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HERMAPHRODITE MESONS AND QCD SUM RULES

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A B S T R A C T

We present a new evaluation of the two-point function built from the colourless operators $g\bar{\psi}\lambda^a\gamma_\nu(\gamma_5)\psi F_a^{\mu\nu}$. Previous calculations for the dimension-six vacuum condensate contributions were incomplete. We give new predictions for the masses and decay amplitudes of 1^{-+} and 0^{--} hermaphrodite mesons.

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QCD sum rules have nowadays become a powerful method in the analysis of the hadron parameters as well as in the phenomenological estimate of the QCD and chiral symmetry-breaking parameters. An earlier attempt to study the properties of hermaphrodite (or meikton) mesons within the framework of QCD sum rules can be found elsewhere[1]. However, it has been noticed later on[2] that in Ref. 1 there are some mistakes in the evaluation of the QCD contributions making, unfortunately, their prediction invalid for the masses and decay amplitudes of the hermaphrodite mesons. Given the complexity and the non-triviality of the QCD calculations, it becomes necessary to provide a completely independent calculation. We shall be concerned with the two-point functions,

$$\Pi_i^{\mu\nu}(q) \equiv i \int d^4x \, e^{iq.x} \langle \underset{\sim}{0} | T \left(O_i^{\mu}(x) \, O_i^{\nu}(0) \right) | \underset{\sim}{0} \rangle =$$

$$= - \left(q^2 g^{\mu\nu} - q^{\mu} q^{\nu} \right) \Pi_i^{(1)}(q^2) + q^{\mu} q^{\nu} \Pi_i^{(0)}(q^2) \tag{1}$$

built from the colourless, local gauge-invariant operators

$$O_V^{\mu}(x) \equiv \; : g \, \bar{\psi}(x) \, \lambda_a \, \gamma_{\nu} \, \psi(x) \, F_a^{\mu\nu}(x) : \tag{2a}$$

$$O_A^{\mu}(x) \equiv \; : g \, \bar{\psi}(x) \, \lambda_a \, \gamma_{\nu} \, \gamma_5 \, \psi(x) \, F_a^{\mu\nu}(x) : \tag{2b}$$

which are the unique lowest-dimension operators that can be used to study the properties of the hermaphrodite mesons with quantum numbers 1^{-+} and 0^{--}, respectively.

The evaluation of the two-point functions have been carried out using the method of Shifman, Vainshtein and Zakharov[3] which takes into account the contributions of the non-trivial values of the vacuum condensates. We choose to work in the same gauge as previous authors[1,2], which is the so-called coordinate Schwinger gauge[4] ($x_{\mu}B^{\mu}(x) = 0$). As is well known, this choice simplifies considerably the evaluation of the coefficients of the vacuum condensates in the expansion of the two-point functions. We present our result in the Table. For the diagrams I to IV we get the same results as in Ref. 2 (hereafter called G^2VW), but for a spurious normalization factor. A discrepancy appears for the coefficient of the triple gluon condensate, since G^2VW have missed diagram V and we disagree with the sign of diagram VI. For the mixed condensate we realize that G^2VW have only drawn the diagrams VII, VIII, and IX, and we get for them the same total contribution. However, one should notice that diagrams X to XIV give a non-vanishing contribution. The last two (as well as V) are due to the propagation of the quark condensate*).

Collecting all the results given in the Table and writing only those terms that will give a contribution when the Laplace (Borel) transformation is carried out, we obtain that the two-point functions projected in the 1^{+-} and 0^{--} channels are, respectively,

$$\Pi_V^{(1)}(q^2) = -\left\{ \frac{\alpha_s}{60\pi^3}(q^2)^3 + \frac{1}{9\pi}\left[\alpha_s\langle F^2\rangle + 8\alpha_s m\langle\bar\psi\psi\rangle\right]q^2 - \frac{1}{8\pi^2} g^3 f_{abc}\langle F^3_{abc}\rangle\right\} \ln(-q^2/\nu^2) \tag{3a}$$

$$\Pi_A^{(0)}(q^2) = -\left\{ \frac{\alpha_s}{120\pi^3}(q^2)^3 - \frac{1}{6\pi}\left[\alpha_s\langle F^2\rangle - 8\alpha_s m\langle\bar\psi\psi\rangle\right]q^2 + \frac{1}{8\pi^2} g^3 f_{abc}\langle F^3_{abc}\rangle - \frac{11}{18}\frac{\alpha_s}{\pi} m g\langle\bar\psi F\psi\rangle\right\} \ln(-q^2/\nu^2) \tag{3b}$$

Notice that we have neglected the contribution of O_2 since it is of the same order of magnitude as that of other neglected radiative correction terms.

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*) In Ref. 5, he corrects his previous result for the baryonic sector due to the presence of condensate propagation. A useful method for the evaluation of such a contribution can be seen in Ref. 5.

Spectral function sum rules

We relate the above QCD expression of the two-point function to the lowest ground state mass M_H and decay amplitude f_H via a spectral function sum rule approach. We parametrize the spectral function using the duality ansatz:

$$\frac{1}{\pi} \operatorname{Im} \Pi_i(t) = 2 f_H^2 M_H^6 \, \delta(t - M_H^2) + A(t)\, \Theta(t - t_c)$$

where $\sqrt{t_c}$ is the continuum threshold which should be interpreted as an average of the multiparticle thresholds; A(t) corresponds to the discontinuity coming from the QCD[1] diagrams. One can notice that contrary to the case of the ρ and π channels, the QCD continuum receives here also some contributions coming from the non-perturbative terms. We can work for the analysis with various sum rules discussed in the literature[6,7]. As here we shall be more interested in a sum rule which is directly sensitive to the lowest ground state mass, we find it convenient to work with the moment sum rules ratios[3,7b]:

$$R_i^{(1,0)}(\tau) = -\frac{d}{d\tau} \ln \int_0^\infty dt\, e^{-t\tau} \frac{1}{\pi} \operatorname{Im} \Pi_i^{(1,0)}(t) \tag{4}$$

where τ is the "imaginary time" sum rule variable. The QCD expression of the moment is, for instance*),

$$R_V^{(1)}(\tau) = 4\tau^{-1} \Big\{ 1 + O(\bar{\alpha}_s/\pi) - \tau^2 \left(\frac{\pi}{\bar{\alpha}_s}\right) \left[\frac{5\pi}{9} \alpha_s \langle F^2 \rangle - \frac{15}{16} g^3 f_{abc} \langle F^3_{abc} \rangle \tau \right] + O(\tau^4) \Big\} \tag{5}$$

We give its behaviour in Fig. 1 where we have used $\alpha_s \langle F^2 \rangle = (0.04 \pm 0.01)\ \mathrm{GeV}^4$ [6,8] $g^3 f_{abc} \langle F^3_{abc} \rangle \simeq (1.1\ \mathrm{GeV}^2)\ \alpha_s \langle F^2 \rangle$[9] and $\bar{\alpha}_s/\pi = -4/(9 \log \tau\Lambda^2)$.

Our estimate of the uncertainties on R comes from the radiative corrections for small τ and from an assumed uncertainty of 25% in the estimate of $g^3 f_{abc} \langle F^3_{abc} \rangle$

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*) The analogue of Eqs. (5 to 9) for the 0^{--} can be easily obtained[6] from Eq. (3b).

from the lattice calculations for larger τ. Then, the estimated uncertainties on R are

$$\Delta R_V^{(1)} = -\tau^{-1}/\ln\tau\Lambda^2 \quad - \tau^2(\ln\tau\Lambda^2)\ 0.1\ \mathrm{GeV}^6 \tag{6}$$

The phenomenological expression of the moments is

$$R_V^{(1)}(\tau) = \frac{4}{\tau}\ \frac{2f_H^2 M_H^8 e^{-M_H^2\tau} + 24 C_1^{(1)}\tau^{-5}\rho_4 + 2C_4^{(1)}\langle O_4\rangle\tau^{-3}\rho_2 - C_6^{(1)}\langle O_6\rangle\tau^{-2}\rho_1}{2f_H^2 M_H^6 e^{-M_H^2\tau} + 6C_1^{(1)}\tau^{-4}\rho_3 + C_4^{(1)}\langle O_4\rangle\tau^{-2}\rho_1 - C_6^{(1)}\langle O_6\rangle\tau^{-1}\rho_0} \tag{7}$$

with

$$C_1^{(1)} \equiv \frac{\alpha_s}{60\pi^3} \quad ; \quad C_4^{(1)}\langle O_4\rangle \equiv \frac{1}{9\pi}\left[\alpha_s\langle F^2\rangle + 8\bar{\alpha}_s m\langle\bar{\psi}\psi\rangle\right] ;$$

$$C_6^{(1)}\langle O_6\rangle = \frac{1}{8\pi^2}\ g^3 f_{abc}\langle F^3_{abc}\rangle$$

$$\rho_i = e^{-t_c\tau}\left\{1 + \frac{(t_c\tau)}{1!} + \dots + \frac{(t_c\tau)^i}{i!}\right\} \tag{8}$$

The asymptotic coincidence of the QCD and phenomenological sides of the sum rule gives the constraint for small τ:

$$2f_H^2 M_H^6 = C_1^{(1)} t_c^4 + 2C_4^{(1)}\langle O_4\rangle t_c^2 - 4C_6^{(1)}\langle O_6\rangle t_c + \left(C_4^{(1)}\langle O_4\rangle\right)^2/3C_1^{(1)} \tag{9}$$

Such a contraint allows us to eliminate one of the three parameters (f_H, M_H^2, t_c) in the fitting procedure. For instance, we eliminate f_H. Then we confront Eqs. (5) and (7) for the values of τ less than a critical value τ_{max} using a two-parameter fit, with the help of the CERN library subroutine FUMILI [10] *). We give our results for the 1^{-+} and 0^{--} channels in Figs. 2 and 3 respectively. For

*) More details on the uses of such a program within the sum rule are discussed in Ref. 11.

instance, in Fig. 2a, we analyse the effect of t_c for given values of τ_{max} and Λ. The best fit corresponds to the minimum of χ^2/NDF, which gives a mass of the resonance at 1.66 GeV. In Fig. 2b, we analyse the effect of τ_{max} for a given value of t_c which comes from the best fit of Fig. 2a. We see in Fig. 2c that the effect of the value of Λ in the range of 100-200 MeV is irrelevant. Our best fit, which takes into account the uncertainties coming from the input parameters, gives for the 1^{-+} channel

$$M_{1^-} = (1.7 \pm 0.1)\ \mathrm{GeV} \simeq \sqrt{t_c}$$

and

$$f_{1^-} \simeq 46.4\ \mathrm{MeV} \tag{10}$$

We summarize the analysis of the 0^{--} mass in Fig. 3. The fit is less good than for the 1^{-+} channel, as a lesser stability is obtained. We deduce the predictions

$$M_{0^-} = (3.1 \pm 0.2)\ \mathrm{GeV} \simeq \sqrt{t_c}$$

and

$$f_{0^-} \simeq 23.4\ \mathrm{MeV} \tag{11}$$

If our results are compared with those of G^2VW, one can see that the one in Eq. 10 is higher than 400 MeV whilst that in Eq. (11) is lower with about the same amount*). The result in Eq. (10) is in the range of the bag model predictions[12]. Owing to the unorthodox quantum numbers of the 1^{-+} and 0^{--} hermaphrodites, a better scanning of the heavy quarkonia, $p\bar{p}$, or other hadronic data should show evidence of the existence of these mesons.

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*) The effects of dimension 8 operators are not clear because they are renormalization scheme dependent. However, they are numerically irrelevant at the τ-values where we are working.

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Figure captions

Fig. 1 : Behaviour of the moments versus τ for given values of the QCD parameters.

Fig. 2 : a) Variation of the mass squared M_H^2 and of χ^2/NDF for the 1^{-+} meson at given values of Λ and τ_{max} for various values of t_c.

b) Behaviour of M_H^2 and χ^2/NDF versus τ_{max} for $\Lambda = 0.15$ GeV and $t_c = 3$ GeV^2.

c) Behaviour of M_H^2 versus Λ for $\tau_{max} = 1.5$ GeV^2 and $t_c = 3$ GeV^2.

Fig. 3 : a) Variation of M_H^2 and of χ^2/NDF for the 0^{--} meson at given values of Λ and τ_{max} for various values of t_c.

b) Behaviour of M_H^2 and χ^2/NDF versus τ_{max} for $\Lambda = 0.15$ GeV and $t_c = 9$ GeV^2.

Table

QCD contributions to the two-point function in Eq. (1)

Lowest order:

I)

$$\Pi_V^{\mu\nu}(q) = \Pi_A^{\mu\nu}(q) = \frac{\alpha_s}{120\pi^3} q^4 \Big\{ \Big(\frac{1}{\hat\varepsilon} + \gamma + \ln\Big(-\frac{q^2}{4\pi\nu^2}\Big) - \frac{117}{20}\Big) q^2 g^{\mu\nu} - \frac{3}{2}\Big(\frac{1}{\hat\varepsilon} + \gamma + \ln\Big(-\frac{q^2}{4\pi\nu^2}\Big) - \frac{331}{60}\Big) q^\mu q^\nu \Big\}$$

$1/\hat\varepsilon \equiv 1/\varepsilon + \gamma + \ln(-q^2/4\pi\nu^2)$; $\alpha_s \equiv g^2/4\pi$

$D \equiv 4 + 2\varepsilon$ space-time dimensions

Dimension-four:

II)

$$\Pi_V^{\mu\nu}(q) = \frac{4\alpha_s}{9\pi} m\langle\bar\psi\psi\rangle \Big\{ \Big(\frac{2}{\hat\varepsilon} - \frac{7}{3}\Big) q^2 g^{\mu\nu} + \Big(\frac{1}{\hat\varepsilon} - \frac{13}{6}\Big) q^\mu q^\nu \Big\}$$

$$\Pi_A^{\mu\nu}(q) = \frac{4\alpha_s}{9\pi} m\langle\bar\psi\psi\rangle \Big\{ \Big(\frac{4}{\hat\varepsilon} - \frac{17}{3}\Big) q^2 g^{\mu\nu} + \Big(-\frac{7}{\hat\varepsilon} + \frac{43}{6}\Big) q^\mu q^\nu \Big\}$$

III)

$$\Pi_V^{\mu\nu}(q) = \Pi_A^{\mu\nu}(q) = \frac{\alpha_s}{18\pi} \langle F^2\rangle \Big\{ \Big(\frac{2}{\hat\varepsilon} - \frac{4}{3}\Big) q^2 g^{\mu\nu} + \Big(\frac{1}{\hat\varepsilon} - \frac{5}{3}\Big) q^\mu q^\nu \Big\}$$

; $\langle F^2\rangle \equiv \langle \underset{\sim}{0}|:F^{\mu\nu}_a(0) F^a_{\mu\nu}(0):|\underset{\sim}{0}\rangle$

Dimension-six

a) Four-fermion:

IV)

$$\Pi_V^{\mu\nu}(q) = -\Pi_A^{\mu\nu}(q) = -\frac{16\pi\alpha_s}{9q^2}\langle\bar\psi\psi\rangle^2 \big[q^2 g^{\mu\nu} + 2q^\mu q^\nu \big]$$

b) Triple gluon:

V)

$$\Pi_V^{\mu\nu}(q) = \Pi_A^{\mu\nu}(q) = \frac{1}{36\pi^2 q^2} \Big\{ \Big[\Big(-\frac{9}{\hat\varepsilon} + \frac{15}{2}\Big) q^2 g^{\mu\nu} - 6q^\mu q^\nu\Big] O_1 + \Big[\Big(-\frac{3}{\hat\varepsilon} + \frac{5}{2}\Big) q^2 g^{\mu\nu} - 6q^\mu q^\nu\Big] O_2$$

$O_1 \equiv \langle \underset{\sim}{0}|:\mathrm{Tr}\,[F_{\sigma\tau}(0) F^{\tau\rho}(0) F_\rho{}^\sigma(0)]:|\underset{\sim}{0}\rangle \equiv \frac{1}{4} g^3 f_{abc}\langle F^3_{abc}\rangle$

$O_2 \equiv \langle \underset{\sim}{0}|:\mathrm{Tr}\,\{[D^\sigma(0), F_{\sigma\rho}(0)][D_\tau(0), F^{\tau\rho}(0)]\}:|\underset{\sim}{0}\rangle$

$F_{\mu\nu} \equiv \frac{i}{2} g \lambda_a F^a_{\mu\nu}$; $D_\mu \equiv \partial_\mu - ig\frac{\lambda_a}{2} B^a_\mu$

VI) 

$$\Pi_V^{\mu\nu}(q) = \Pi_A^{\mu\nu}(q) = -\frac{1}{24\pi^2 q^2}\left\{\left(\frac{6}{\hat\varepsilon} - 7\right) q^2 g^{\mu\nu} + 4 q^\mu q^\nu\right\} O_1$$

c) <u>Mixed condensate</u>:

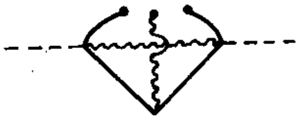
VII) 

$$\Pi_V^{\mu\nu}(q) = -\Pi_A^{\mu\nu}(q) = \frac{\alpha_s}{8\pi q^2}\, m g \langle\bar\Psi F \psi\rangle \left\{\left(\frac{3}{2\hat\varepsilon} - \frac{5}{4}\right) q^2 g^{\mu\nu} - q^\mu q^\nu\right\} \quad ; \langle\bar\Psi F\psi\rangle \equiv \langle 0 | : \bar\psi(0)\sigma^{\mu\nu}\lambda_a F^a_{\mu\nu}(0)\psi(0) : | 0\rangle$$

VIII) 

$$\Pi_V^{\mu\nu}(q) = \frac{\alpha_s}{8\pi q^2}\, m g \langle\bar\Psi F\psi\rangle\left\{\left(-\frac{3}{\hat\varepsilon} + 7\right) q^2 g^{\mu\nu} + 2 q^\mu q^\nu\right\}$$

$$\Pi_A^{\mu\nu}(q) = \frac{\alpha_s}{8\pi q^2}\, m g \langle\bar\Psi F\psi\rangle\left\{\left(\frac{6}{\hat\varepsilon} - \frac{17}{2}\right) q^2 g^{\mu\nu} - 8 q^\mu q^\nu\right\}$$

IX) 

$$\Pi_V^{\mu\nu}(q) = \frac{\alpha_s}{144\pi q^2}\, m g \langle\bar\Psi F\psi\rangle\left\{\left(-\frac{3}{\hat\varepsilon} + \frac{1}{2}\right) q^2 g^{\mu\nu} - 14 q^\mu q^\nu\right\}$$

$$\Pi_A^{\mu\nu}(q) = \frac{\alpha_s}{144\pi q^2}\, m g \langle\bar\Psi F\psi\rangle\left\{\left(-\frac{3}{\hat\varepsilon} - \frac{11}{2}\right) q^2 g^{\mu\nu} + 10 q^\mu q^\nu\right\}$$

X) 

$$\Pi_V^{\mu\nu}(q) = \frac{\alpha_s}{24\pi q^2}\, m g \langle\bar\Psi F\psi\rangle\left\{\left(\frac{1}{\hat\varepsilon} - \frac{1}{2}\right) q^2 g^{\mu\nu} + 2 q^\mu q^\nu\right\}$$

$$\Pi_A^{\mu\nu}(q) = \frac{\alpha_s}{24\pi q^2}\, m g \langle\bar\Psi F\psi\rangle\left\{\left(\frac{3}{\hat\varepsilon} - \frac{13}{6}\right) q^2 g^{\mu\nu} + \frac{2}{3} q^\mu q^\nu\right\}$$

XI) 

$$\Pi_V^{\mu\nu}(q) = \Pi_A^{\mu\nu}(q) = 0$$

XII) 

$$\Pi_V^{\mu\nu}(q) = \frac{\alpha_s}{3\pi q^2}\, m g \langle\bar\Psi F\psi\rangle\left\{\left(\frac{1}{\hat\varepsilon} - \frac{29}{18}\right) q^2 g^{\mu\nu} + \left(\frac{4}{3\hat\varepsilon} - \frac{28}{9}\right) q^\mu q^\nu\right\}$$

$$\Pi_A^{\mu\nu}(q) = \frac{\alpha_s}{3\pi q^2}\, m g \langle\bar\Psi F\psi\rangle\left\{\left(\frac{5}{3\hat\varepsilon} - \frac{55}{18}\right) q^2 g^{\mu\nu} + \left(-\frac{4}{3\hat\varepsilon} + \frac{4}{3}\right) q^\mu q^\nu\right\}$$

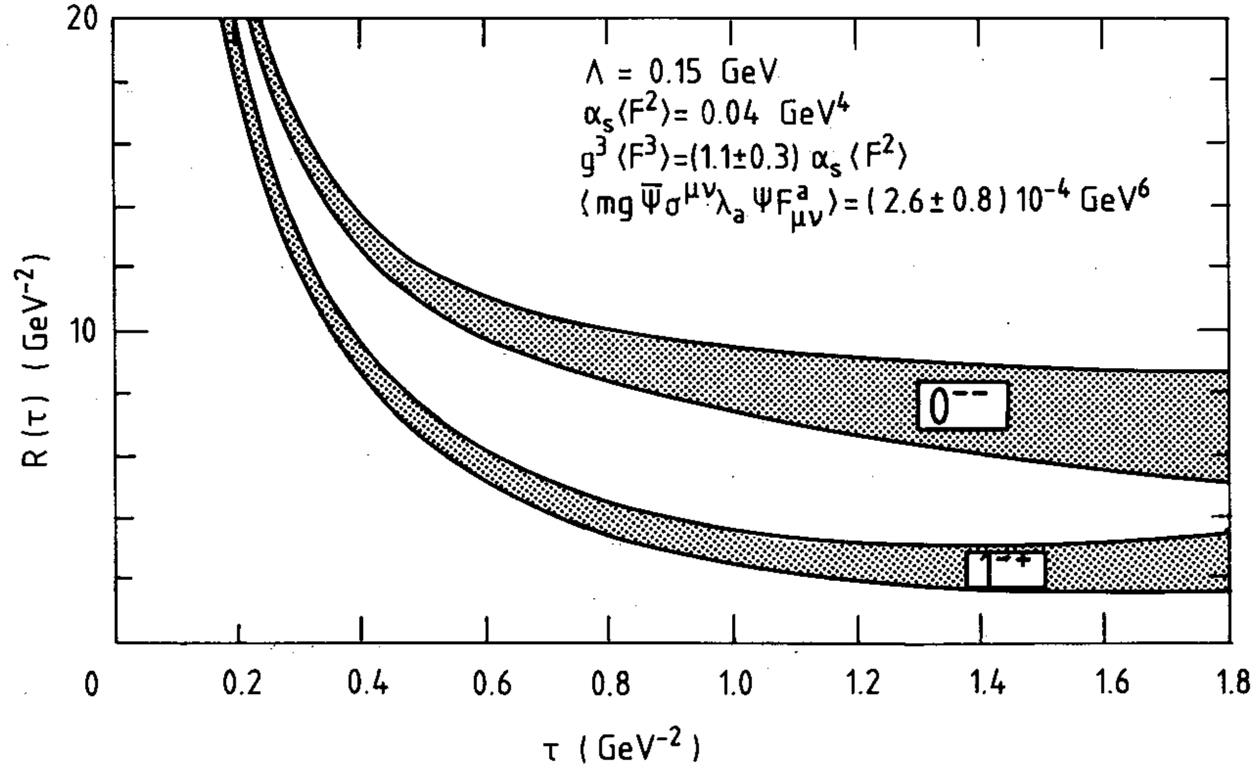
XIII) 

$$\Pi_V^{\mu\nu}(q) = \frac{\alpha_s}{6\pi q^2}\, m g \langle\bar\Psi F\psi\rangle\left\{\left(-\frac{1}{\hat\varepsilon} + \frac{1}{6}\right) q^2 g^{\mu\nu} - 2 q^\mu q^\nu\right\}$$

$$\Pi_A^{\mu\nu}(q) = \frac{\alpha_s}{6\pi q^2}\, m g \langle\bar\Psi F\psi\rangle\left\{\left(-\frac{1}{\hat\varepsilon} - \frac{11}{6}\right) q^2 g^{\mu\nu} + 6 q^\mu q^\nu\right\}$$

XIV)

 $\Pi^{\mu\nu}_{V}(q) = \Pi^{\mu\nu}_{A}(q) = 0$



20
10
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
R (τ) (GeV-2)
τ (GeV-2)
Λ = 0.15 GeV
αs⟨F2⟩= 0.04 GeV4
g3 ⟨F3⟩=(1.1±0.3) αs ⟨F2⟩
⟨mg Ψ̄σμν λa ΨFaμν⟩ = (2.6 ± 0.8) 10-4 GeV6
0--
1-+

Fig. 1

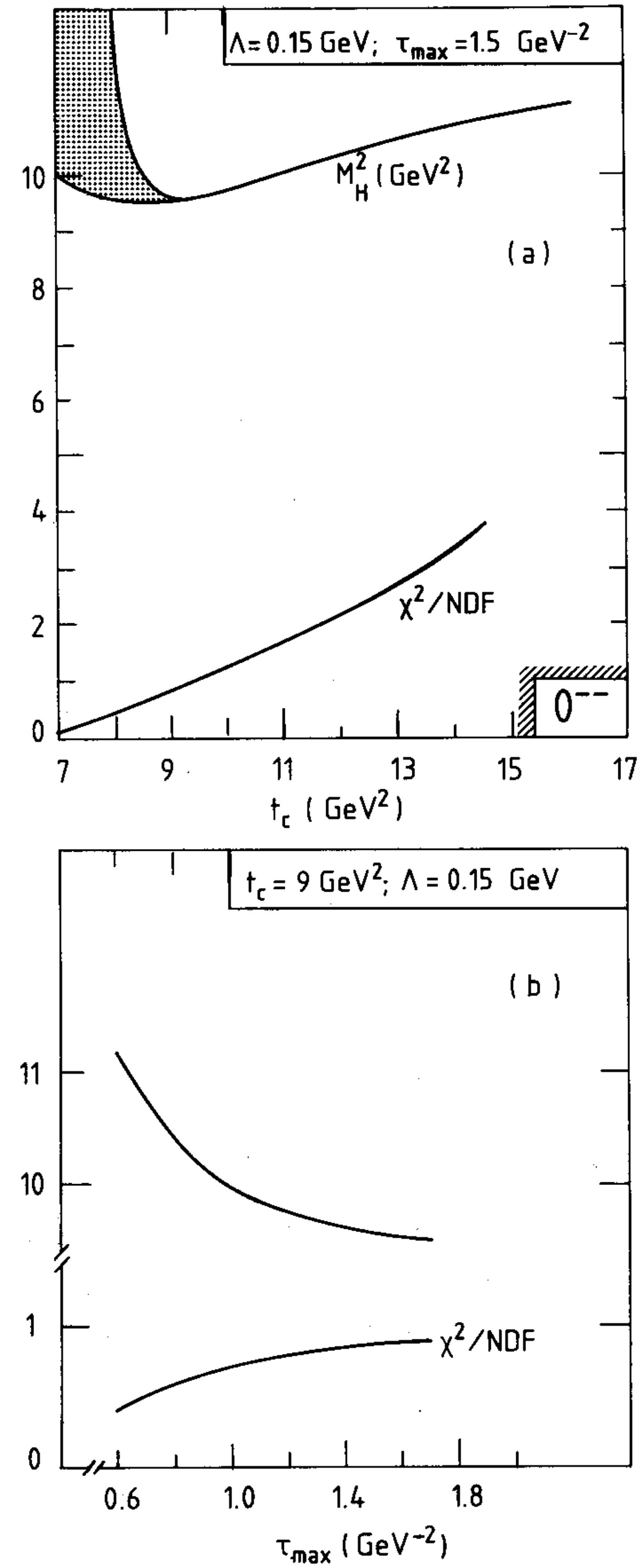

Λ= 0.15 GeV; τmax =1.5 GeV-2
10
8
6
4
2
0
M2H (GeV2)
(a)
χ2/NDF
0--
7
9
11
13
15
17
tc (GeV2)
tc = 9 GeV2; Λ = 0.15 GeV
(b)
11
10
1
0
χ2/NDF
0.6
1.0
1.4
1.8
τmax (GeV-2)

Fig. 2

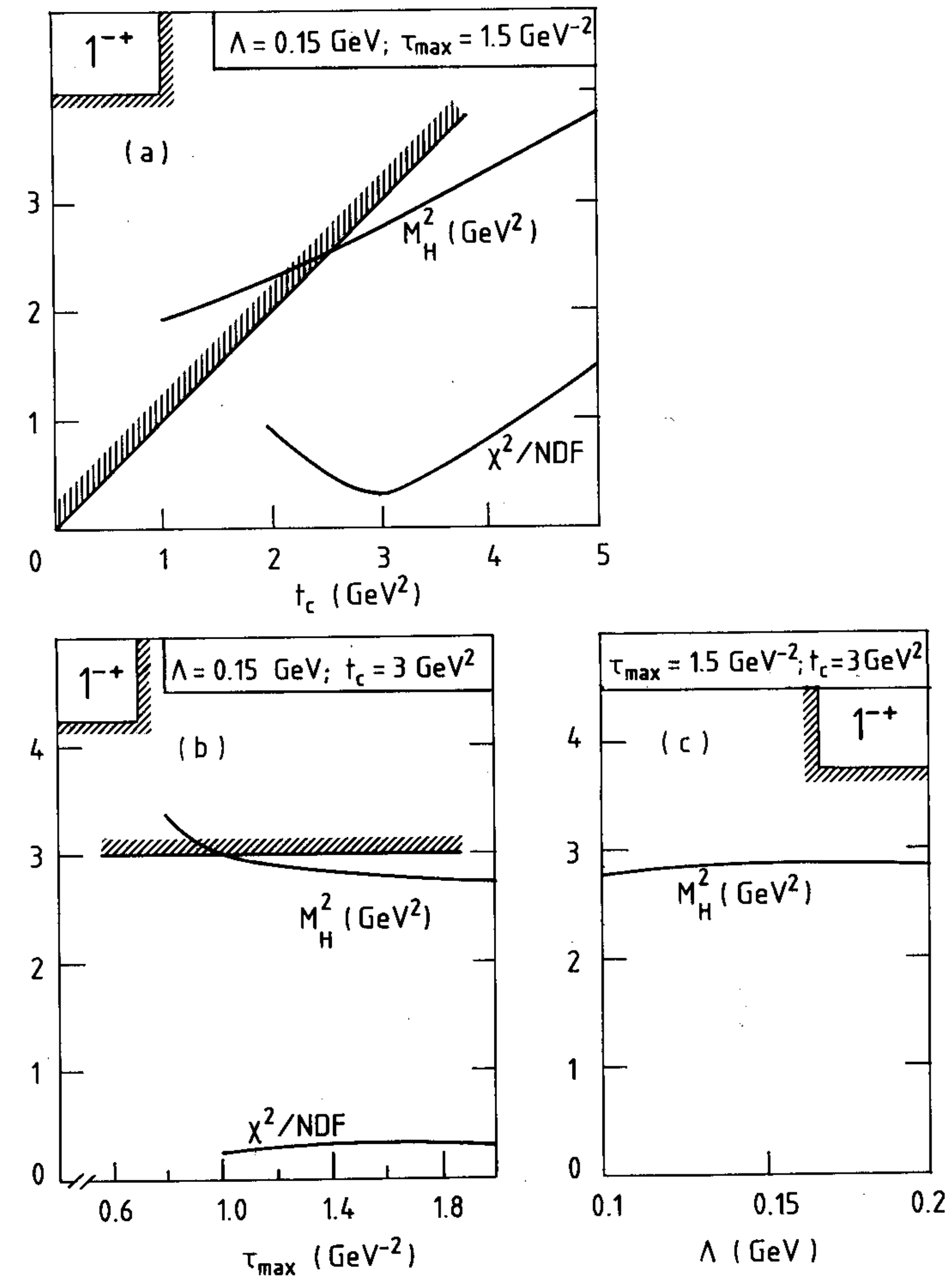

1⁻⁺
Λ = 0.15 GeV; τmax = 1.5 GeV⁻²
(a)
M_H² (GeV²)
χ²/NDF
0
1
2
3
4
5
tc (GeV²)
1⁻⁺
Λ = 0.15 GeV; tc = 3 GeV²
(b)
M_H² (GeV²)
χ²/NDF
0.6
1.0
1.4
1.8
τmax (GeV⁻²)
τmax = 1.5 GeV⁻²; tc = 3 GeV²
(c)
1⁻⁺
M_H² (GeV²)
0.1
0.15
0.2
Λ (GeV)

Fig. 3