CDCC analysis for breakup of three-body projectiles

T. Matsumoto[1], *K. Minomo*[1], *K. Ogata*[2], *M. Yahiro*[1], *and K. Kato*[3]
[1]Department of Physics, Kyushu University, Fukuoka 812-8581, Japan
[2]Research Center for Nuclear Physics (RCNP), Osaka University, Osaka 567-0047, Japan
[3]Division of Physics, Graduate School of Science, Hokkaido University, Sapporo 060-0810, Japan

Abstract
We present a new method of smoothing discrete breakup cross sections calculated by the continuum-discretized coupled-channels method based on the complex-scaling method. One of advantages of this approach is applicable to many-body breakup reaction systems. In this work, we apply the new smoothing method to analyses of ^{12}C(^{6}He, nn^4He) and ^{208}Pb(^{6}He, nn^4He) reactions at 240 MeV/A.

1 INTRODUCTION

Exploring unstable nuclei far from the stable line is one of the most important subjects in nuclear physics. Two-neutron halo nuclei near the neutron drip line such as ^{6}He and ^{11}Li have exotic properties, i.e., soft dipole excitation and a di-neutron correlation. These properties can be investigated via breakup reactions, where the projectile breaks up into three fragments (core + n + n). One of the most reliable methods for treating the projectile breakup processes is the method of continuum-discretized coupled channels (CDCC) [1–3], which has been proposed as solving three-body scattering problems. Recently, we have developed CDCC as a method of treating four-body breakup processes in scattering of a three-body projectile [4–9].

Breakup cross sections include properties of continuum and resonance states of a projectile, and are obtained by the T-matrix elements in theoretically. The T-matrix elements estimated by CDCC, T_i, are discrete in the excitation energy ε of the projectile, although the exact ones $T(\varepsilon)$ are continuous. Thus one needs a way of smoothing T_i to analyze breakup reactions. For three-body breakup reactions, we have proposed the smoothing function method and confirmed the validity [6,7]. However, it is quite hard to adopt it to four-body breakup processes. Thus, it is highly expected that an accurate and practical method of smoothing T_i will be proposed.

In this work, we propose a new method to obtain the differential breakup cross section as a continuous function of ε accurately and practically, by using CDCC and the complex-scaling method (CSM) [10,11]. The new method is applied to the ^{12}C(^{6}He, nn^4He) reaction at 229.8 MeV/A. A merit of the present smoothing method is that one can see fast convergence of the calculated breakup cross section with respect to extending the model space. The method is also applied to ^{12}C(^{6}He, nn^4He) and ^{208}Pb(^{6}He, nn^4He) reactions at 240 MeV/A and compared with the experimental data. In principle, this method is applicable not only for four-body breakup reactions but also for many-body breakup reactions.

2 FORMULATION

We consider scattering of a projectile B from a target A. The scattering is described by the Schrödinger equation with outgoing boundary conditions,

$$[H - E_{\rm tot}]|\Psi^{(+)}\rangle = 0, \tag{1}$$

where the total energy $E_{\rm tot}$ is related to the corresponding incident energy of the center-of-mass system $E_{\rm in}^{\rm CM}$ as $E_{\rm tot} = E_{\rm in}^{\rm CM} + \varepsilon_0$ with the ground-state energy ε_0 of B. The total Hamiltonian H of this system is defined as

$$H \;=\; K_R + U(\boldsymbol{\xi}, \boldsymbol{R}) + H_{\rm B}, \tag{2}$$

where $\boldsymbol{R}$ is a coordinate between B and A, and $\boldsymbol{\xi}$ is a set of internal coordinates in B. The kinetic energy operator for $\boldsymbol{R}$ and internal Hamiltonian of B are represented by K_R and H_B, respectively, and U is a sum of nuclear and Coulomb potentials between constituents in B and A.

The most fundamental assumption in CDCC is that the scattering takes place in a modelspace,

$$\mathcal{P} = \sum_{\gamma} |\Phi_\gamma\rangle\langle\Phi_{n\gamma}|, \tag{3}$$

where Φ_γ is a γth eigenstate obtained by diagonalizing $H_{\rm B}$ with L^2-type basis functions. Therefore, the Schrödinger equation is solved in the modelspace:

$$\mathcal{P}[H - E_{\rm tot}]\mathcal{P}|\Psi^{(+)}_{\rm CDCC}\rangle = 0. \tag{4}$$

The T matrix amplitude for breakup processes, in which the final state of B has an excitation energy ε, is

$$T(\varepsilon) = \langle\psi^{(-)}_\varepsilon(\boldsymbol{\xi})\chi^{(-)}_\varepsilon(\boldsymbol{R})|\hat{U}|\Psi^{(+)}(\boldsymbol{\xi},\boldsymbol{R})\rangle_{\boldsymbol{\xi R}}, \tag{5}$$

$$\hat{U} = U(\boldsymbol{\xi},\boldsymbol{R}) - V^{\rm Coul}_{\rm B}(R), \tag{6}$$

where $V^{\rm Coul}_{\rm B}$ is the Coulomb interaction between B and A. The exact final channel wave function $\psi^{(-)}_\varepsilon(\boldsymbol{\xi})\chi^{(-)}_\varepsilon(\boldsymbol{R})$ with incoming boundary conditions satisfies

$$\left[K_R + V^{\rm Coul}_B(R) - (E_{\rm tot} - \varepsilon)\right]|\chi^{(-)}_\varepsilon(\boldsymbol{R})\rangle = 0, \tag{7}$$

$$[H_{\rm B} - \varepsilon]\,|\psi^{(-)}_\varepsilon(\boldsymbol{\xi})\rangle = 0. \tag{8}$$

Using Eq. (5), the differential cross section as a function of ε can be calculated as

$$\frac{d\sigma}{d\varepsilon} = \int d\varepsilon'\delta(\varepsilon - \varepsilon')|T(\varepsilon')|^2 = \frac{1}{\pi}{\rm Im}\mathcal{R}(\varepsilon) \tag{9}$$

with the response function

$$\mathcal{R}(\varepsilon) = \int d\boldsymbol{\xi}d\boldsymbol{\xi}'\mathcal{O}^\dagger(\varepsilon,\boldsymbol{\xi})\mathcal{G}^{(-)}(\varepsilon,\boldsymbol{\xi},\boldsymbol{\xi}')\mathcal{O}(\varepsilon,\boldsymbol{\xi}'), \tag{10}$$

where the Green's function $\mathcal{G}^{(-)}$ and operator $\mathcal{O}$ are defined by

$$\mathcal{G}^{(-)}(\varepsilon,\boldsymbol{\xi},\boldsymbol{\xi}') = \lim_{\eta\to+0}\langle\boldsymbol{\xi}|\frac{1}{\varepsilon - H_{\rm B} - i\eta}|\boldsymbol{\xi}'\rangle, \tag{11}$$

$$\mathcal{O}(\varepsilon,\boldsymbol{\xi}) = \langle\chi^{(-)}_\varepsilon(\boldsymbol{R})|\hat{U}|\Psi^{(+)}(\boldsymbol{\xi},\boldsymbol{R})\rangle_{\boldsymbol{R}}. \tag{12}$$

In order to evaluate $\mathcal{R}(\varepsilon)$, we use the complex scaling method (CSM), where the scaling transformation operator is represented by $C(\theta)$. The scaled Green's function is written as

$$\mathcal{G}^{(-)}_\theta(\varepsilon,\boldsymbol{\xi},\boldsymbol{\xi}') = \lim_{\eta\to+0}\langle\boldsymbol{\xi}|\frac{1}{\varepsilon - H^\theta_{\rm B} - i\eta}|\boldsymbol{\xi}'\rangle, \tag{13}$$

with the complex-scaled Hamiltonian

$$H^\theta_{\rm B} = C(\theta)H_{\rm B}C^{-1}(\theta). \tag{14}$$

The scaled Green's function $\mathcal{G}^\theta$ is a L^2-type operator when $-\pi < \theta < 0$, so that it can be expanded with L^2-type basis functions with high accuracy:

$$\mathcal{G}^\theta(\varepsilon,\boldsymbol{\xi},\boldsymbol{\xi}') \approx \sum_i \frac{|\phi^\theta_i\rangle\langle\tilde{\phi}^\theta_i|}{\varepsilon - \varepsilon^\theta_i}, \tag{15}$$

where ϕ_i^θ is a i-th eigenstate obtained by diagonalizing $H_{\rm B}^\theta = C(\theta) H_{\rm B} C^{-1}(\theta)$ in a modelspace spanned by L^2-type basis functions, $\langle \tilde{\phi}_i^\theta | H_{\rm B}^\theta | \phi_{i'}^\theta \rangle = \varepsilon_i^\theta \delta_{ii'}$. Note that the scaling angle should be taken as negative, because the Green's function $\mathcal{G}^{(-)}$ satisfies an incoming boundary condition.

Furthermore the modelspace approximation is applied to the Green's function and total wave function, that is, $\mathcal{G}^{(-)}$ and Ψ are replaced by $\mathcal{P}\mathcal{G}^{(-)}\mathcal{P}$ and $\Psi^{(+)}_{\rm CDCC}$, respectively. This leads to

$$\begin{aligned} \mathcal{R}(\varepsilon) \;\approx\; & \sum_i \sum_{\gamma',\gamma} \langle \Psi^{(+)}_{\rm CDCC} | \hat{U}^* | \chi^{(-)}_\gamma \Phi_{\gamma'} \rangle \\ & \frac{\langle \Phi_{\gamma'} | C^{-1}(\theta) | \phi_i^\theta \rangle \langle \tilde{\phi}_i^\theta | C(\theta) | \Phi_\gamma \rangle}{\varepsilon - \varepsilon_i^\theta} \\ & \times \langle \Phi_\gamma \chi^{(-)}_\gamma | \hat{U} | \Psi^{(+)}_{\rm CDCC} \rangle. \end{aligned} \tag{16}$$

Noting that $\langle \Phi_\gamma \chi^{(-)}_\gamma | \hat{U} | \Psi^{(+)}_{\rm CDCC} \rangle$ is a T-matrix element of CDCC, T_γ, to Φ_γ, we define scaled T-matrix elements by

$$\tilde{T}_i^\theta \equiv \sum_{\gamma'} \langle \tilde{\phi}_i^\theta | C(\theta) | \Phi_{\gamma'} \rangle T_{\gamma'}, \tag{17}$$

$$T_i^\theta \equiv \sum_{\gamma} T_\gamma^* \langle \Phi_n | C^{-1}(\theta) | \phi_i^\theta \rangle. \tag{18}$$

The final form of the differential cross section is then obtained by

$$\frac{d\sigma}{d\varepsilon} \;=\; \frac{1}{\pi} {\rm Im} \sum_i \frac{T_i^\theta \tilde{T}_i^\theta}{\varepsilon - \varepsilon_i^\theta}. \tag{19}$$

For the diagonalization of $H_{\rm B}$ and $H_{\rm B}^\theta$, we adopt the Gaussian expansion method (GEM) [12]. In GEM, the state of the ^{4}He + n + n system is described by a superposition of three channels, each channel with a different set of Jacobi coordinates, $(\boldsymbol{y}_c, \boldsymbol{r}_c)$. For each c (channel), the radial parts of the internal wave functions regarding $\boldsymbol{y}_c$ and $\boldsymbol{r}_c$ are expanded by a finite number of Gaussian basis functions

$$\begin{aligned} \varphi_{j\lambda}(\boldsymbol{y}_c) &= y_c^\lambda e^{-(y_c/\bar{y}_j)^2} Y_\lambda(\Omega_{y_c}), \\ \varphi_{i\ell}(\boldsymbol{r}_c) &= r_c^\ell e^{-(r_c/\bar{r}_i)^2} Y_\ell(\Omega_{r_c}), \end{aligned} \tag{20}$$

respectively. Here λ (ℓ) is the angular momentum regarding $\boldsymbol{y}_c$ ($\boldsymbol{r}_c$), and the range parameters are taken to lie in geometric progression:

$$\bar{y}_j \;=\; (\bar{y}_{\rm max}/\bar{y}_1)^{(j-1)/j_{\rm max}}, \tag{21}$$

$$\bar{r}_i \;=\; (\bar{r}_{\rm max}/\bar{r}_1)^{(i-1)/i_{\rm max}}. \tag{22}$$

The parameters depend on c, but we omitted the dependence in Eqs. (21) and (22) for simplicity; see Ref. [4] for the details of the diagonalization and the definition of Jacobi coordinates. As interactions V_{nn} and $V_{n\alpha}$ in H_B, we take the so-called GPT [13] and KKNN [14] potentials, respectively. These potentials with a Gaussian form reproduce well data of low-energy nucleon-nucleon and nucleon-^{4}He scattering, respectively. The particle exchange between valence neutrons and neutrons in ^{4}He is treated approximately with the orthogonality condition model [15].

3 RESULTS AND DISCUSSIONS

First, we prepare the three sets of parameters of basis functions shown in Table 1 to confirm the convergence of the breakup cross section. For the 0^+ and 1^- states, maximum internal angular momenta

Table 1: Gaussian range parameters.

Set	c	$j_{\max}$	$\bar{y}_1$ (fm)	$\bar{y}_{\max}$ (fm)	$i_{\max}$	$\bar{r}_1$ (fm)	$\bar{r}_{\max}$ (fm)
I	3	10	0.1	10.0	10	0.5	10.0
	1, 2	10	0.5	10.0	10	0.5	10.0
II	3	15	0.1	20.0	15	0.5	20.0
	1, 2	15	0.5	20.0	15	0.5	20.0
III	3	20	0.1	50.0	20	0.5	50.0
	1, 2	20	0.5	50.0	20	0.5	50.0

$\ell_{\max}$ and $\lambda_{\max}$ are both set to unity. For the 2^+ states, we take $\ell_{\max} = \lambda_{\max} = 1$ for $c = 1$ and 2, and $\ell_{\max} = \lambda_{\max} = 2$ for $c = 3$. Figure 1 shows the breakup cross sections $d\sigma/d\varepsilon$ to the 0^+, 1^-, and 2^+ continua for ^{12}C(^{6}He,nn^4He) reaction at 229.8 MeV. For all the cross sections, sets II and III yield the same result, whereas the result of set I is somewhat different from it. The convergence of CDCC solution with respect to expanding the model space is thus obtained with set II. Here, we take $\theta = -14^\circ$ as the scaling angle, since the converged spectra are obtained at this angle.

In Fig. 2, the breakup cross section $d\sigma/d\varepsilon$ calculated by the present method is compared with the experimental data for ^{6}He + ^{12}C and ^{6}He + ^{208}Pb reactions at 240 MeV/A [16]. These data have already been analyzed by four-body distorted-wave Born approximation (DWBA) [17] and the eikonal approximation [18]. In the present analysis, we estimate optical potentials for n-target and ^{4}He-target based on the double-folding model the Melbourne nucleon-nucleon g-matrix interaction [19] with the densities obtained by the spherical Hartree-Fock (HF) calculation with the Gogny D1S interaction. [20, 21]. For the result of the ^{12}C(^{6}He, nn^4He) reaction, one sees clear peak of the 2^+ resonance around 1 MeV as shown in Fig. 2(a). In this analysis, we found that Coulomb breakup effects are negligible and the present theoretical result is consistent with the experimental data except for the peak of the 2^+ resonance around $\varepsilon = 1$ MeV. On the other hand, Coulomb breakup to the 1^- continuum is dominant for the ^{208}Pb(^{6}He, nn^4He) reaction as shown in Fig. 2(b). For ^{208}Pb target, the present method underestimates the experimental data at $\varepsilon \geq 2$ MeV. A possible origin of this underestimation is that the inelastic breakup reactions are not included in the present calculation. As mentioned in Ref. [17], the inelastic breakup effect is not negligible, and the elastic breakup cross section calculated with four-body DWBA also underestimates the data.

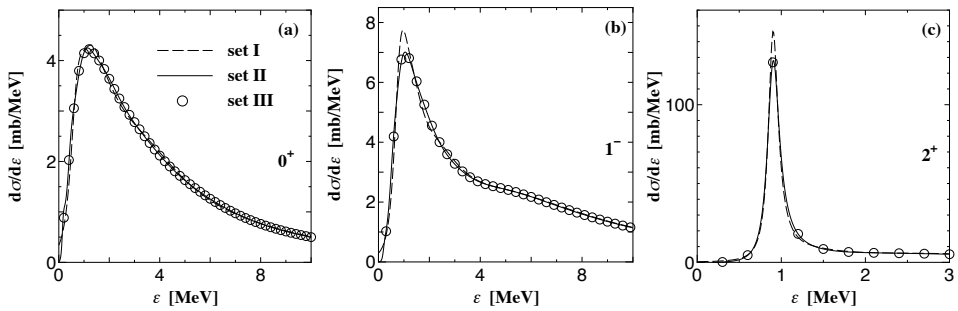

Fig. 1: Convergence of the breakup cross sections to the 0^+ (a), 1^- (b), and 2^+ (c) continua. In each panel, the dashed line, the solid line, and the open circles correspond to results of sets I, II, and III, respectively. The dotted line in (b) shows the result when Coulomb breakup processes are switched off.

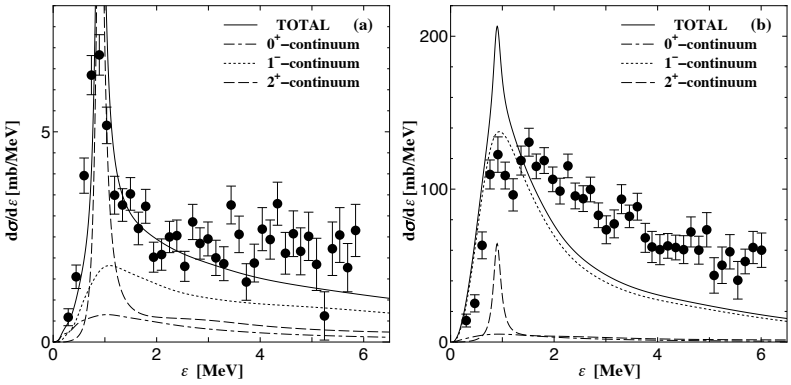

Fig. 2: Comparison of the breakup cross section calculated by CDCC (solid line) with experimental data for (a) the ^{6}He + ^{12}C scattering at 240 MeV/nucleon and (b) the ^{6}He + ^{208}Pb scattering at 240 MeV/nucleon. The dash-dotted, dotted, and dashed lines correspond to the contributions of the 0^+, 1^-, and 2^+ breakup, respectively, and the solid line is the sum of them. The experimental data are taken from Ref. [16].

4 SUMMARY

In summary, we have proposed a practical method of calculating the differential breakup cross section as a continuous function of the excitation energy of a projectile, by combining CDCC and CSM. One of advantages of this method is that we do not require to calculate the exact continuum wave functions of the projectile. In the present formalism, we have to do is just diagonalize the projectile Hamiltonian and the scaled Hamiltonian with L^2-type basis functions. Furthermore, the scaling operator $C(\theta)$ operates only on spatially damping functions and hence the differential breakup cross section converges quickly as the model space is extended. The method is successful in reproducing the data on ^{12}C(^{6}He, nn^{4}He) and ^{208}Pb(^{6}He, nn^{4}He) reactions at 240 MeV/A. In principle, the present formalism is applicable for many-body breakup reaction, if the diagonalization of the projectile Hamiltonian and the scaled Hamiltonian is feasible.

References

[1] M. Kamimura, M. Yahiro, Y. Iseri, Y. Sakuragi, H. Kameyama and M. Kawai, Prog. Theor. Phys. Suppl. **89**, 1 (1986).

[2] N. Austern, Y. Iseri, M. Kamimura, M. Kawai, G. Rawitscher and M. Yahiro, Phys. Rep. **154**, 125 (1987).

[3] M. Yahiro, K. Ogata, T. Matsumoto, and K. Minomo, Prog. Theor. Exp. Phys. **1**, 01A209 (2012); arXiv:1203.5392 (2012).

[4] T. Matsumoto, E. Hiyama, K. Ogata, Y. Iseri, M. Kamimura, S. Chiba, and M. Yahiro, Phys. Rev. C **70**, 061601(R) (2004).

[5] T. Matsumoto, T. Egami, K. Ogata, Y. Iseri, M. Kamimura, and M. Yahiro, Phys. Rev. C **73**, 051602 (2006).

[6] T. Matsumoto, T. Kamizato, K. Ogata, Y. Iseri, E. Hiyama, M. Kamimura, and M. Yahiro, Phys. Rev. C **68**, 064607 (2003).

[7] T. Egami, K. Ogata, T. Matsumoto, Y. Iseri, M. Kamimura, and M. Yahiro, Phys. Rev. C **70**, 047604 (2004).

[8] T. Matsumoto, E. Hiyama, M. Yahiro, K. Ogata, Y. Iseri and M. Kamimura, Nucl. Phys. **A 738c**, 471 (2004).

[9] M. Rodríguez-Gallardo, J. M. Arias, J. Gómez-Camacho, R. C. Johnson, A. M. Moro, I. J. Thompson, and J. A. Tostevin, Phys. Rev. C **77**, 064609 (2008).

[10] J. Aguilar and J.M. Combes, Commun. Math. Phys., 22, 1971, 269.
E. Balslev and J.M. Combes, Commun. Math. Phys., 22, 1971, 280.

[11] S. Aoyama, T. Myo, K. Kato, and K. Ikeda, Prog. Theor. Phys. 116, 1 (2006).

[12] E. Hiyama, Y. Kino and M. Kamimura, Prog. Part. Nucl. Phys. **51**, 223 (2003).

[13] D. Gogny *et al*., Phys. Lett. **B32**, 591 (1970).

[14] H. Kanada, *et al*., Prog. Theor. Phys. **61**, 1327 (1979).

[15] S. Saito, Prog. Theor. Phys. **41**, 705 (1969).

[16] T. Aumann *et al*., Phys. Rev. C **59**, 1252 (1999).

[17] S. N. Ershov, B. V. Danilin, and J. S. Vaagen, Phys. Rev. C **62**, 041001(R) (2000).

[18] D. Baye, P. Capel, P. Descoubemont, and Y. Suzuki, Phys. Rev. C **79**, 024607 (2009).

[19] K. Amos, P. J. Dortmans, H. V. von Geramb, S. Karataglidis, and J. Raynal, in *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt(Plenum, New York, 2000) Vol. 25, p. 275.

[20] J. Decharge and D. Gogny, Phys. Rev. C **21** (1980), 1568.

[21] J. F. Berger, M. Girod, and D. Gogny, Comp. Phys. Comm. **63** (1991), 1365.