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N = 1 AND N = 2 SUPERGRAVITIES COUPLED TO MATTER: SUPER-HIGGS EFFECT AND GEOMETRICAL STRUCTURE

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A B S T R A C T

In these lectures, we describe the spontaneous breakdown of supersymmetry in N = 1 and N = 2 supergravity theories coupled to matter multiplets (super-Higgs effect). Particular emphasis is devoted to those supergravity models which undergo spontaneous supersymmetry breaking with vanishing cosmological constant. Properties of the scalar potential and of the σ-model structure of extended supergravity theories are also discussed.

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ABSTRACT

In these lectures, we describe the spontaneous breakdown of supersymmetry in N = 1 and N = 2 supergravity theories coupled to matter multiplets (super-Higgs effect). Particular emphasis is devoted to those supergravity models which undergo spontaneous supersymmetry breaking with vanishing cosmological constant. Properties of the scalar potential and of the σ-model structure of extended supergravity theories are also discussed.

1. Super-Higgs effect in N = 1 supergravity

Spontaneously broken supersymmetric theories have recently received much attention as possible candidates for unified theories of the fundamental forces of Nature. In particular, they offer an interesting extension of the standard model with testable predictions if supersymmetry is broken at moderate energies, in the region of the Fermi scale[1]. The possibility of an approximated supersymmetry in the energy domain near the weak interaction scale has often been advocated, in connection with the so-called hierarchy problem[2] one encounters when vastly different scales of gauge symmetry breaking appear in a unified theory. It has been further observed that supergravity corrections to globally supersymmetric Lagrangians can be important at "low energies" ($E \sim 100$ GeV), if the primordial supergravity breaking scale M_S is intermediate between the Fermi scale M_W and the Planck mass M_P [3],

$$M_S \simeq O(\sqrt{M_W M_P}).$$

In particular, one may avoid unwanted mass relations of spontaneously broken globally supersymmetric theories, since the supergravity couplings give corrections to the scalar masses typically of the form

$$\delta m_i \simeq O(\kappa M_S^2).$$

The phenomenon yielding to this non-trivial interplay between gravity and "low-energy" physics is called the super-Higgs effect[4].

It is known, from the algebra of supersymmetry, that gravity coupled to global supersymmetry implies local supersymmetry. The gauge field of local supersymmetry is a spin 3/2 field $\psi_{\mu\alpha}$, which in absence of

supersymmetry breaking, is supposed to describe a massless spin $\frac{3}{2}$ particle, the gravitino. If supersymmetry is spontaneously broken, the super-Higgs effect occurs, namely the Goldstone mode is eaten up by the gravitino which becomes massive, with mass[4]

$$m_{3/2} = \sqrt{\frac{8\pi}{3}}\,\frac{M_S^2}{M_P}.$$

The gravitino mass depends on M_S quadratically. If $M_S = O((M_W M_P)^{\frac{1}{2}})$, then $m_{3/2} = O(M_W)$, and supergravity effects can become important in the region of the Fermi scale.

The relation between $m_{3/2}$ and M_S is valid in absence of a cosmological constant. This is possible in local supersymmetry, since the scalar potential is no longer positive definite, as it is the case in global supersymmetry.

The aim of this section is to discuss in some detail this phenomenon and its implications for the low-energy effective Lagrangian at energies comparable to the weak vector boson masses. It will appear that at these energies one can still neglect gravitational corrections, but not supergravity corrections which will manifest as a soft explicit breaking of global supersymmetry.

The general coupling of chiral and vector multiplets to N = 1 supergravity is specified by two functions of the complex scalar fields z^i contained in chiral multiplets[5,6]. An analytic function $f_{ab}(z) = f_{ba}(z)$, related to the Yang-Mills part of the Lagrangian, gives for the kinetic terms of the gauge fields,

$$-\frac{1}{4}(\mathrm{Re}\, f_{ab})F^a_{\mu\nu}F^{b\mu\nu} + \frac{i}{4}(\mathrm{Im}\, f_{ab})F^a_{\mu\nu}\tilde{F}^{b\mu\nu}. \tag{1.1}$$

(a,b are indices of the adjoint representation of the gauge group G). Then, a real function $\mathcal{G}(z,z^*)$, the Kähler potential, defines the scalar kinetic terms, given by*)

$$-\mathcal{G}^i_j\,(\partial_\mu z^j)(\partial^\mu z^*_i). \tag{1.2}$$

This function is gauge invariant:

$$\mathcal{G}_i (T^a)^i_j z^j = \mathcal{G}^i (T^a)^j_i z^*_j. \tag{1.3}$$

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*) We use the notation

$$\mathcal{G}_i = \frac{\partial\mathcal{G}}{\partial z^i},\quad \mathcal{G}^i = \frac{\partial\mathcal{G}}{\partial z^*_i},\quad \mathcal{G}^i_j = \frac{\partial^2\mathcal{G}}{\partial z^*_i\,\partial z^j},\ \ldots$$

The kinetic terms have a form characteristic of supersymmetric non-linear σ-models. They are invariant under Kähler transformations

$$\mathcal{G} \longrightarrow \mathcal{G} + F(z) + F^*(z^*) \,, \tag{1.4}$$

where F(z) is a gauge invariant analytic function. The scalar fields in N = 1 supergravity span a Kähler manifold. Coset spaces corresponding to Kähler manifolds are of the form $H/K \otimes U(1)$, where $K \otimes U(1)$ is a maximal (compact) subgroup of H. It is convenient to split the Kähler potential in two parts:

$$\mathcal{G}(z,z^*) = -J(z,z^*) + \ln|f(z)|^2 \,. \tag{1.5}$$

The gauge invariant superpotential f(z) appears in the part of the Lagrangian which is not invariant under (1.4), induced by the coupling to gravitation. $\mathcal{G}$ is, of course, invariant under the transformations

$$J \longrightarrow J + F'(z) + F'^*(z^*) \,,$$
$$f \longrightarrow \exp\left(F'(z)\right) \cdot f \,.$$

To discuss symmetry and supersymmetry breaking, we need the scalar potential V. The potential has two terms:

$$V = V_c + V_g \,. \tag{1.6}$$

The gauge potential V_g reads

$$V_g = \tfrac{1}{2}\left(\mathrm{Re}\, f_{ab}^{-1}\right) D^a D^b \,, \tag{1.7}$$

where the real [see eq. (1.3)] functions D^a are

$$D^a = g_a \mathcal{G}_j (T^a)^j_i \, z^i \,. \tag{1.8}$$

(g_a is the gauge coupling constant associated to the normalized generator T^a). V_g is invariant under Kähler transformations (1.4), and thus does not depend on the superpotential. The "chiral" potential[5,6] is

$$\begin{aligned} V_c &= \exp\mathcal{G}\left(\mathcal{G}_i \mathcal{G}^j (\mathcal{G}_i{}^j)^{-1} - 3\right) \\ &= -\exp(-J)\left((fJ_i - f_i)(f^*J^j - f^{*j})(J_i^j)^{-1} + 3|f|^2\right) . \end{aligned} \tag{1.9}$$

The Lagrangian contains also an interaction between the gravitino ψ_μ and a particular spin ½ state ψ_L of the form

$$-\bar{\psi}_{\mu R}\gamma^\mu \psi_L + \text{c.c.} \tag{1.10}$$

ψ_L is given by:

$$\psi_L = -\exp(\mathcal{G}/2)\mathcal{G}_i \chi_L^i + \tfrac{i}{2} D^a \lambda_L^a \,, \tag{1.11}$$

where χ_L^i and λ_L^a are the spin ½ states of chiral and vector multiplets, respectively. When the vacuum expectation value of the scalar fields (denoted by ⟨...⟩) is such that

$$\langle \exp(\mathcal{G}/2)\mathcal{G}_i \rangle \neq 0 \,, \tag{1.12}$$

or

$$\langle D^a \rangle \neq 0 \,, \tag{1.13}$$

the theory contains a Goldstone spinor η_L

$$\eta_L = -\langle \exp(\mathcal{G}/2)\mathcal{G}_i \rangle \chi_L^i + \tfrac{i}{2}\langle D^a \rangle \lambda_L^a \,,$$

which can be rotated away using a local supersymmetry transformation. The super-Higgs effect is effective and supersymmetry is spontaneously broken.

It is apparent from the potential (1.9) that, unlike in the global case, spontaneously broken supersymmetry does not imply $\langle V \rangle > 0$. This is fortunate since one can now obtain the super-Higgs effect in Minkowski space ($\langle V \rangle = 0$) as well as in de Sitter ($\langle V \rangle > 0$) or anti-de Sitter ($\langle V \rangle < 0$) space. The theory contains also a gravitino mass term, $m_{3/2}\, \bar{\psi}_{\mu L}\sigma^{\mu\nu}\psi_{\nu L}$, with

$$m_{3/2} = \langle \exp(\mathcal{G}/2) \rangle \,. \tag{1.14}$$

For unbroken supersymmetry, $m_{3/2}^2 = -\langle V \rangle/3$; when $\langle V \rangle \neq 0$, the parameter $m_{3/2}$ is not a physical mass; a massless gravitino in de Sitter or anti-de Sitter space does not mean $m_{3/2} = 0$.

A flat Kähler manifold, with

$$\mathcal{G}^i{}_j = -J^i{}_j = \delta^i_j \tag{1.15}$$

will lead to canonical scalar kinetic terms. Equation (1.15) implies

$$\mathcal{G} = z^i z_i^* + \ln|f(z)|^2 \,. \tag{1.16}$$

Vector kinetic terms can also be canonical, with the choice

$$f_{ab} = \delta_{ab} \,. \tag{1.17}$$

The minimal coupling, with canonical kinetic terms leads to the mass formula[5]:

$$\text{Supertrace } \mathcal{M}^2 = \sum_{J=0}^{3/2} (-)^{2J}(2J+1)m_J^2$$
$$= (n-1)\left(2m_{3/2}^2 - \kappa^2 D^a D^a\right) + 2g_a D^a \mathrm{Tr}\, T^a . \qquad (1.18)$$

The first term is a supergravity correction to the mass formula of global $N = 1$ supersymmetry, where only the last term is present. Equation (1.18) implies that the "mean squared mass" of the scalar fields is $m_{3/2}^2$. The phenomenological requirement of having scalars heavier than fermions is then naturally satisfied. In particular, in the attractive possibility where the gravitino scale is identified to the weak interaction scale, scalar partners of quarks and leptons will have masses of order M_W [1]). Notice, however, that the low-energy spectrum, when renormalization effects are taken into account, may be drastically different from the tree-level masses. One-loop corrections on scalar masses have been shown to be able to induce $SU(2)_L \times U(1)$ symmetry breaking at low energy, even if all scalars have positive tree-level "mean" squared masses of order $m_{3/2}^2 \sim M_W^2$ [3]).

The simplest example of a Kähler potential (for one single neutral chiral field z) for which the super-Higgs mechanism occurs is the Polony model[7]), for which the superpotential is

$$f(z) = \mu(z+\beta)/\kappa . \qquad (1.19)$$

The kinetic terms are canonical, i.e. $\mathcal{G}$ and f_{ab} are given by eqs. (1.16) and (1.17). The corresponding potential $V = V_c$ has a stationary point, which is a local minimum, with $\langle V \rangle = 0$ when β is tuned to take the value

$$\beta = (2-\sqrt{3})/\kappa . \qquad (1.20)$$

The gravitino mass is then related to the parameter μ by

$$m_{3/2}^2 = \mu^2 \cdot \exp(\sqrt{3}-1)^2 , \qquad (1.21)$$

and the two real scalars ($z = a + ib$) have masses

$$m_a^2 = 2\sqrt{3}\, m_{3/2}^2 ,$$
$$m_b^2 = 2(2-\sqrt{3})\, m_{3/2}^2 , \qquad (1.22)$$

which verify

$$m_a^2 + m_b^2 = 4m_{3/2}^2 , \qquad (1.23)$$

as derived from Eq. (1.18), when $n = 1$.

The simple form (1.19) can be easily generalized when several chiral multiplets are present. The superpotential reads[8])

$$f(z,y^i) = f(z)\left[1 + f(z)^{q-2}/\langle f(z)\rangle^{q-1}\right] \cdot h(y) ,$$
$$q = 1, 2, \ldots , \qquad (1.24)$$

where z is the field which induces the super-Higgs effect and y^i the other scalars (which verify $\langle y^i\rangle \ll \langle z\rangle \sim M_P$), $f(z)$ is the Polony potential [Eq. (1.19)] and $h(y)$ an arbitrary gauge invariant superpotential. One can now take the limit $\kappa \to 0$, with $m_{3/2}$ fixed, to get an effective Lagrangian valid at scales lower than M_P. With this class of superpotential, the field z consistently decouples in the effective theory; the interactions of the "hidden sector" z with the "visible sector" y^i are governed by κ. The effective potential has the form[8])

$$V_{eff}(y,y^*) = |\tilde{h}_i|^2 + (A-3)m_{3/2}(\tilde{h} + \tilde{h}^*)$$
$$+ \frac{1}{2} D^a D^a , \qquad (1.25)$$

with $A - 3 = \sqrt{3}(q-2)$, $\tilde{h}(y) = \exp\left[(\sqrt{3}-1)^2/2\right] h(y)$ and

$$\tilde{h}_i = \frac{d\tilde{h}}{dy^i} + m_{3/2}\, y_i^* . \qquad (1.26)$$

The effective theory is a supersymmetric gauge theory with chiral multiplets y^i, interacting through a superpotential $\tilde{h}$, softly broken by scalar interactions contained in V_{eff}. The effective theory is renormalizable as long as the superpotential $\tilde{h}(y)$ is a polynomial at most cubic in fields y^i. Notice that gaugino masses can be introduced simply, using non-minimal kinetic terms for the gauge fields; one could also obtain this way CP-violating $F_{\mu\nu}\tilde{F}^{\mu\nu}$ terms. Various $SU(3) \times SU(2) \times U(1)$ or unified models coupled to $N = 1$ supergravity have been constructed[1,3,8]), resulting in an effective theory of this form at scales below M_P.

Up to now, we have discussed Kähler potentials for which the gravitino mass is essentially a free parameter μ. One has then to choose $\mu \sim O(M_W)$ to obtain a stable hierarchy of particle interactions. Moreover, the cosmological constant $\langle V \rangle = \Lambda$ is zero due to a fine tuning of the parameters of G. An additional parameter (β in the Polony case) is in general needed only to obtain a vanishing Λ.

There is, however, an elegant way to circumvent these two unsatisfactory points: there exist non-trivial Kähler potentials for which the chiral potential V_c is identically zero[9,10]). Supersymmetry is, however, broken. Vacuum expectation values are not determined by the classical theory; the same holds for the gravitino mass. The cosmological constant is naturally zero. One can then take the low-energy limit ($\kappa \to 0$ with $m_{3/2}$ undetermined but fixed) to get an effective theory with softly broken supersymmetry. This limit exists as long as the "hidden sector" (for which $V_c \equiv 0$) effectively decouples from the visible sector which does not participate in the super-Higgs effect. Radiative corrections are then applied in the effective theory to determine the various scales of gauge symmetry breaking[11], which in general will be closely related to the gravitino mass. These scales are in fact proportional to $m_{3/2}$, but the proportionality constants are strongly model-dependent and can be many orders of magnitude away from one. The consistency of this scheme is submitted to the very strong assumption that one can apply radiative corrections on an effective theory whose main properties (mainly the potential) are not affected by possible quantum gravitational effects.

Since it is, in principle, sufficient to require zero chiral potential only in the direction of the singlet scalar z, let us first discuss "flat potentials" in the case of a single chiral multiplet coupled to N = 1 supergravity[9]. The chiral potential in terms of the Kähler potential can be written

$$V_c = 9\exp\left(\tfrac{4}{3}\mathcal{G}\right)\mathcal{G}_{z\bar z}^{-1}\frac{d^2}{dz\,dz^*}\exp\left(-\tfrac{1}{3}\mathcal{G}\right), \qquad \left(\mathcal{G}_{z\bar z} = \frac{d^2}{dz\,dz^*}\mathcal{G}\right) \tag{1.27}$$

and $V_c \equiv 0$ implies

$$\frac{d^2}{dz\,dz^*}\exp\left(-\tfrac{1}{3}\mathcal{G}\right) = 0. \tag{1.28}$$

The solution is

$$\mathcal{G} = -\ln\left(\varphi(z) + \varphi^*(z^*)\right)^3. \tag{1.29}$$

The scalar kinetic term $G_{z\bar z}(\partial_\mu z)(\partial_\mu z^*)$ is never canonical, and the gravitino mass is

$$m_{3/2} = \langle\left(\varphi(z) + \varphi^*(z^*)\right)^{-3/2}\rangle. \tag{1.30}$$

$m_{3/2}$ is undetermined but non-zero since

$$\langle\mathcal{G}_{z\bar z}\rangle = 3\langle|\varphi_z|^2\rangle\left(m_{3/2}\right)^{4/3} \neq 0. \tag{1.31}$$

The geometry of the Kähler manifold when $V_c \equiv 0$ is particular. From Eq. (1.28) one obtains

$$R_{z\bar z} = \frac{2}{3}\mathcal{G}_{z\bar z}, \tag{1.32}$$

for the curvature, defined by

$$R_{z\bar z} = \frac{d^2}{dz\,dz^*}\ln\mathcal{G}_{z\bar z}. \tag{1.33}$$

Equation (1.32) means that the Kähler manifold is an Einstein space (maximally symmetric space), i.e. that the scalar field z is a coordinate on the coset space SU(1,1)/U(1)*). The non-compact SU(1,1) invariance can be checked explicitly in the whole Lagrangian apart from the gravitino mass term. A simpler way to understand its appearance is to consider the superfield formulation of the theory, in which SU(1,1) is a linear symmetry acting on the chiral superfield and on the compensating multiplet, when condition (1.28) is applied[12].

The condition (1.32) can indeed be derived from the mass sum rules which holds for any spontaneously broken N = 1 supergravity theory with zero cosmological constant[13]

$$\begin{aligned}\text{Supertrace } \mathcal{M}^2 &= 2m_{3/2}^2\langle\mathcal{G}_z\mathcal{G}_{\bar z}R_{z\bar z}/\mathcal{G}_{z\bar z}^2\rangle \\ &= 2m_{3/2}^2\langle\left(\exp(-\mathcal{G})V + 3\right)R_{z\bar z}/\mathcal{G}_{z\bar z}\rangle,\end{aligned} \tag{1.34}$$

for one-chiral multiplet. When $V \equiv 0$ (and then $m_a = m_b = 0$), all values of the field z (n = 1) are stationary points of the potential, so (1.34) holds for all values of z, with the corresponding gravitino mass $m_{3/2}^2(z) = \exp\mathcal{G}(z)$. Equation (1.34) leads then immediately to the curvature constraint (1.32), since Supertrace $m^2 = 4m_{3/2}^2$.

It is interesting to notice[9] that the N = 1 Lagrangian for one chiral multiplet with vanishing potential corresponds, up to the gravitino mass term, to a particular truncation of N = 4 supergravity, which is known to possess an SU(1,1) non-compact global symmetry[14].

Vanishing chiral potentials for an arbitrary number n of chiral multiplets also exist. The generalization of the single field case is, however, not trivial due to the matrix structure of the Kähler metric $\mathcal{G}_i^j$.

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*) Or equivalently U(1,1)/U(1) × U(1).

Let us first consider two cases[31] which will be useful also in the $N = 2$ case we will discuss in the next section. It will be convenient to use the definition*)

$$\mathcal{G} = -\ln \tilde{Y}(z, z^*), \tag{1.35}$$

for the Kähler function. Then, one can easily rewrite the chiral potential (1.9) in terms of $\tilde{Y}$

$$V_c = \frac{1}{\tilde{Y}}\left(\frac{W}{W-1} - 3\right), \tag{1.36}$$

where

$$W = \frac{1}{\tilde{Y}} \frac{d\tilde{Y}}{dz^i}\left(\frac{d^2\tilde{Y}}{dz^i dz_j^*}\right)^{-1} \frac{d\tilde{Y}}{dz_j^*} \,. \tag{1.37}$$

The condition for vanishing potential is then

$$W = 3/2 \,. \tag{1.38}$$

The first class of solutions to this condition is obviously obtained when

$$\tilde{Y} = \sum_{i=1}^{n} \tilde{y}_{(i)}(z^i, z_i^*), \tag{1.39}$$

which implies

$$W \cdot \tilde{Y} = \sum_{i=1}^{n} \frac{d\tilde{y}_{(i)}}{dz^i} \frac{d\tilde{y}_{(i)}}{dz_i^*}\left(\frac{d^2\tilde{y}_{(i)}}{dz^i dz_i^*}\right)^{-1}$$
$$\equiv \sum_{i=1}^{n} W_{(i)} \tilde{y}_{(i)} \,.$$

The potential will be zero if

$$\sum_{i=1}^{n} W_{(i)} \tilde{y}_{(i)} = \frac{3}{2} \sum_{i=1}^{n} \tilde{y}_{(i)} \,.$$

This is the case when each function of one field $\tilde{y}_{(i)}$ gives itself a vanishing potential in the single field case, i.e. when $\tilde{y}_{(i)}$ has the form [see Eq. (1.29)]:

$$\tilde{y}_{(i)}(z^i, z_i^*) = \left(\varphi_{(i)}(z^i) + c.c.\right)^3. \tag{1.40}$$

*) $\tilde{Y}$ is related to the function Y, defined by Eq. (2.10) of Section 2, by $\tilde{Y} = Y/|h(z)|^2$, h(z) being given in Eq. (2.14).

Another class of solutions to condition (1.38) is obtained in the following way. Assume that $\tilde{Y}$ is a homogeneous function of degree δ of the combinations $\alpha z^i + \beta z_i^*$. ($\tilde{Y}$ should in fact also be real, but the following derivation is independent of that constraint.) Homogeneity means that

$$\frac{\partial^2 \tilde{Y}}{\partial z^i \partial z_j^*}(\alpha z^j + \beta z_j^*) = \beta(\delta - 1)\frac{\partial \tilde{Y}}{\partial z^i} \,,$$

$$\frac{\partial^2 \tilde{Y}}{\partial z^i \partial z_j^*}(\alpha z^i + \beta z_i^*) = \alpha(\delta - 1)\frac{\partial \tilde{Y}}{\partial z_j^*} \,,$$

$$\frac{\partial^2 \tilde{Y}}{\partial z^i \partial z_j^*}(\alpha z^i + \beta z_i^*)(\alpha z^j + \beta z_j^*) = \alpha\beta\delta(\delta - 1)\tilde{Y}.$$

Then:

$$W = \frac{1}{\tilde{Y}} \frac{(\alpha z^i + \beta z_i^*)(\alpha z^j + \beta z_j^*)}{\alpha\beta(\delta-1)^2} \frac{\partial^2 \tilde{Y}}{\partial z^i \partial z_j^*} = \frac{\delta}{\delta - 1} \,, \tag{1.41}$$

and the chiral potential is

$$V_c = \frac{\delta - 3}{\tilde{Y}} \tag{1.42}$$

V_c then vanishes when the function $\tilde{Y}$ is an arbitrary, homogeneous (real) function of degree 3, of a linear combination of z^i and z_i^*, like for instance $z^i + z_i^*$ or $i(z^i - z_i^*)$.

The last class of couplings of an arbitrary number of chiral multiplets with zero chiral potential we will mention here is in fact the most natural from a superfield point of view[12]. In this formalism, the condition $V_c \equiv 0$ can be solved using only one superfield, with scalar z, the coupling of the other fields, with scalars y^i, remaining unconstrained. This leads to the following class of Kähler potentials[11,12]:

$$\mathcal{G} = -\ln\left(\varphi(z) + \varphi^*(z^*) + g(y^i, y_i^*)\right)^3. \tag{1.43}$$

g is an arbitrary real function and φ is an analytic function of z only. One checks easily that this natural extension of Eq. (1.29) gives also $V_c \equiv 0$.

The possibility of vanishing potentials in the hidden sector is attractive, since it allows naturally to break supergravity with zero cosmological constant. Subsequent scales of gauge symmetry breakings in the effective "low-energy" theory are then obtained through "dimensional transmutation", induced by the radiative corrections to the potential in the flat directions. The structure of the soft breaking terms, resulting from the choice of G, is then crucial to fix the scales. In particular, no scale larger than $m_{3/2}$ (like M_P, for instance) should appear in the effective theory, even when radiative corrections are included. A necessary condition for the consistency of the mechanism is that the soft breaking terms are such that

$$\mathrm{Supertrace}\ \mathcal{M}^2 = 0 \tag{1.44}$$

in the effective theory. Such a condition is obviously not satisfied if one requires zero potential for the visible sector also. A class of models in which Eq. (1.44) is fulfilled has been constructed[15]. The corresponding Kähler manifold is of the type $U(N,1)/U(N) \times U(1)$, but the potential is no more flat.

2. Super-Higgs Effect in N = 2 Supergravity

Compared with the N = 1 case. N = 2 supergravity[16] coupled to N = 2 matter multiplets (i.e. multiplets containing states of spin up to one) shows several new features, which appear in general in extended supergravities. Extended supersymmetry algebras possess an internal global SU(N) symmetry acting on the spinorial charges which is a symmetry of the supergravity theory. There is then the possibility to enlarge the local symmetry of the theory be using either the vector fields contained in the graviton supermultiplet, which are sufficient to gauge the SO(N) subgroup [$SO(6) \times U(1)$ for N = 6], or additional vector multiplets or a combination of both options. The gauge group of a general N = 2 theory will then have the form $G \times G_{int}$, the internal part G_{int} being $O(2) \sim U(1)$ [17], or eventually $SU(2)$ [18] using additional vector multiplets. We will see that the gauging of this internal symmetry plays a crucial role when vector multiplets induce the super-Higgs effect.

The N = 2 supergravity and gravitino multiplets (with maximal helicity 2 and 3/2 respectively) do not contain scalar fields. N = 2 offers then a convenient way to study the scalar sector and supersymmetry breaking pattern of larger N theories, which will correspond to some coupling (fixed by invariances) of N = 2 matter multiplets. An important aspect of the N = 2 theory, in particular if one is concerned with the σ-model structure[19] of the scalar sector of extended supergravities, is that it admits irreducible PCT self-conjugate multiplets. Scalars of such multiplets have an almost complex structure only, and the scalar manifold will not be Kählerian.

N = 2 matter multiplets, like in the N = 1 case, are of two kinds. Both, however, contain scalar fields and generate a characteristic scalar potential. Supersymmetry and gauge symmetry breaking can then be induced by using both sorts of matter multiplets.

To be more specific, vector multiplets[20] contain, as partners of each massless gauge field, two Majorana spinors (gauginos) and a complex scalar:

$$(A_\mu^a,\ \Omega_\pm^a,\ z^a)\ ,\qquad a = 1, \ldots, \dim G$$

All these fields belong to the adjoint representation of the gauge group G. Hypermultiplets[21] can be constructed from the action of supersymmetric charges on a Clifford vacuum having helicity $+\frac{1}{2}$ and belonging to some irreducible representation $\underline{r}$ of G. Two cases arise[22]: if $\underline{r}$ is pseudoreal, then PCT invariance is satisfied without doubling of states. The hypermultiplet contains two-component spinors in representation $\underline{r}$ and real scalars transforming according to $(\underline{r},\underline{2})$ of $G \times SU(2)$ [$SU(2)$ is the invariance of N = 2 supersymmetry algebra]. Notice that the dimension of pseudoreal representations is always even. Pseudoreal hypermultiplets do not admit N = 2 supersymmetric mass terms; this fact allows to construct finite Yang-Mills theories with some massless fermions[23]. On the contrary, if $\underline{r}$ is real or complex, PCT invariance will require doubling the states. The hypermultiplet will then contain Dirac fermions in the representation $\underline{r}$ and real scalars transforming according to $(\underline{r}+\bar{\underline{r}},\underline{2})$ of $G \times SU(2)$.

The scalar fields of the two sorts of matter multiplets will be coordinates on two classes of manifolds. Scalars from the vector multiplets, which are complex, will live on a Kähler manifold. Cosets corresponding to Kähler manifold have the form $G/H \otimes U(1)$, where $H \otimes U(1)$ is a maximal (compact, to avoid ghosts) subgroup of G. Scalars in hypermultiplets are in SU(2) doublets. They will span quaternionic manifolds[25,26]. The corresponding coset spaces have the structure $G/H \otimes SU(2)$; $H \otimes SU(2)$ is a maximal (compact) subgroup of G such that coset coordinates transform according to $(\underline{R}_H,\underline{2})$ of $H \otimes SU(2)$, which is precisely what is required for the scalar fields. The list of quaternionic coset spaces can be found in Ref. 26. Notice that $\underline{R}_H$ reads $\underline{r} + \bar{\underline{r}}$, with $\underline{r}$ complex, only for Grassmannian manifolds $SU(n,2)/SU(n) \otimes U(1) \otimes SU(2)$. These manifolds are also Kählerian.

What are the implications of the multiplet structure and transformation properties of the theory for supersymmetry breaking pattern? Gravitinos are in an SU(2) doublet. That is also the case for the scalars of hypermultiplets. The complex scalars in vector multiplets are, however, SU(2) singlets. This fact implies that the number of unbroken supersymmetries allowed by vacuum expectation values of scalars belonging to vector multiplets is zero of two, since SU(2) remains unbroken. The massive gravitino multiplet of N = 1 Poincaré supersymmetry contains two massive vector fields. Then, if we want to end up with unbroken N = 1 supersymmetry we need to couple both hypermultiplets and vector multiplets to N = 2 supergravity[27]. There is, however, a general argument which forbids the breaking of N = 2 supergravity into Poincaré N = 1 supergravity [with zero cosmological constant[28]]. The only allowed breaking pattern is then into N = 0. Notice, however, that the masses of the two gravitinos will be in general different in presence of hypermultiplets.

We now want to investigate some aspects of the super-Higgs effect for general couplings of n vector multiplets to N = 2 supergravity.

The action for such couplings has been established by de Wit et al.[24, 29], using N = 2 tensor calculus. The scalar sector we are interested in can be obtained in the following way. The action is based on a gauge invariant function $F(X^I)$ of n + 1 variables (I = 0, 1, ..., n), required to be homogeneous of degree two. Conformal invariance is fixed by the gauge choice

$$X^I N_{IJ} X^{*J} = 1, \tag{2.1}$$

where

$$N_{IJ} = \frac{1}{4}\left(F_{IJ} + F^*_{IJ}\right) = \frac{1}{2} \mathrm{Re}\left(\frac{d^2 F}{dX^I dX^J}\right). \tag{2.2}$$

It is then convenient to define scalar fields z^I by

$$z^I = X^I / X^0, \tag{2.3}$$

so that $z^0 = 1$. We further define

$$Y(z,z^*) = (X^0)^{-2} = z^I N_{IJ} z^{*J}. \tag{2.4}$$

Notice that Y and N_{IJ} remain invariant under the transformation

$$F \longrightarrow F + i C_{IJ} X^I X^J \tag{2.5}$$

where C_{IJ} are real constants. This remains true for the whole theory. [X^0 can certainly be chosen real due to Eq. (2.1).] Alternatively, one can define the coupling in terms of the fields $z^a = z^I$, a = 1, ..., n, with an arbitrary function f(z) related to F by:

$$X^{0\,-2} F(X^I) = F(z^I) = F(1, z^a) \equiv f(z^a). \tag{2.6}$$

Then:

$$N_{00} = \frac{1}{2}\left(f - f_a z^a + \frac{1}{2} f_{ab} z^a z^b\right) + \text{c.c.}, \tag{2.7}$$

$$N_{0a} = \frac{1}{4}\left(f_a - f_{ab} z^b\right) + \text{c.c.}, \tag{2.8}$$

$$N_{ab} = \frac{1}{4} f_{ab} + \text{c.c.}, \tag{2.9}$$

$$Y = \frac{1}{2}\left(f + f^* - \frac{1}{2}\left(f_a - f_a^*\right)\left(z^a - z^{*a}\right)\right). \tag{2.10}$$

Since the fields z^a are partners of the gauge bosons in the vector multiplets, they transform according to the adjoint representation of the gauge group.

As we have already mentioned, the SO(2) symmetry of the N = 2 theory can be gauged*). The gauge field is in general an arbitrary linear combination of all abelian vector fields of the theory, including the one belonging to the gravity multiplet. The corresponding scalar combination is

$$g_I X^I = X^0 \left(g_0 + g_a z^a\right). \tag{2.11}$$

The real coefficients g_I play the role of gauge coupling constants. The case

$$g_I = g' \delta_{I0} \tag{2.12}$$

means that the SO(2) gauge field belongs to the supergravity multiplet.

Scalar fields z^a, like scalars of N = 1 chiral multiplets, are coordinates on a Kähler manifold. The scalar kinetic Lagrangian and the potential will then be obtained from the Kähler function $\mathcal{G}(z,z^*)$. In the N = 2 case, we find[30]

$$\mathcal{G}(z,z^*) = -\ln Y(z,z^*) + \ln |h(z)|^2 \tag{2.13}$$

*) More general gaugings, in particular of the whole SU(2), can also be envisaged[18, 29].

and the superpotential h(z) is

$$h(z) = 2\sqrt{2}\, g_I z^I = 2\sqrt{2}\,(g_0 + g_a z^a)\,; \tag{2.14}$$

h(z) is then constant when the SO(2) gauge field is member of the gravity multiplet and vanishes when SO(2) is not gauged. The chiral part of the potential is then

$$\begin{aligned} V_c &= -8 g_I g_J (N^{-1})^{IJ} - 16 |g_I X^I|^2 \\ &= -8 g_I g_J (N^{-1})^{IJ} - 16\, Y^{-1} |g_0 + g_a z^a|^2 . \end{aligned} \tag{2.15}$$

To obtain the gauge potential we first need the kinetic terms of the gauge fields. They are given by

$$-\frac{1}{4}\, \mathrm{Re}\, \tilde{f}_{IJ}\, F^I_{\mu\nu} F^{J\mu\nu}\,, \tag{2.16}$$

with

$$\tilde{f}_{IJ} = -\frac{1}{4} F_{IJ} + \frac{(N_{IK} X^{*K})(N_{JL} X^{*L})}{X^{*K} N_{KL} X^{*L}}\,. \tag{2.17}$$

$F^0_{\mu\nu}$ is the field strength of the vector field in the gravity multiplet. Notice that the function $\tilde{f}_{IJ}$ is now dependent on z and z^*. The z^* dependence, which was not present in N = 1, is related to the effect of the second N = 1 gravitino multiplet necessary to form the N = 2 theory. The second term, however, does not contribute to the gauge potential V_g, due to gauge invariance and transformation properties of the scalar fields. One finds

$$V_g = \frac{1}{2}\, \mathrm{Re}\, \tilde{f}^{-1}_{ab} D^a D^b\,, \tag{2.18}$$

with, like in the N = 1 case,

$$\begin{aligned} D^a &= g \frac{dG}{dz^c} C^a{}_{cd} z^d \\ &= g C^a{}_{cd} N_{cI} X^{*I} X^d . \end{aligned} \tag{2.19}$$

Then:

$$V_g = g^2 N_{ab} (C^a{}_{cd} X^c X^{*d})(C^b{}_{ek} X^e X^{*k}). \tag{2.20}$$

g is the coupling constant of the non-abelian part of the gauge group (assumed simple for simplicity) and $C^a{}_{bc}$ are its structure constants. The scalar fields which are partners of abelian gauge fields do not appear in V_g, due to the vanishing of the corresponding structure constants.

Notice that the N = 1 formulation[30] of the N = 2 theory is simply given by Eqs. (2.13), (2.14) and (2.17).

It will be useful to remark that one can reparametrize the theory in such a way that $g_I = 0$ except for g_0 [31]. This is done with the help of real, linear transformations, acting on the scalar partners of abelian vector fields, according to

$$\begin{aligned} X^I &\longrightarrow U^I{}_J X^J\,, \\ g_I &\longrightarrow g_J (U^{-1})^J{}_I\,, \\ N_{IJ} &\longrightarrow (U^{-1})_I{}^K N_{KL} (U^{T-1})^L{}_J\,. \end{aligned} \tag{2.21}$$

Such a tranformation does not act on the gauge potential since abelian multiplets do not appear in V_g. These transformations leave in fact F(x) and $g_I X^I$ form invariant. Theories related by (2.21) are then equivalent, although expressed in terms of different parameters. This will allow us to consider in general the simpler case $g_I = g' \delta_{I0}$ for which the chiral potential is

$$V_c = -8 g'^2 \left((N^{-1})^{00} + 2 Y^{-1} \right). \tag{2.22}$$

We still need to specify what are the conditions for unbroken supersymmetry[29] at a stationary point of the potential. It is convenient to split the potential into two parts

$$V = V_+ + V_-\,, \tag{2.23}$$

with

$$V_+ = V_g + g_I \mathcal{H}^I\,, \tag{2.24}$$

and

$$V_- = -24|g_I X^I|^2. \tag{2.25}$$

$\mathcal{H}^I$ is given by

$$\mathcal{H}^I = 8\left(X^I(g_J X^{*J}) - (N^{-1})^{IJ} g_J\right). \tag{2.26}$$

If negative-metric states are not present, one easily shows that

$$V_+ \geq 0.$$

Notice that V_-, in the N = 1 formulation given by Eqs. (2.13) and (2.14), reads

$$V_- = -3\exp \mathcal{G} \leq 0, \tag{2.27}$$

and V_+ is then the first term in the N = 1 chiral potential given by Eq. (1.9). Then, supersymmetry is preserved if

$$\langle \mathcal{H}^I \rangle = 0, \tag{2.28}$$

which implies as usual $\langle V_+ \rangle = 0$. In terms of fields z^a, these conditions read:

$$\langle \mathcal{H}^0 \rangle = 0 : \langle g_0 + g_a z^a \rangle = \langle Y (N^{-1})^{0I} g_I \rangle, \tag{2.29}$$

$$\langle \mathcal{H}^a \rangle = 0 : \qquad \langle z^a \rangle = \frac{(N^{-1})^{aI} g_I}{(N^{-1})^{0J} g_J}. \tag{2.30}$$

The vacuum expectation values of z^a are real when supersymmetry is not broken. The second set of conditions,

$$\langle D^a \rangle = g C^a{}_{bc} \langle X^b \rangle \langle X^{*c} \rangle = 0, \tag{2.31}$$

are then automatically satisfied. The condition $\langle D^a \rangle = 0$ is, however, weaker than Eq. (2.28). It only implies that $C^a{}_{bc}(\mathrm{Re}X^a)(\mathrm{Im}X^b) = 0$. In terms of the Kähler function $\mathcal{G}$, one finds

$$\mathcal{H}^I = -8(g_L z^{*L})(N^{-1})^{IK} \frac{d\mathcal{G}}{dz^{*K}}. \tag{2.32}$$

Notice also that the cosmological constant Λ is given by

$$\Lambda = \langle V_- \rangle = -24(g_0 + g_a \langle z^a \rangle)^2 / \langle Y \rangle, \tag{2.33}$$

at a supersymmetry preserving point. Λ is then negative for unbroken N = 2 supersymmetry when the vector field of the gravity multiplet participates in SO(2) gauging.

N = 2 supersymmetry will be broken (into N = 0) when $\langle \mathcal{H}^I \rangle \neq 0$ for some value of I. The corresponding gravitino mass term is, like in N = 1 case:

$$-m_{3/2}\, \delta_{ij}\, \overline{\psi}_{\mu i}\, \sigma^{\mu\nu} \psi_{\nu j}, \quad ij = 1,2, \tag{2.34}$$

with

$$m_{3/2} = \langle \exp(\mathcal{G}/2) \rangle = |2\sqrt{2}\, g_I \langle X^I \rangle|. \tag{2.35}$$

Our aim now is to analyse some of the aspects of the Higgs and super-Higgs effects in the N = 2 theory we have just described.

As a first example, consider the so-called minimal coupling defined by

$$F(X) = (X^0)^2 - X^a X^a, \tag{2.36}$$

for which

$$N_{IJ} = 2\delta_{I0}\delta_{J0} - \delta_{IJ}, \tag{2.37}$$

or equivalently by

$$f(z) = 1 - z^a z^a. \tag{2.38}$$

F(x) is certainly invariant under SO(1,n). The gauge choice (2.1) is then

$$|X^0|^2 - X^a X^{*a} = 1, \tag{2.39}$$

which is precisely the SU(1,n)-invariant constraint of non-compact CP^n models. The corresponding Kähler manifold is the coset space SU(1,n)/SU(n) × U(1). It follows from (2.4) that

$$Y = 1 - z^a z^{*a} = \frac{1}{1 + X^a X^{*a}}. \tag{2.40}$$

Choosing the SO(2) gauge field to belong to the gravity multiplet, i.e. $g_a = 0$, $g_0 = g'$, we get the following scalar potential

$$V = -8g'^2\left[1 + 2(1 - z^a z^{*a})^{-1}\right] + g^2(1 - z^a z^{*a})^{-2}\left(C^a{}_{bc} z^b z^{*c}\right)^2 \tag{2.41}$$

deduced from the Kähler function [see Eq. (2.13)]

$$G(z, z^*) = \ln\left(\frac{8g'^2}{1 - z^a z^{*a}}\right). \tag{2.42}$$

Equivalently, using X variables in a matrix notation:

$$V = -24g'^2 - 16g'^2 \mathrm{Tr}(XX^+) + g^2 \mathrm{Tr}([X, X^+]^2). \tag{2.43}$$

The stationary points of this polynomial potential can be analysed[32]. We first split the complex matrix X into

$$X = A + iB \,, \tag{2.44}$$

with A and B hermitean. Then, from the minimum equation, we get:

$$\mathrm{Tr}(\langle A\rangle\langle B\rangle) = 0 \,, \tag{2.45}$$

$$\mathrm{Tr}(\langle A\rangle^2) = \mathrm{Tr}(\langle B\rangle^2) \,, \tag{2.46}$$

$$g^2 \mathrm{Tr}([\langle X\rangle, \langle X^+\rangle]^2) = 16g'^2 \mathrm{Tr}(\langle A\rangle^2), \tag{2.47}$$

$$\langle V\rangle = -24g'^2\left[1 + \frac{2}{3}\mathrm{Tr}(\langle A\rangle^2)\right]. \tag{2.48}$$

All stationary points have negative cosmological constant. Equation (2.47) indicates that all supersymmetry breaking solutions break simultaneously the gauge symmetry. For the only supersymmetric vacuum, $\langle z\rangle = 0$, the potential can be written

$$V = -24g'^2\left(1 + \frac{2}{3} z^a z^{*a}\right) + O(z^4). \tag{2.49}$$

All scalar fields receive a negative "mass squared" term, $-2/3\,\Lambda$, in term of the cosmological constant Λ. This is precisely the conformal "mass term" required for effectively massless scalar fields in Anti-de Sitter space. This stationary point is, however, (classically and locally) stable due to the stability condition[33]

$$\frac{m^2}{\Lambda} < \frac{3}{4} \,, \tag{2.50}$$

for the eigenvalues m^2 of the "mass matrix" obtained from the quadratic terms of the potential.

The gauge symmetry breaking patterns corresponding to stationary points (recall that N = 2 is always broken into N = 0) can be characterized in the following way[32]. For a gauge group G = SU(N), there is a stationary point of V with residual symmetry

$$SU(m_1) \times SU(m_2) \times \ldots \times SU(m_N) \times U(1)^{p-1} \,,$$

where $m_1, \ldots, m_N$ are all possible sets of non-negative integers such that

$$\sum_{n=1}^{N} n\, m_n = N \,,$$

and p is the number of non-zero m_n's. The case $m_1 = N$ corresponds to $\langle z\rangle = 0$ and for $m_N = 1$, all gauge symmetries and supersymmetries are broken. Stability of stationary points can, in general, be ensured, the condition (2.50) leading to a lower bound of order 1 on the ratio g'^2/g^2. In particular, for $N \geq 7$, $N \neq 10$, there is always an SU(3) × × SU(2) × U(1)-invariant stationary point.

Notice that for the minimal coupling case, positivity of the kinetic energies is automatically satisfied due to the CP^n structure of the theory.

The minimally coupled theory does not possess any classical vacuum with zero cosmological constant. This is, however, not the case when more complicated couplings are considered. Demanding that, at a stationary point of the potential:

$$\left\langle \frac{dV}{dz^a} \right\rangle = 0 \,,$$

the cosmological constant $\Lambda = \langle V\rangle$ is also zero leads to severe constraints on the possible coupling functions f(z), in particular when the stability requirements are also imposed. A general analysis of this problem can be found in Ref. 31, which is the basis for the rest of this section.

Before coming to examples of couplings which exhibit N = 2 super-Higgs effect with zero cosmological constant, let us mention a general property of the chiral potential, Eq. (2.15). For all stationary points of the chiral potential, one finds that:

$$\left(\frac{d^2V}{dz^a dz^{*b}} - 2V\mathcal{G}_{ab}\right)\left(\mathcal{H}^b - z^{*b}\mathcal{H}^0\right) = 0 \qquad (2.51)$$

($\mathcal{G}_{ab} = d^2\mathcal{G}/dz^a dz^{*b}$ is the metric of scalar kinetic terms). This simple result indicates that, for instance, all flat potentials which break supersymmetry are zero potentials, and that a stationary point with $V = 0$ is stable only if some scalars are massless. These two observations reinforce our interest in flat potentials, as a necessary ingredient for a "realistic" N = 2 supergravity breaking. In the case of vanishing chiral potential, the trace of the (squared) mass matrix of the 2n spin ½ states in the n vector multiplets is

$$\mathrm{Tr}\, m^2_{1/2} = 4(n-1)\, m^2_{3/2} + \text{gauge terms}. \qquad (2.52)$$

This indicates that these Majorana states receive a mean squared mass $2m^2_{3/2}$ as a consequence of the super-Higgs effect. (Recall that two spin ½ states are eaten up by the two massive gravitinos.)

Couplings to N = 2 supergravity of a single vector multiplet, with scalar field z, and giving rise to vanishing potentials are easily found. As we learned from N = 1, we need a Kähler function $\mathcal{G}$ of the form

$$\mathcal{G} = -\ln\left(Y/8g'^2\right), \qquad (2.53)$$

in the "gauge" $g_0 = g'$, $g_1 = 0$, with

$$Y = \left(\varphi(z) + \varphi^*(z^*)\right)^3 \qquad (2.54)$$

Thus, we have to find all function f(z) able to give such a form to the corresponding Y. The first remark is that

$$\frac{d^2Y}{dz\,dz^*} = \frac{1}{4}\frac{d^2f(z)}{dz^2} + \text{c.c.}$$
$$= 6\frac{d\varphi}{dz}\frac{d\varphi^*}{dz^*}(\varphi+\varphi^*). \qquad (2.55)$$

This equation leads to only two solutions, which are apparent using:

$$\frac{d^4Y}{dz^2dz^{*2}} = 0 = 3\frac{d^2\varphi}{dz^2}\frac{d^2\varphi^*}{dz^{*2}}\left(\varphi + \left(\frac{d\varphi}{dz}\right)^2\Big/\frac{d^2\varphi}{dz^2} + \text{c.c.}\right). \qquad (2.56)$$

We have either

$$\frac{d^2\varphi}{dz^2} = 0, \qquad (2.57)$$

leading to

$$Y_1 = -ia(z-z^*)^3, \qquad (2.58)$$

where a is a real constant, or

$$\frac{d}{dz}\left(\varphi\frac{d\varphi}{dz}\right) = \text{imaginary constant}, \qquad (2.59)$$

leading to

$$Y_2 = \left(\sqrt{bz+c} + \sqrt{bz^*+c}\right)^3, \qquad (2.60)$$

where b and c are real constants. These two solutions are obtained respectively from functions f(z) given by

$$f_1(z) = 4iaz^3, \qquad (2.61)$$

and

$$f_2(z) = 8(bz+c)^{3/2}, \qquad (2.62)$$

up to a second order polynomial with imaginary coefficient which does not contribute to Y [see Eq. (2.5)]. With a general gauging of SO(2), the two couplings with vanishing potentials are given by

$$F_1(X^0, X^1) = i\frac{(\alpha_I X^I)^3}{g_I X^I} + iC_{IJ}X^I X^J, \qquad (2.63)$$

and

$$F_2(X^0, X^1) = 4(g_I X^I)^{1/2}(\alpha_J X^J)^{3/2} + iC_{IJ}X^I X^J, \qquad (2.64)$$

respectively, (α_I, g_I and C_{IJ} are real constants). In both cases, there exists a domain for which the kinetic energies are positive. Notice that, while in the N = 1 case the constraint of zero potential was solved with an arbitrary function of z, the N = 2 case is much more contrived: there are only two solutions, the only arbitrariness being the value of the real parameters a, b, c. The SU(1,1) invariance of the action found for N = 1 is also present in the N = 2 case, and the scalar manifold is also SU(1,1)/U(1).

Vanishing chiral potentials can also be found when an arbitrary number of vector multiplets are coupled to N = 2 supergravity. We will consider here two classes of such couplings which make use of the results discussed in the case of N = 1 supergravity.

For the first class, observe that if the function f(z) is the sum of functions $f_{(a)}$ of only one field z^a,

$$f = \sum_{a=1}^{n} f_{(a)}(z^a), \tag{2.65}$$

then

$$Y = \sum_{a=1}^{n} Y_{(a)}, \tag{2.66}$$

with

$$Y_{(a)} = \frac{1}{2}\left[f_{(a)} + f^*_{(a)} - \frac{1}{2}\left(\frac{df_{(a)}}{dz^a} - \frac{df^*_{(a)}}{dz^{*a}}\right)(z^a - z^{*a})\right]$$

like in Eq. (2.10). We can then use the result derived in Section I [see Eqs. (1.39) and (1.40)]: the chiral potential will vanish when each $f_{(a)}$ is one of the two solutions to the single field case, given in Eqs. (2.61) and (2.62), assuming $g_a = 0$, $g_0 \neq 0$. Alternatively, using X^I variables, the chiral potential will vanish when

$$F(X^I) = \sum_{a=1}^{n} F_{(a)}(X^0, X^a), \tag{2.67}$$

each function $F_{(a)}$ being either Eq. (2.63) or Eq. (2.64).

We have also shown in Section 1 that zero potentials occur when the function exp $(-\mathcal{G})$ is homogeneous of order 3 in the combinations $i(z^j - z^*_j)$. Such a function can easily be obtained in our N = 2 case if we choose ($g_a = 0$, $g_0 \neq 0$):

$$f(z) = 4i d_{abc} z^a z^b z^c, \tag{2.68}$$

where d_{abc} are real coefficients. We obtain

$$Y = -i d_{abc}(z^a - z^{*a})(z^b - z^{*b})(z^c - z^{*c}), \tag{2.69}$$

and the chiral potential is zero. For a general SO(2) gauging, this second class of couplings is generated by:

$$F(X) = \frac{i d_{IJK} X^I X^J X^K}{g_L X^L}. \tag{2.70}$$

This form of F generalizes the single field solution given in Eq. (2.63).

The coefficients d_{abc} are, in principle, arbitrary. However, f and Y have to be gauge invariant. There are different possibilities: if we underline indices corresponding to abelian vector multiplets, $d_{\underline{abc}}$ is arbitrary, $d_{\underline{ab}c}$ is zero, $d_{\underline{a}bc}$ is proportional to the Killing metric η_{bc} of the non-abelian group; $\bar{d}_{abc}$ can be non-zero only if the three indices correspond to gauge multiplets belonging to the adjoint representation of the same simple group. The adjoint representations of SU(N) groups only possess a cubic symmetric invariant. The coefficients d_{abc} exist then for each simple factor G_i = SU(N) of the gauge group. The invariances of f and Y can, however, be larger than the gauge group, the vector multiplets being embedded in some representation possessing a cubic invariant.

This is in particular the case in the very restricted class of models in which the gauge choice (2.1), with a cubic function $F(X^I)$, gives rise to a non-linear σ-model for the scalars. The coset space G/H are obtained[34] from the requirement that G possesses an irreducible representations $\underset{\sim}{R}$ which decomposes into

$$\underset{\sim}{R} = \underset{\sim}{1} + \underset{\sim}{1} + \underset{\sim}{r}$$

with respect to the maximal compact subgroup H of G, $\underset{\sim}{r}$ being the representation of H classifying the elements of the coset $\tilde{G}/H$; $\underset{\sim}{r}$ is further required to possess a cubic H-invariant tensor. $\underset{\sim}{R}$ will classify the fields X^I. Interestingly enough, these models can be obtained by dimensional reduction of N = 2 supergravity in 5 dimensions coupled to abelian vector multiplets[34]. The resulting four-dimensional theory has a scalar potential given by Eq. (2.15), with Y as given in Eq. (2.69). If a symmetric coefficient c^{abc} exist such that[31]

$$d_{i(ab} c^{ifg} d_{cd)f} = \delta^g_{(a} d_{bcd)}, \tag{2.71}$$

then the potential can be written:

$$V = \frac{1}{3} c^{abc} g_a g_b h_c, \tag{2.72}$$

with

$$h_c = d_{cab}(z^a - z^{*a})(z^b - z^{*b})/Y. \tag{2.73}$$

Equation (2.72) is precisely the potential obtained in Ref. 34. It can be made zero, if the condition

$$C^{abc} g_a g_b = 0 \tag{2.74}$$

can be satisfied.

3. On the Sigma Model Structure of Extended Supergravities

A very intriguing property of extended supergravity theories ($N \geq 4$) is the "automatic" appearance of hidden, non-compact symmetries[19]. These global invariances, acting on the bosonic sector, manifest themselves as duality transformations of the abelian field strengths, and are non-linearly realized in the scalar sector which shows the structure of non-linear σ-models. These non-compact symmetries are broken in gauged supergravity. The scalar fields are then coordinates on a coset manifold G/H, G being the non-compact global symmetry group and H its maximal compact subgroup. H is then a local symmetry, with composite, non-propagating, gauge fields. In N-extended supergravities, $H = U(N)$, $N \leq 6$ and $H = SU(8)$ for $N = 8$. The whole theory is found to be locally H-invariant, since all fields in gravity multiplets can, in fact, be classified in SU(N) representations, due to the supersymmetry algebra. The appearance of such local symmetries could in fact be crucial if "low-energy" particle interactions somehow rely on extended supergravity, at least if the composite gauge fields become propagating due to the dynamics of the theory.

$N = 1$ and 2 supergravities coupled to matter multiplets show many resemblances with the $N \geq 4$ theories. The scalar sector exhibits also a σ-model structure. As discussed in previous sections, the scalar manifold is Kählerian for $N = 1$ chiral multiplets and $N = 2$ vector multiplets, and quaternionic for $N = 2$ hypermultiplets. The supersymmetry algebra is, however, not large enough to restrict further the scalar sector. Non-compact symmetries, like SU(1,1), also occur in these theories. Duality properties of the $N = 2$ theory have also been investigated[29].

The appearance of hidden symmetries is not understood. σ-models naturally occur in theories obtained via dimensional reduction. There is, however, no reason to believe in the physical significance or necessity of this mechanism. Moreover, invariances of the supergravity theories are much larger than those obtained by dimensional reduction.

Kaluza-Klein theories have not shed up to now light on this problem. In this section, we just want to follow a purely group theoretical approach to classify the invariances of the scalar sector, accepting as a matter of life that the σ-model structure has to exist.

States of a given helicity in a massless supermultiplet of N-extended supersymmetry are classified in SU(N) representations. This SU(N) group arises from the supersymmetry algebra, which, in a light-like frame, has an SO(2N) group of automorphism[22]. SO(2N) contains $SU(N) \otimes U(1)$ and the U(1) factor is essentially the helicity of the states. Supersymmetric charges transform according to the fundamental representation $\underset{\sim}{N}$ of SU(N). The gravitinos will also be classified in the $\underset{\sim}{N}$ representation of SU(N). If we consider the massless multiplets where the maximal helicity (λ_{max}) state is an SU(N)-singlet, scalar fields will transform under SU(N) according to the representation

$$[2\lambda_{MAX}]_N + [N - 2\lambda_{MAX}]_N \, , \tag{3.1}$$

where $[k]_N$ is the k-fold irreducible antisymmetric SU(N)-tensor ($[0]_N = [N]_N = \underset{\sim}{1}$). The second term is due to the doubling of states required by PCT-invariance. It will not be present when $[2\lambda_{max}]_N$ is a real representation, i.e. when $4\lambda_{max} = N$ and $N/2$ is an even integer. If such multiplets transform under a symmetry group G (a gauge group for instance) which commutes with supersymmetry, then the representation of scalar fields is

$$\left(\underset{\sim}{R}, [2\lambda_{MAX}]_N\right) + \left(\overline{\underset{\sim}{R}}, [N - 2\lambda_{MAX}]_N\right), \tag{3.2}$$

with respect to $G \times SU(N)$. $\underset{\sim}{R}$ is arbitrary, and the second term does not appear if

$$\lambda_{max} = N/4 \text{ and } \underset{\sim}{R} \text{ is real}$$

for N/2 integer and even, or if

$$\lambda_{max} = N/4 \text{ and } \underset{\sim}{R} \text{ is pseudoreal}$$

for N/2 integer and odd.

The SU(N) group of classification is promoted to a symmetry of the ungauged supergravity theories, the only difficulty coming from the (real) massless vector fields. States of helicity λ and $-\lambda$ transform like conjugate SU(N) representations. SU(N) can then be realized straightforwardly on fields, for complex quantities like fermions, or for scalars ($\lambda = 0$). For vector fields, however, except for $N = 4$ and $\lambda_{max} = 2$, one has to invoke duals of field strengths to span the SU(N) representation.

There are then two kinds of supermultiplets as long as scalars are considered. Multiplets for which the second term in Eq. (3.2) is present have intrinsically complex scalar fields. This is the case for instance, of chiral $N = 1$ multiplets, vector $N = 2$ multiplets and $N = 4, 5, 6$ gravity multiplets. The SU(N) invariance can always be trivially extended to U(N), as a consequence of the reality of the Lagrangian. The second class concerns multiplets for which the second term in Eq. (3.2) does not appear. Their scalar fields do not possess a complex structure and remain irreducible under $G \times SU(N)$. We will call them "irreducible PCT-conjugate multiplets". Examples of such multiplets are $N = 2$ hypermultiplets in pseudoreal representations of the gauge group[23], $N = 4$ Yang-Mills multiplets, or $N = 8$ gravity multiplet.

While in the first class, the geometry of the scalar manifold will be Kählerian due to the complexity of the fields, irreducible PCT-conjugate multiplets will show non-Kählerian geometry.

Let us now consider, in general, the coset spaces which can occur as scalar manifolds in supergravity theories. They will be assumed to have the form $G/H \times SU(N)$, where $H \times SU(N)$ is a maximal compact subgroup of G. The coset elements have necessarily to transform under $H \times SU(N)$ according to Eq. (3.2). The list of these coset spaces is given in Table 1. This list is to be compared with Table II, which gives the transformation properties of all scalars appearing in massless multiplets of supergravity theories ($\lambda_{max} \leq 2$, $N \leq 8$) (in Table II: G is the symmetry group, commuting with supersymmetry and R the representation of the multiplet).

In Table 1, coset spaces of Class I, which are Grassmannian manifolds, are suitable for multiplets with $\lambda_{max} = (N-1)/2$ of N-extended supersymmetry. They are Kählerian, and, since they exist for all values of N and M, the number of such multiplets is not limited. Class II cosets can occur for multiplets with $\lambda_{max} = (N-2)/2$. Recall, however, that since $SO^*(8) \sim SO(6,2)$ (Class X), $SO^*(6) \sim SU(3,1)$ (Class I), and $SO^*(4)/SU(2) \times U(1) \sim SU(1,1)/U(1)$, these Kählerian manifolds are of interest for $N \geq 4$ only. These manifolds can accommodate one multiplet only ($\underset{\sim}{R} = 1$). Coset spaces of classes III to IX (and Class I with $N = 2$) are all quaternionic coset spaces[26]. They appear for $N = 2$ hypermultiplets. Notice that $E_{6,2}/SU(6) \times SU(2)$ is also suitable as scalar manifold for two $[\underset{\sim}{R} = \underset{\sim}{2}$ of $SU(2)]$ gravitino multiplets of $N = 6$.

Coset spaces of Class X will appear when M vector multiplets are coupled to $N = 4$ supergravity, and $E_{7,7}/SU(8)$ is known to appear in $N = 8$ supergravity[19].

We are then led to the following conclusions. The coset spaces suitable for the gravity multiplet of $N = 5, 6, 8$ supergravities are uniquely[19] $SU(5,1)/SU(5) \times U(1)$, $SO^*(12)/U(6)$ and $E_{7,7}/SU(8)$, respectively. For $N = 4$, the gravity multiplet contains one complex scalar. The coset is then obviously $SU(1,1)/U(1)$. Notice that there is no coset candidate for $N = 7$ or $N > 8$ supergravities. If we consider gravitino

multiplets ($\lambda_{max} = 3/2$), the possible cosets are $E_{6,2}/SU(6) \times SU(2)$ for $N = 6$, $SO^*(10)/U(5)$ for $N = 5$, and $SU(n,4)/SU(n) \times SU(4) \times U(1)$ for $N = 4$. It is remarkable that $E_{6,2}/SU(6) \times SU(2)$, which corresponds to two gravitinos is precisely obtained truncating the $N = 8$ theory into $N = 6$ [30]. The same holds for $SO^*(10)/U(5)$ (one gravitino) which occurs uniquely when $N = 6$ is truncated into $N = 5$. Truncations of $N = 5, 6, 8$ into $N = 4$ would lead respectively to $SU(4,1)/SU(4) \times U(1)$, $SU(4,2)/SU(4) \times SU(2) \times U(1)$ and $SU(4,4)/SU(4) \times SU(4) \times U(1)$ for the scalars of $N = 4$ gravitino multiplets. The $N = 3$, $\lambda_{max} = 3/2$ multiplet is simply found to be Kählerian. As already mentioned, Kähler manifolds appear for $N = 1$ chiral multiplets and $N = 2$ vector multiplets, while $N = 2$ hypermultiplets have scalar living on quaternionic manifolds. The last cases of interest are the $N = 3$ and 4 vector multiplets. For $N = 4$, the coset will be $SO(n,6)/SO(n) \times SO(6)$ when n vector multiplets are coupled to supergravity. For $N = 3$, the corresponding coset is $SU(n,3)/SU(n) \times SU(3) \times U(1)$.

If we consider n $N = 4$ vector multiplets coupled to $N = 4$ supergravity, the scalar fields of the whole theory will span the manifold $[SO(n,6)/SO(n) \times SO(6)] \times [SU(1,1)/U(1)]$. For instance, truncating the $N = 8$ theory, one gets 6 $N = 4$ vector multiplets. The non-compact $E_{7,7}$ invariance of $N = 8$ has a maximal subgroup $SO(6,6) \times SU(1,1)$ which will be the non-compact symmetry group of the resulting $N = 4$ theory. The vector field strengths of the $N = 4$ theory and their duals will then transform irreducibly under $SO(n,6) \times SU(1,1)$ according to $(\underset{\sim}{n+6}, \underset{\sim}{2})$, $SO(n,6) \times SU(1,1)$ acting as the group of duality transformations[35].

It is remarkable that requiring the existence of a coset fixes the number of gravitinos for $N = 5, 6$ to result from truncations of the $N = 6, 8$ theories respectively. Also, the possible cosets are unique for $N = 6$ and 8 supergravities. In fact, this complete list of the possible coset spaces candidates for the scalar manifolds of supergravity theories shows an intriguing matching with the theories known to exist.

Table 1

All coset spaces $G/H \times SU(N)$ with coset elements in $(R,[k]_N)$ + c.c. of $H \times SU(N)$

	$G/H \otimes SU(N)$	Coset transformation
I	$SU(M,N)/SU(M) \otimes SU(N) \otimes U(1)_Q$	$(\bar{M},N,Q)$ + c.c.
II	$SO^*(2N)/SU(N) \otimes U(1)_Q$	$([2]_N,Q)$ + c.c.
III	$SO(M,4)/SO(M) \otimes SU(2) \otimes SU(2)$	$(M,2,2)$
IV	$Sp(2M,2)/Sp(2M) \otimes SU(2)$	$(2M,2)$
V	$G_{2,2}/SU(2) \otimes SU(2)$	$(4,2)$
VI	$F_{4,4}/Sp(6) \otimes SU(2)$	$(14,2)$
VII	$E_{6,2}/SU(6) \otimes SU(2)$	$(20,2) = ([3]_6,2)$
VIII	$E_{7,-5}/SO(12) \otimes SU(2)$	$(32,2)$
IX	$E_{8,-24}/E_7 \otimes SU(2)$	$(56,2)$
X	$SO(M,6)/SO(M) \otimes SU(4)$	$(M,6)$
XI	$E_{7,7}/SU(8)$	$70 = [4]_8$

Table 2

Transformation properties of scalar fields in massless multiplets of supergravity theories

N	λ_{max}	$G \times SU(N)$ representations of scalars
1	1/2	$R + \bar{R}$
2	1	$(R+\bar{R},1)$
	1/2	$(R,2)$, R is pseudoreal
3	3/2	$(R+\bar{R},1)$
	1	$(R,\bar{3}) + (\bar{R},3)$
4	2	$2(1,1)$
	3/2	$(R,\bar{4}) + (\bar{R},4)$
	1	$(R,6)$, R is real
5	2	$(1,5+\bar{5})$
	3/2	$(R,\overline{10}) + (\bar{R},10)$
6	2	$(1,15+\overline{15})$
	3/2	$(R,20)$, R is pseudoreal
7	2	$(1,35+\overline{35})$
8	2	$(1,70)$

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