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A NEW EMPIRICAL REGULARITY FOR MULTIPLICITY DISTRIBUTIONS IN PLACE OF KNO SCALING

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ABSTRACT

Charged particle multiplicity distributions for the inelastic, non single-diffractive component of $\bar{p}p$ reactions at 540 GeV c.m.s. energy and of pp reactions above about 10 GeV are shown to be remarkably well described by a negative binomial distribution at each energy. The two parameters of the distribution depend on energy in a regular manner, which affords the possibility of predicting multiplicity distributions at high energies not yet experimentally available. In this description there is a complete absence of any tendency to approach asymptotic KNO scaling below 540 GeV. The approximate scaling in the c.m. energy range 10-62 GeV is shown, in this framework, to be accidental.

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INTRODUCTION

Asymptotic scaling of multiplicity distributions in hadron collisions was predicted in 1971 by Koba, Nielsen and Olesen [1] by assuming the validity of Feynman scaling [2]. If P_n is the probability for n charged particles in the final state, and $\langle n \rangle$ the mean multiplicity, KNO scaling states that $\langle n \rangle P_n = \psi(z)$ is a function of the scaled multiplicity $z = n/\langle n \rangle$ with no further energy dependence other than that implied from the relation between $\langle n \rangle$ and $\sqrt{s}$. Another derivation of scaling in multiplicities has been made, for which there is no need to assume Feynman scaling, being based instead on the principle of geometric scaling [3].

The moments in terms of the scaled variable $n/\langle n \rangle$ are energy independent constants when KNO scaling holds. Approximate scaling behaviour of the second moment was pointed out by A. Wroblewski [4], whose relation between the dispersion D and the average multiplicity $\langle n \rangle$ $D = \omega(\langle n \rangle - \alpha)$, where ω and α are empirical constants, suggests asymptotic scaling when $\langle n \rangle \gg \alpha$. Various forms of the scaling function $\psi(z)$ for hadron-hadron inelastic collisions have been published, either empirical fits or based on theoretical arguments [5].

When reliable data for inelastic collisions with single diffraction excluded became available over a wide range of energies, including the ISR energy range, the constancy of the measured moments in terms of the scaled multiplicity variable improved [6], suggesting that KNO scaling is valid in the energy range above $\sqrt{s} \simeq 10$ GeV for this component in pp collisions. This may seem surprising, since it is now known that both Feynman scaling and geometric scaling are violated [7]. However, a third derivation of KNO scaling, using only general statistical arguments within the framework of a branching mechanism for particle production, has recently been claimed [8].

The large step in energy taken by the CERN antiproton-proton collider ($\sqrt{s}$ = 540 GeV) has opened up new possibilities for testing multiplicity scaling. The UA5 Collaboration with its streamer chamber detector, covering 95% of the full solid angle, measured in detail the charged particle multiplicity distribution. Clear violation of scaling was observed [9], manifested most clearly by an increase in the proportion of high multiplicity events. Recently, the UA5 Collaboration has completed a detailed study of multiplicity distributions at $\sqrt{s}$ = 540 GeV in a set of limited regions of phase space, defined by cuts in pseudorapidity [10]. In this context it was found that a negative binomial distribution with its two free parameters adjusted to fit the data points, described each measured distribution with its many degrees of freedom remarkably well, including the high multiplicity tail of the distribution. The two parameters $\{\bar{n},k\}$ of the negative binomial distribution, given by

$$P_n = \binom{n+k-1}{n} \left(\frac{\bar{n}/k}{1+\bar{n}/k}\right)^n \frac{1}{(1+\bar{n}/k)^k}, \tag{1}$$

were found to vary smoothly with the size of the pseudorapidity range. We denote by $\bar{n}$ the average of the negative binomial distribution and by $\langle n \rangle$ the average of the measured sample.

The purpose of this paper is to report on results obtained in fitting negative binomial distributions to full phase space data in pp and $\bar{p}p$ collisions in the c.m. energy range 11–540 GeV. We have used only inelastic, non single-diffractive events, since it is this component which shows good scaling behaviour in the first part of the selected energy region and which then violates scaling at 540 GeV. We have used fixed target pp data at p_{lab} = 69 [11], 102 [12], 205 [13], 303 [14,16], and 405 GeV/c [15,16], ISR pp data at $\sqrt{s}$ = 30.5, 44.5, 52.6, and 62.2 GeV [6] and the UA5 $\bar{p}p$ sample from the 1982 run at 540 GeV, now about 15% larger than in Ref. [9].

METHOD

A fitting procedure was used which involved varying the two parameters $\{\bar{n},k\}$ of the negative binomial distribution until a best fit was achieved, using the minimum χ^2 method. The goodness of the fit was assessed in two ways. Firstly, by inspecting the contribution to χ^2_{min} from each bin in multiplicity. Secondly, the difference was computed between $\sqrt{2\chi^2_{min}}$ and its expected value $\sqrt{2DF-1}$, where DF is the number of degrees of freedom. For the numbers of degrees of freedom in the present study, the difference $y = \sqrt{2\chi^2} - \sqrt{2DF-1}$ should be approximately normally distributed with zero mean and standard deviation unity. Since large values of χ^2_{min} indicate unsatisfactory fits, cases for which $y < 2$ may be taken as representing very good fits, and $y < 3$ as acceptable fits. However, systematic errors in the bin populations may cause problems. At the lowest multiplicities corrections for inefficiencies are generally more uncertain than at higher multiplicities as are corrections for contaminations from diffractive and elastic events. Only at two of the energies was the contribution to χ^2_{min} from the first bin (n=2) excessively large, still leaving the overall fit, however, very good. New fits at all energies while leaving out the first bin, gave similar or sometimes, of course, improved fits. The resulting changes in the values of k indicate that systematic errors are generally small but may be as large as the statistical ones.

When dealing with data from the whole of phase space one has only even multiplicities due to charge conservation, whereas the negative binomial distribution is defined for all integer values of n. In the fitting we have taken only even integers (n ≥ 2) from the negative binomial distribution with parameters $\{\bar{n},k\}$, and renormalized.

The usual equations for the moments as functions of the parameters can be written in the following way:

$$C_2-1 \equiv \gamma_2 \equiv \left(\frac{D}{\langle n\rangle}\right)^2 = \frac{1}{\bar{n}} + \frac{1}{k}\ ; \tag{2}$$

the two next higher C_q moments, defined by $C_q = \langle n^q\rangle/\langle n\rangle^q$ are

$$C_3 = 1+3\left(\frac{1}{\bar{n}}+\frac{1}{k}\right) + \left(\frac{1}{\bar{n}}+\frac{1}{k}\right)^2 + \frac{1}{k}\left(\frac{1}{\bar{n}}+\frac{1}{k}\right) \tag{3}$$

$$C_4 = 1+6\left(\frac{1}{\bar{n}}+\frac{1}{k}\right) + 7\left(\frac{1}{\bar{n}}+\frac{1}{k}\right)^2 + \left(\frac{1}{\bar{n}}+\frac{1}{k}\right)^3 + \tag{4}$$

$$+\frac{4}{k}\left(\frac{1}{\bar{n}}+\frac{1}{k}\right) + \frac{4}{k}\left(\frac{1}{\bar{n}}+\frac{1}{k}\right)^2 + \frac{1}{k^2}\left(\frac{1}{\bar{n}}+\frac{1}{k}\right) .$$

When only even integers are taken there will be small corrections to these formulae. For the sets of parameters which fit the data, these corrections are found to be generally smaller than the experimental errors.

RESULTS

The fit parameters $\{\bar{n},k\}$ are collected in Table 1 together with the goodness of fit variable $y = \sqrt{2\chi^2_{min}}-\sqrt{2DF-1}$. All fits are seen to be very good, since $y < 2$. Higher moments can be calculated from $\{\bar{n},k\}$ and are compared to those calculated directly from the data in Table 2. The agreement is generally excellent which reflects the finding that a negative binomial distribution describes the data well.

In Fig. 1 the resulting values of the inverse of k are plotted against energy and are seen to increase approximately linearly with lns over the entire range $10 < \sqrt{s} < 540$ GeV. A best fit to the data points of the form

$$k^{-1} = \alpha + \beta \ln s \tag{5}$$

gives $\alpha = -0.098 \pm 0.008$ and $\beta = 0.0282 \pm 0.0009$ with s in GeV^2.
The other parameter n agrees well with the average multiplicity $\langle n \rangle$ computed directly from the data, and of course increases with $\sqrt{s}$. The average multiplicity is usually parametrized as below in Eqs. (6a) or (6b)

$$\langle n \rangle = a + b \cdot \ln s + c \cdot \ln^2 s \tag{6a}$$

$$\langle n \rangle = a' + b' s^{c'} . \tag{6b}$$

Both relations give a good representation of the data points when $a = 1.97 \pm 0.96$, $b = 0.21 \pm 0.29$, $c = 0.148 \pm 0.022$, $a' = -7.4 \pm 2.2$, $b' = 7.6 \pm 1.7$ and $c' = 0.124 \pm 0.014$ with s in GeV^2.

CONCLUSIONS AND DISCUSSION

We conclude that a negative binomial distribution with adjusted parameters at each energy describes remarkably well the measured multiplicity distributions for the inelastic, non single-diffractive component of pp ($\bar{p}p$) reactions at high energies. Although this does not exclude the possibility that the agreement is only approximate, the negative binomial distribution provides a convenient framework for a discussion of the energy variation of the shape of the multiplicity distribution in terms of only two energy dependent parameters, in particular the parameter k.

Since the value $k^{-1} = 0$ corresponds to a Poisson distribution and the value $k = 1$ to a geometric (exponential) distribution one sees from Fig. 1 that actual multiplicity distributions for inelastic, non-single diffractive events are in between and that the deviation from a Poisson distribution becomes progressively more pronounced as the energy is increased. Even if the rate of increase of k^{-1} with energy diminishes at c.m. energies above 540 GeV, very broad multiplicity distributions may have to be considered at energies typical for cosmic ray air showers.

At very high energy, such that the average multiplicity is large ($\bar{n} >> k$), the negative binomial, Eq. (1), is well approximated in terms of the scaled quantities $z = n/\bar{n}$ and $\bar{n}P_n$, by

$$\bar{n}P_n = \frac{k^k}{\Gamma(k)} z^{k-1} e^{-kz} . \qquad (7)$$

The first four moments are $C_1 = 1$, $C_2 = 1 + 1/k$, $C_3 = 1 + 3/k + 2/k^2$ and $C_4 = 1 + 6/k + 11/k^2 + 6/k^3$, obtainable directly from Eq. (7) or from Eqs. (2) to (4). KNO scaling would obviously require a constant k^{-1}. Instead k^{-1} increases almost linearly with lns and the UA5 point at 540 GeV supports this behaviour. As seen from Fig. 1 there is no sign of an approach to KNO scaling at c.m. energies below 0.54 TeV. Since Feynman scaling and geometric scaling are both violated, KNO scaling is in any case not on a secure foundation. In view of the results presented here, the statistical derivation of KNO scaling is clearly undermined, since the tacit assumption made in [8] is equivalent to k = constant. Several detailed models of particle production have been published [17], which do not obey KNO scaling. We are, however, not aware of any model which leads to the negative binomial distribution with an energy dependence of its parameters as found here. It is known, however, that the negative binomial distribution can be generated at least by two mechanisms of particle production: i) by stimulated emission of bosons from k independent and identical chaotic sources [18], or ii) by cluster production and decay [19]. In either case the naïve expectation is that k increases with energy, which is in contradiction to our experimental finding.

The scaling observed in the energy region $10 < \sqrt{s} < 62$ GeV is reproduced in Fig. 2, where the two curves represent the energy dependence of the inverse parameters, $\bar{n}^{-1}$ and k^{-1}, respectively. The sum ($\bar{n}^{-1} + k^{-1}$) is equal to the second moment of the scaled variable as shown in Eq. (2). It has a broad minimum at about 20 GeV. At nearby energies the decrease of one of the terms in the sum $(\bar{n}^{-1} + k^{-1})$ is approximately compensated by the increase of the other, leading fortuitously to an almost constant second moment of the scaled variable and therefore to an almost perfect linear relation $D = 0.44 \langle n\rangle$. The higher moments C_q are also approximately constant at these energies, since they are dominated by terms containing the sum $(\bar{n}^{-1} + k^{-1})$ as seen in Eqs. (3) and (4). The energy variation of the term $k^{-1}(\bar{n}^{-1} + k^{-1})$ in C_3, for example, is not detectable within experimental errors. Viewed in the context of this paper early scaling is approximate and accidental.

A new basis for predicting multiplicity distributions at higher energies in place of KNO scaling is provided by the negative binomial distribution together with the empirical regularity represented by Fig. 1 and Eq. (5) for the parameter k and Eq. (6) for the average multiplicity $\langle n\rangle$. We present in Fig. 3 predicted multiplicity distributions, based on Eqs. (5) and (6a) for the non single-diffractive component in pp ($\bar{p}p$) collisions at c.m. energies of 0.9, 2, and 40 TeV, which correspond to the energies of the CERN SPS $\bar{p}p$ collider in pulsed mode, the Tevatron $\bar{p}p$ collider at the Fermi National Laboratory and the proposed future pp collider, SSC, respectively. Although the step to 40 TeV is large, this extrapolation seems to be the best available. Using Eq. (5) we find $k \simeq 2.0$, whereas $\langle n\rangle \simeq 70$ from (6a) and $\langle n\rangle \simeq 95$ from (6b) at 40 TeV. In either case the corresponding multiplicity distribution can be written $\langle n\rangle P_n = 4ze^{-2z}$, since the condition for Eq. (7) to be valid is fulfilled. The relative cross-section for high multiplicities is predicted to be quite large as shown in Fig. 4. The probability for events with $z > 3$ is about 1.8×10^{-2}, for $z > 6$ about 8×10^{-5}, which is many orders of magnitude larger than a prediction based on the continued validity of KNO scaling.

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Table 1

Parameters of negative binomial distributions fitted to published data and the resulting χ^2_{min} and $y = \sqrt{2\chi^2_{min}} - \sqrt{2DF-1}$, where DF is the number of degrees of freedom

Energy	Parameters of fit		Goodness of fit		Number of events (inelastic sample)
$\sqrt{s}$ (GeV)	$\bar{n}$	k	χ^2_{min}/DF	y	
11.5	6.23 ± 0.04	19 ± 2	9.8/6	1.1	29965
13.8	7.11 ± 0.06	21 ± 2	15.7/7	2.0	4345
19.7	8.48 ± 0.09	15 ± 1	8.6/9	0.0	8810
23.9	9.2 ± 0.1	12 ± 1	6.5/10	-0.8	10054
27.6	9.6 ± 0.1	8.5 ± 0.8	13.6/10	0.9	2594
30.4	10.7 ± 0.1	11.0 ± 0.7	23.9/13	1.9	37069
44.5	12.2 ± 0.1	9.4 ± 0.5	12.5/17	-0.7	61455
52.6	12.8 ± 0.1	7.9 ± 0.3	4.9/17	-2.6	26842
62.2	13.6 ± 0.1	8.2 ± 0.4	23.8/17	1.2	58196
540*)	28.3 ± 0.2	3.69 ± 0.09	67.6/66	0.2	7344

*) The fit to our own 540 GeV data was made with the method of Ref. [10].

Table 2

The average multiplicities, <n>, and the C_q moments, defined by $C_q = \langle n^q \rangle / \langle n \rangle^q$, calculated from the data samples. Within brackets the corresponding quantities computed from the two parameters of the fitted negative binomial distributions

$\sqrt{s}$ (GeV)	<n>	C2	C3	C4	C5
11.5	6.35±0.08 (6.23±0.04)	1.192±0.009 (1.21 ±0.01)	1.63±0.03 (1.70±0.02)	2.49±0.08 (2.68±0.06)	4.2±0.2 (4.6±0.2)
13.8	7.21±0.06 (7.11±0.06)	1.175±0.006 (1.19 ±0.01)	1.57±0.02 (1.61±0.02)	2.33±0.04 (2.42±0.06)	3.8±0.1 (4.0±0.1)
19.7	8.56±0.11 (8.48±0.09)	1.174±0.010 (1.19 ±0.01)	1.57±0.03 (1.61±0.03)	2.34±0.08 (2.43±0.07)	3.8±0.2 (4.0±0.2)
23.9	9.25±0.20 (9.2 ±0.1)	1.19±0.02 (1.19±0.01)	1.62±0.06 (1.64±0.03)	2.47±0.14 (2.52±0.08)	4.2±0.3 (4.3±0.2)
27.6	9.77±0.16 (9.6 ±0.1)	1.21±0.01 (1.22±0.01)	1.72±0.05 (1.74±0.04)	2.76±0.13 (2.82±0.12)	5.0±0.4 (5.1±0.3)
30.4	10.54±0.14 (10.7 ±0.1)	1.20±0.01 (1.18±0.01)	1.68±0.03 (1.60±0.02)	2.64±0.10 (2.43±0.06)	4.6±0.3 (4.1±0.2)
44.5	12.08±0.13 (12.2 ±0.1)	1.20±0.01 (1.19±0.01)	1.67±0.03 (1.62±0.02)	2.63±0.10 (2.49±0.06)	4.6±0.3 (4.2±0.2)
52.6	12.76±0.14 (12.8 ±0.1)	1.21±0.01 (1.21±0.01)	1.70±0.03 (1.68±0.02)	2.70±0.09 (2.66±0.05)	4.8±0.3 (4.7±0.1)
62.6	13.63±0.16 (13.6 ±0.1)	1.20±0.01 (1.20±0.01)	1.67±0.03 (1.65±0.02)	2.60±0.08 (2.56±0.06)	4.4±0.2 (4.4±0.2)
540	29.1±0.9 (28.3±0.2)	1.31±0.03 (1.31±0.01)	2.12±0.11 (2.10±0.03)	4.05±0.32 (3.98±0.09)	8.8±1.0 (8.7±0.3)

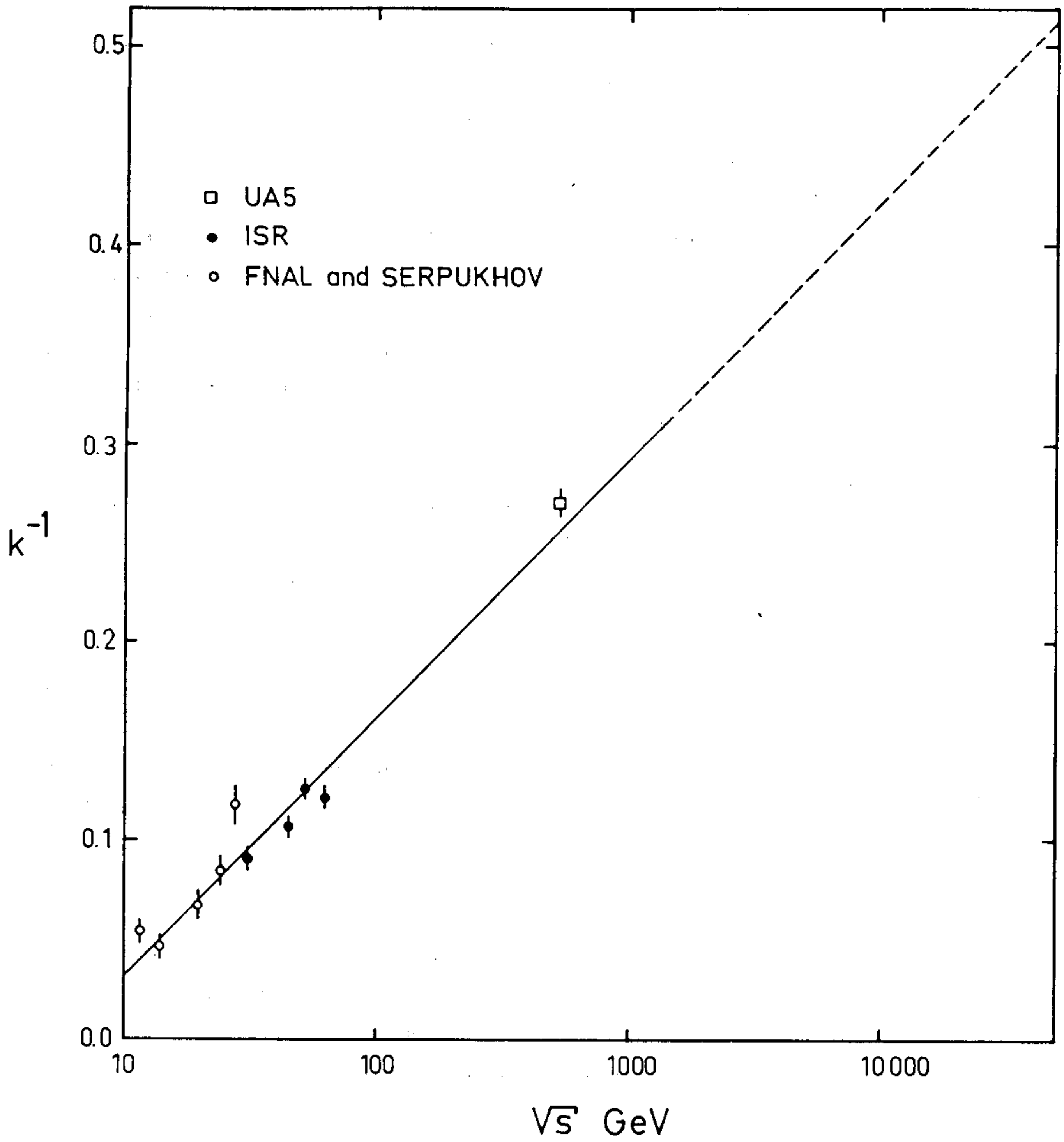
Figure captions

Fig. 1 : The parameter k of the negative binomial distributions fitted to data in references [6,9,11-16] for inelastic, non single-diffractive pp reactions (below 100 GeV) and for $\bar{p}p$ reactions (at 540 GeV). Only statistical errors are shown. The line is a linear fit to the data points. There is no indication of an approach to a constant value of k^{-1} at high energies as would be required if KNO scaling were to hold.

Fig. 2 : Approximate scaling in the energy range $10 \lesssim \sqrt{s} \lesssim 100$ GeV results, within the framework of the negative binomial description, from the addition of an increasing k^{-1} and the decreasing $\bar{n}^{-1}$ ($\bar{n}$ being the mean multiplicity). The straight line, labelled k^{-1}, is taken from Fig. 1, and the curve, labelled $\bar{n}^{-1}$, is obtained from Eq. (6a). Their sum with one standard deviation increase or decrease lies at the boundaries of the shaded area. The second moments, $(D/\bar{n})^2 = (\bar{n}^{-1}+k^{-1})$, are seen to be approximately constant at energies not too far from the minimum at 20 GeV. The points are measured values from Table 2 of $\langle n\rangle^{-1}$ and $(C_2 - 1)$.

Fig. 3 : Predicted multiplicity distributions for the inelastic, non single-diffractive component of pp (or $\bar{p}p$) reactions at c.m.s. energies 0.9, 2, and 40 TeV, based on extrapolations according to Eqs. (5) and (6a). Included is also a curve for 0.54 TeV. The dashed line shows the expected multiplicity distribution at 40 TeV based on KNO scaling.

Fig. 4 : Predicted probabilities for charged particle multiplicities $n \geq z\langle n\rangle$ in inelastic, non single-diffractive pp ($\bar{p}p$) reactions at 40 TeV based on the new regularity (see text) in comparison with a prediction based on KNO scaling.



0.5
0.4
0.3
0.2
0.1
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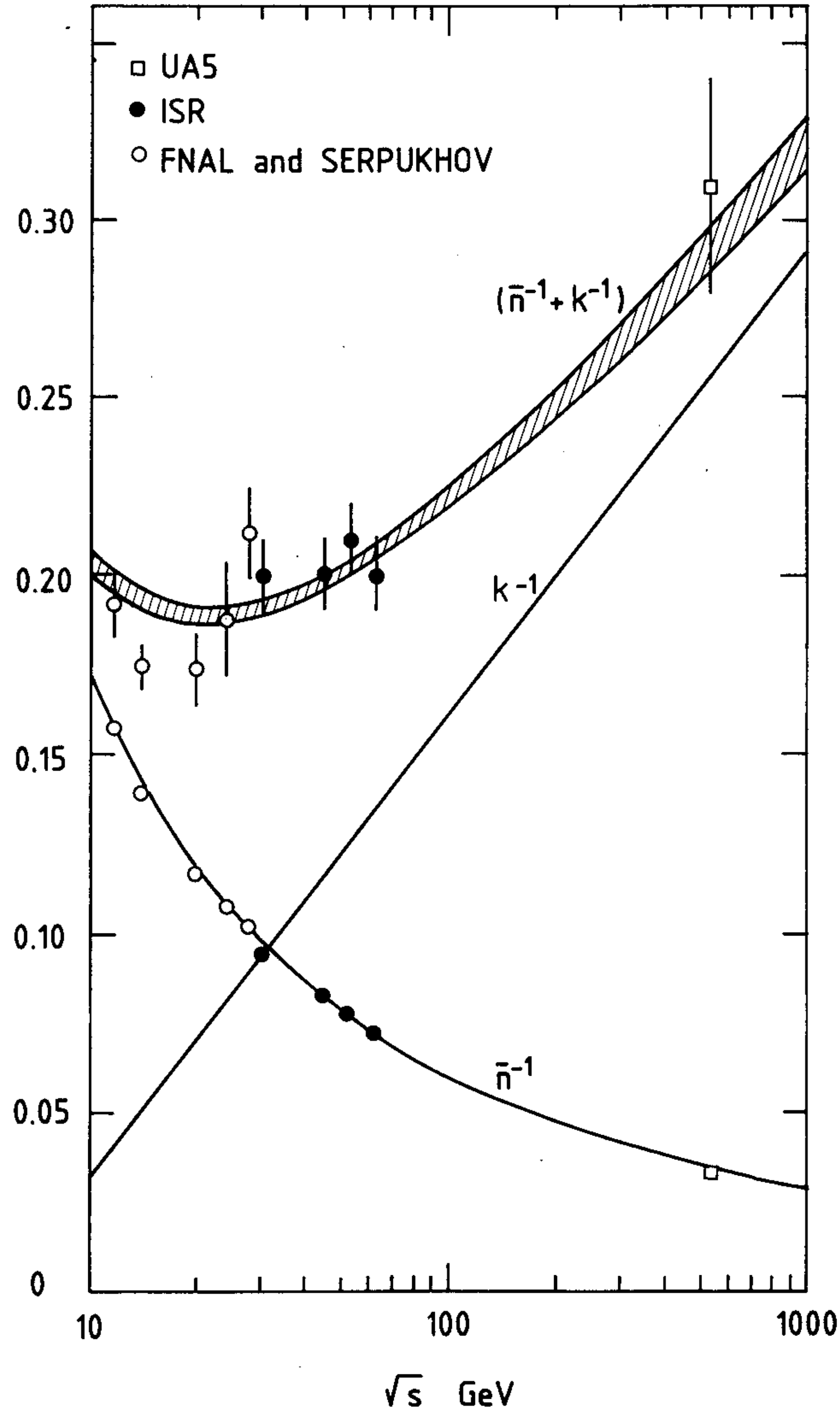
UA5
ISR
FNAL and SERPUKHOV
k^{-1}
10
100
1000
10 000
$\sqrt{s}$ GeV

Fig. 1



UA5
ISR
FNAL and SERPUKHOV
$(\bar{n}^{-1} + k^{-1})$
k^{-1}
$\bar{n}^{-1}$
0.30
0.25
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100
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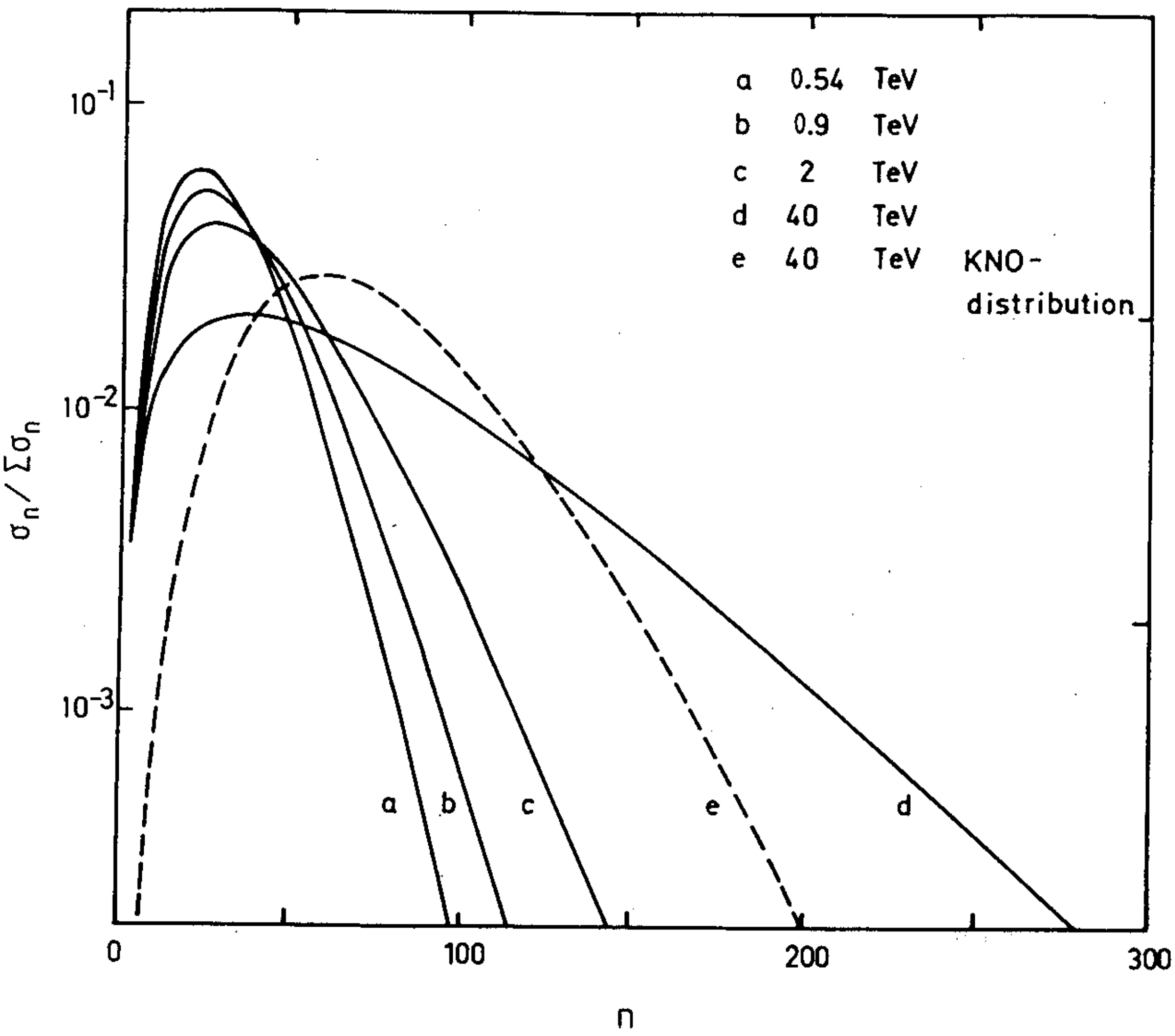
$\sqrt{s}$ GeV

Fig. 2


a 0.54 TeV
b 0.9 TeV
c 2 TeV
d 40 TeV
e 40 TeV KNO-distribution
10^{-1}
10^{-2}
10^{-3}
$\sigma_n / \Sigma\sigma_n$
a
b
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0
100
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Fig. 3

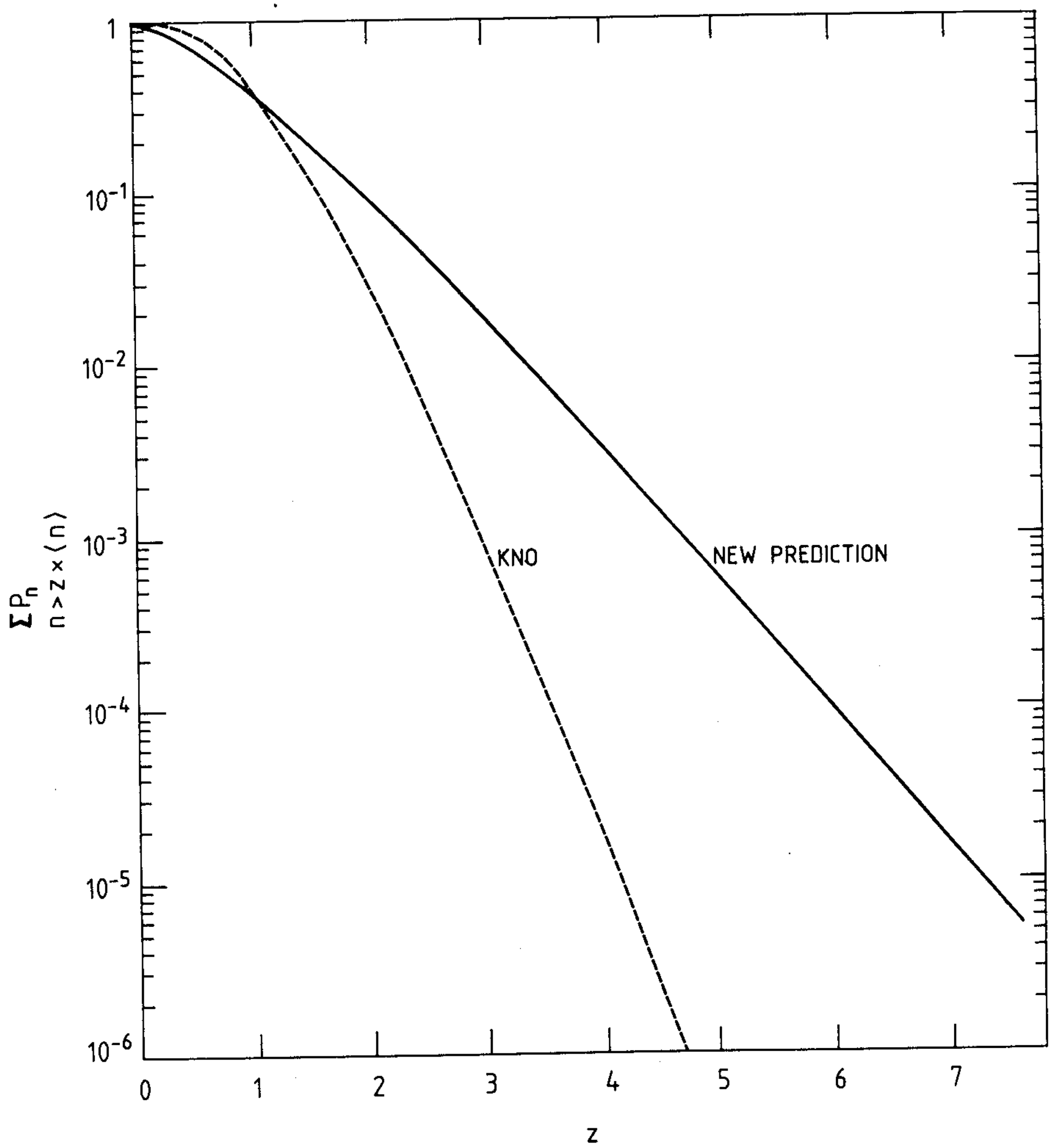
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
$\sum_{n > z \times \langle n \rangle} P_n$
0
1
2
3
4
5
6
7
z
KNO
NEW PREDICTION

Fig. 4