact U(1) lattice gauge theory on a finite volume, some topological excitations (monopole loops and Dirac sheets winding through the lattice) effectively divide the configuration space in separate sectors. Local Monte Carlo algorithms have problems in moving from one such sector to another and may thus produce misleading results. We have observed this phenomenon in two independent programs, one for the Z(128) subgroup and Villain's action near the phase transition, the other for Z(32) and Wilson's action in the cold phase.

*) Supported by Fonds zur Förderung d. Wissenschaftl. Forschung in Österreich, project P5125; by Deutsches Bundesministerium für Wissenschaft und Technologie, Studienstiftung des Deutschen Volkes, and by Deutsche Forschungsgemeinschaft.

Permanent addresses:

- +) Inst. f. Theor. Physik E, Technische Hochschule Aachen, D-5100 Aachen, Fed. Rep. Germany.
- #) Inst. f. Theor. Physik, Universität Graz, A-8010 Graz, Austria.
- ‡) Brookhaven National Laboratory, Upton, New York, USA.

We observed very long-living metastable configurations in MC-simulations of d=4 U(1) gauge theory. As reason for this behaviour we identified objects specific for lattices with finite size topology. These are on one hand monopole current loops (world lines) winding through the lattice and closed only due to the periodic boundary conditions. On the other hand we found Dirac sheets, the world sheets of Dirac strings in three dimensions, without monopole loops as boundary and closed due to periodic b.c.

Both objects are responsible for long living action gaps: the monopole loops at the PT, the Dirac sheets throughout the cold phase. They may obscure the nature of the PT and other properties of the system. This motivates the presentation of our results.

The importance of monopoles for the dynamics of the PT has been discussed in the literature [1,2,3,4,5]. They have been observed in MC simulations of U(1) lattice gauge theory and found to correlate strongly with the free energy at the PT[4]. In d=3 monopoles may be regarded as endpoints of Dirac strings of magnetic flux. They are identified, using Gauss' law, by measuring the total flux leaving a closed surface like e.g. the boundary of a cube. In d=4 one proceeds in an analogous way and we follow Ref.2 for the definition. We introduce plaquette flux variables $\theta_{\rho\sigma}(x)$ as the sum over the link angles $\theta_{x,\mu} \in [-\Pi, \Pi]$

$$\theta_{\rho\sigma}(x) = \theta_{x,\rho} + \theta_{x+\rho,\sigma} - \theta_{x+\sigma,\rho} - \theta_{x,\sigma} \in [-4\Pi, 4\Pi] \quad (1)$$

and the physical flux

$$\bar{\theta}_{\rho\sigma}(x) = \theta_{\rho\sigma}(x) + 2\pi n_p \in [-\pi, \pi]. \quad (2)$$

Here n_p may assume values $-2, -1, 0, 1, 2$; if $n_p \neq 0$ a Dirac string passes through the plaquette. In $d=4$ the plaquette $\theta_{\rho\sigma}(x)$ on the lattice Λ corresponds to a plaquette $\theta_{\mu\nu}(x) = \varepsilon_{\mu\nu\rho\sigma} \theta_{\rho\sigma}(x)$ on the dual lattice Λ^* and the world sheet of the Dirac string is actually a surface of dual plaquettes (which we will henceforth call a Dirac sheet). Its boundary is a closed loop of links on the dual lattice. On Λ these links correspond to monopole 3-cubes and we may construct a monopole current

$$2\pi M_\mu(x) = \varepsilon_{\mu\nu\rho\sigma} \nabla_\nu \bar{\theta}_{\rho\sigma}(x). \quad (3)$$

The operator ∇_ν denotes the lattice derivative. The magnitude of $M_\mu(x)$ is the total net flux out of the cube c at site x .

$$\sum_{p \in \partial c} \bar{\theta}_p = \sum_{p \in \partial c} (\theta_p + 2\pi n_p) = 2\pi \sum_{p \in \partial c} n_p. \quad (4)$$

The loop swept by the monopole on the dual lattice has to be closed because of current conservation

$$\nabla_\mu M_\mu(x) = 0, \quad (5)$$

and is a gauge invariant object, whereas the Dirac sheet itself, except for its boundary, may be deformed by gauge transformations.

In the course of separate investigations we attempted to localize the precise value of β_{crit} for lattice size 16^4 and Villain's action [6]

$$e^{-S_V(\theta_{\mu\nu})} = \sum_{n=-\infty}^{\infty} \exp\left(-\frac{\beta}{2} (\theta_{\mu\nu} - 2n\pi)^2\right). \quad (6)$$

We found two very stable states at $\beta=0.643$, closely above the presumed β_{crit} :

- (a) one state stable at $E \equiv \langle S_V \rangle = 0.50$ for at least 10000 MC-iterations; this after starting from a configuration at $\beta=0.6425$ (hot phase) equilibrated in 25000 MC-iterations;
- (b) one state stable at $E=0.48$ after 19000 MC-iterations from cold start.

For the Wilson action certain signals for first order behaviour have been observed [7] but one would not expect such a behaviour for the Villain action which receives a negative contribution from the charge-2 representation.

A search for monopole loops in the last of the configurations of series (a) and (b) clarified the discrepancy. In both configurations we find many monopole loops. However, we find only in (a) loops wrapping around the lattice, i.e. with a non-vanishing net number of boundary crossings in one or more directions. Such loops belong to the longest loops observed. Their total lengths ℓ vary between 200 and 1000. This may be understood from the relation between the mean value of the squared distance D from start to endpoint of a random walk of length ℓ :

$$\langle D^2 \rangle \sim \ell. \quad (7)$$

In order to wrap once around the lattice one needs $D \geq 16$, resulting in the large values of ℓ observed.

Fig. 1 shows the accumulated total length L of monopole loops

$$L(x) = \sum_{\ell > x} n_{\ell} \ell, \quad (8)$$

where n_{ℓ} is the number of occurrences of loops with length ℓ . We also indicated the steps due to the contributions from the loops, that close because of periodicity only. Subtracting such contributions from the curve for configuration (a) one obtains a curve very similar to that of configuration (b) (cf. Fig. 1b).

Due to our periodic and untwisted b.c. there can be no single periodically closed monopole loop, it has to have at least one partner in the opposite direction. The total number of boundary crossings, taking all periodically closed monopole loops together, is therefore zero.

Although the fluctuations at the PT are of long range the loops winding around the lattice have difficulties to decay. Their only chance is to cooperate with another such loop in order to undo the surplus boundary crossings. For local MC-algorithms this is very unprobable and thus it may take a very long time - hence the apparent stability.

In order to investigate the decay properties the last configuration (a) was subjected to further MC-iterations at $\beta=1$. Within the first 50 iterations the total length $L(0)$ decreases from 14880 down to 2850. This is mainly due to the vanishing of ordinary closed monopole loops. For 900 subsequent iterations one observes essentially just one pair of (periodically closed)

monopole current loops, which do not succeed to annihilate. The last configuration considered had still total length $L(0)=456$ and only this monopole pair left. Had we cooled the system "adiabatically" the monopoles might have had a better chance to drop out.

What happens to the Dirac sheet when a monopole loop closes and disappears? On an infinite lattice without the periodically closed loops the sheet must disappear as well. On a finite lattice, however, there are two possibilities. In Fig.2 we illustrate the situation on a 2-dimensional plane of the dual lattice Λ^* (for linear size 8). Situation (b) is what happens also on an infinite lattice. In case (a) one is left over with a trapped magnetic flux, a Dirac sheet covering a whole surface closed due to the periodic b.c. Details of the shape of the sheet are not gauge invariant but the fact of its existence is.

In independent MC-simulations for $Z(32)$ on lattices of various size and for Wilson's action,

$$e^{-S_W(\theta_{\mu\nu})} = \exp(-\beta(1-\cos\theta_{\mu\nu})), \quad (9)$$

we observed metastable states of enormous lifetime. They occurred frequently at relatively large β -values (in the cold phase), when the start configuration was disordered (hot). The smallest observed action gap varied with the lattice size and was e.g. 0.0127(4) for 4^4 , 0.0026(2) for 6^4 , and 0.0007(1) for 8^4 lattices for $\beta=3.5$. The β -dependence is weak as may be seen in Tab.1. For that gap no decay from the excited state to the ground

state was observed in over 12000 iterations for $\beta > 1.1$.

The gap was noticed originally in thermal cycles of a gauge-Higgs system [8] for fixed β and varied gauge-Higgs coupling κ . The plaquette remained excited in the upward branch (increasing κ) of the hysteresis until far above the Higgs transition. On the downward branch the plaquette variable usually selected the ground state and remained there in further cycles. As this phenomenon occurred in the gauge variable it was not related to a similar effect observed in Ref. [9].

In order to identify the reason for the metastability we performed an iterative gauge transformation procedure for an excited configuration as well as for a ground state configuration. At each site a gauge transformation

$$\theta_{x, \pm\mu} \rightarrow \theta'_{x, \pm\mu} = \theta_{x, \pm\mu} \pm \chi_x \pmod{2\pi} \in [-\pi, \pi] \quad (10)$$

is chosen such as to minimize

$$\sum_{\pm\mu} |\theta'_{x, \mu}| \quad (11)$$

In ambiguous situations the value of χ_x is chosen randomly. Notice that a gauge transformation even with small χ_x can change the value of n_p of a plaquette by flipping some link angle e.g. from a value close to $-\pi$ to a value close to π . Such a gauge transformation is applied to all sites of the lattice which defines one "gauge iteration". The procedure

tends to decrease the number of plaquettes with $n_p \neq 0$ (plaquettes with negative n_p may cancel others with positive n_p). After sufficiently many gauge iterations a minimal number of such plaquettes is obtained. Fig.3 shows the yz -plaquettes with $n_p \neq 0$ (i.e. possibly contributing to a Dirac sheet) as found for an excited configuration (a) and a ground state configuration (b) after zero, one and six gauge iterations. Whereas the distribution of plaquettes with $n_p \neq 0$ before the gauge iterations is quite similar for the excited and the ground state configuration, one finds a remarkable difference after several gauge iterations.

Note that the total number of plaquettes with $n_p \neq 0$ for each xt -plane is always odd for the excited state but even for the ground state. The excitation is identified with a Dirac sheet covering a whole torus on the dual lattice.

Let us estimate the resulting action gap for the case of Wilson's action in the weak coupling situation ($\beta \rightarrow \infty$). For a Dirac sheet in the xt -plane of the dual lattice the minimal surplus flux through every zy -plane of the original lattice is 2π due to one plaquette with $n_p \neq 0$. Eventually it will be distributed over all N^2 plaquettes of each zy -plane (for lattice size N^4). Each of these N^2 plaquettes contributes in average

$$1 - \cos(2\pi/N^2) \approx 2\pi^2/N^4 \quad (12)$$

leading to a minimal gap of $\pi^2/3N^4$ in the plaquette energy averaged

over the lattice. This is in agreement with the numbers observed (cf. the results in Tab.1).

We also reproduced the excited state by starting a MC-run with a configuration completely ordered up to those links that were chosen to build up a Dirac sheet. The same metastability as discussed above was observed with identical action gap. The contributions from the Dirac sheets vanish like the inverse lattice volume and thus seem to be less important for larger lattices. Still one has to keep in mind that one samples in a wrong sector in configuration space, which might cause severe problems.

The decay mechanism for the Dirac sheets is obvious from Fig.2a, read backwards. However, for $\beta \gg \beta_{\text{crit}}$ the spontaneous production of monopole loops is suppressed and the inflation of such loops to the necessary size scarcely possible. In fact we had to decrease β down to 1.04 (closely above β_{crit} for S_W) before the decay into a pair of contracting monopole loops took place.

It is evident that the observed phenomena may severely influence the outcome of a MC-investigation. The discussed objects have to be kept under control to prevent misinterpretation of the results. We find it noteworthy that the discussed effects are of relevance not only at the phase transition but throughout the cold phase.

Acknowledgement: The calculations have been performed at the CYBER 875 at CERN and at the computers centers of Universität Bochum (CYBER 205) and Technische Hochschule Aachen (CYBER 175).

Table 1: The observed minimal action gap for various values of β and lattice size 4^4 and 6^4 in the cold phase (for Wilson's action). The typical errors are 4 and 2 in the last digits for 4^4 and 6^4 results respectively.

β	4^4	6^4
1.3	0.0152	0.0026
1.4	0.0150	0.0023
2.	0.0130	0.0028
2.5	0.0130	0.0024
3.	0.0129	0.0023
3.5	0.0127	0.0026
4.	0.0126	0.0026
5.	0.0126	0.0025

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FIGURE CAPTIONS

Fig.1 (a) The integrated total length of monopole loops $L(x)$ for a configuration (denoted by a) with higher and a configuration (denoted by b) with lower average plaquette energy E at $\beta=0.643$; the arrows indicate the steps due to contributions from periodically closed monopole current loops.

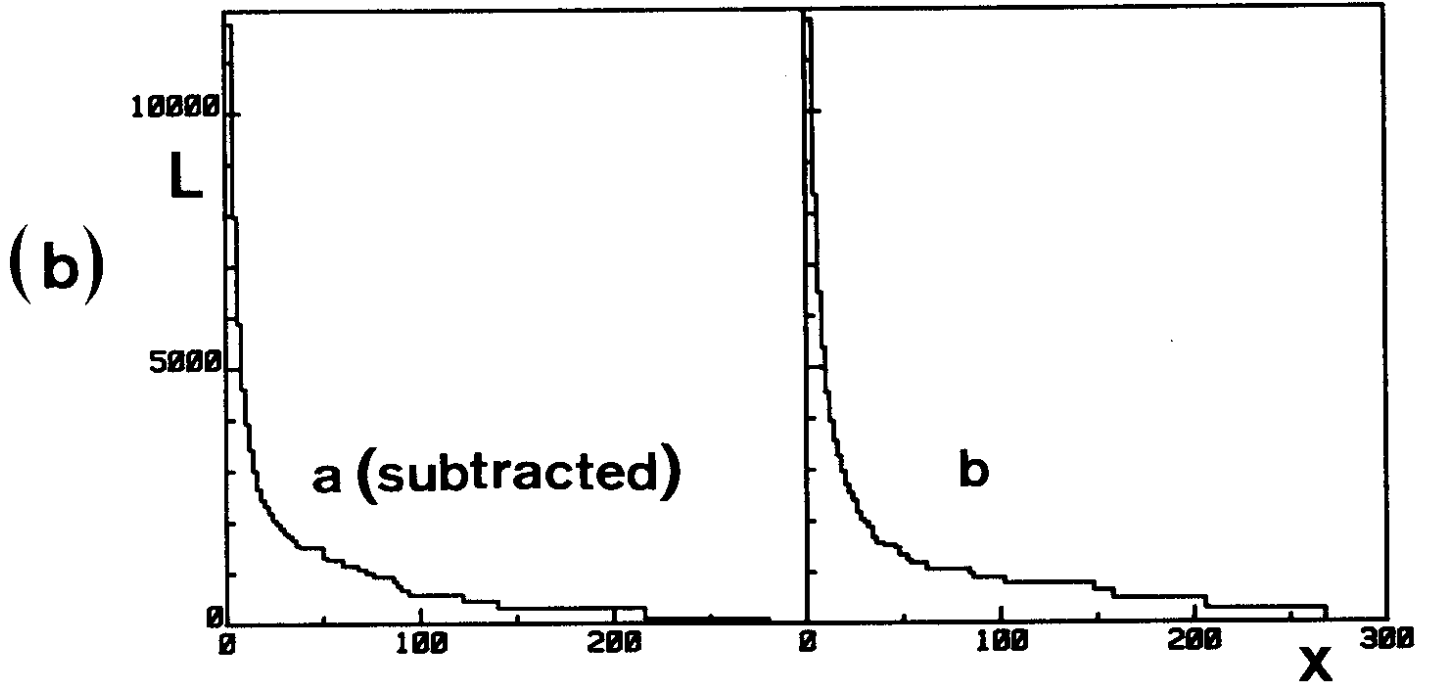
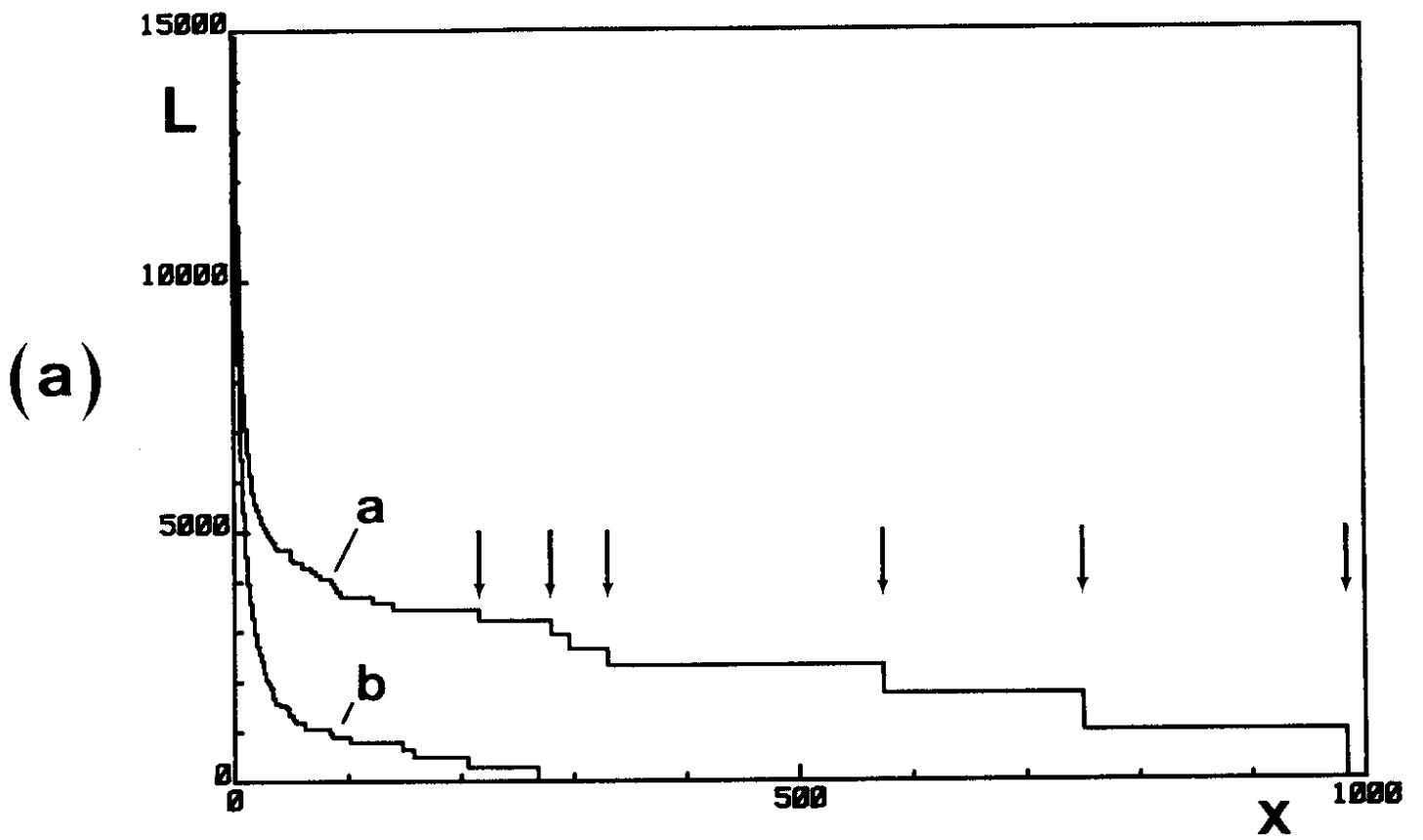
(b) Here we plot again the same curves as in (a) on a different scale after subtracting off the contributions from periodically closed loops from curve a.

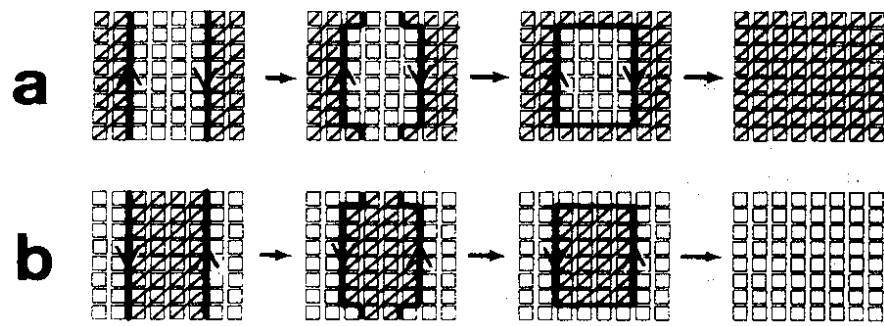
Fig.2 We illustrate the possible history of a pair of periodically closed monopole loops depicted on the dual lattice.

(a) Here a Dirac sheet (shaded area) is left over after the disappearance of the loops, and

(b) no Dirac sheet is left.

Fig.3 We compare an excited (a) and a ground state (b) configuration (obtained at $\beta=3.5$ from hot and cold starts after several hundred MC-iterations); the x , y , z , and t coordinates may be identified as indicated, the squares denote zy -plaquettes without (open square) and with (full square) 2π -flux running through. The first column gives the configurations before gauge iterations, the second and third column after one and six gauge iterations, as discussed in the text.





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