Coherence and the Successive Contribution in Two–Neutron Transfer reactions

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Abstract
In this short letter we briefly address two important points of two–neutron transfer reactions, namely the successive nature of the process and the role of pairing correlations during this sequential transfer. The discussion is made within the 2–step DWBA formalism. The calculations were carried out making use of software specifically developed for this purpose, which includes successive, simultaneous and non–orthogonality contributions to the process. Microscopic form factors are used which take into account the relevant structure aspects of the process, such as the nature of the single–particle wavefunctions, the spectroscopic factors, and the interaction potential responsible for the transfer.

1 Introduction

The specific probe to study the superconducting state is Cooper pair tunneling. Therefore, important progress in the understanding of pairing in atomic nuclei may arise from the systematic study of two–particle transfer reactions. Although this subject of research started about the time of the BCS papers, the quantitative calculation of absolute cross sections taking properly into account the full non–locality of the Cooper pairs (correlation length much larger than nuclear dimensions) is still an open question, which we address here within the 2–step DWBA reaction mechanism (see, for example, [1,5,7,10]). This method have been successfully applied to obtain absolute values of the two–neutron transfer differential cross sections without free parameters [1, 7–9].

In the two following sections we will stress the sequential nature of two–neutron transfer processes, and how the pairing correlations are kept during the whole process despite the separation of the two neutrons forming the correlated Cooper pair. We will do so in the context of the study of the reaction $A + a(= b + 2) \to B(= A + 2) + b$, which will virtually populate several states of the intermediate nuclei $f(= b + 1)$ and $F(= A + 1)$.

2 Successive process

Let us consider the exact eigenfunction $\left|\Psi^{(+)}(\xi_b, \xi_A, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \mathbf{R})\right\rangle$, with energy E, of the Hamiltonian

$$H = H_a(\xi_b, \mathbf{r}_{b1}, \mathbf{r}_{b2}) + H_A(\xi_A) + T_{aA}(R) + V(\xi_b, \xi_A, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \mathbf{R}), \tag{1}$$

written in the *prior* representation. In the above expression, ξ_b, ξ_A stand for the spatial coordinates of the nucleons in the cores b, A, while $\mathbf{r}_{b1}, \mathbf{r}_{b2}$ are the coordinates of neutrons $1, 2$ with respect to core b and $\mathbf{R}$ is the relative coordinate between the cores. Spin degrees of freedom are not explicitly taken into account for the sake of simplicity. Since we are interested in the two–neutron transfer process from core

b (i.e., nucleus a) to core A (nucleus B), we need to evaluate the transition amplitude

$$T_{2NT} = \left\langle \chi_\beta(\mathbf{R})\phi_b(\xi_b)\psi_B(\xi_A, \mathbf{r}_{A1}, \mathbf{r}_{A2}) \right| [V(\xi_b, \xi_A, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \mathbf{R}) - U(\mathbf{R})] \left| \Psi^{(+)}(\xi_b, \xi_A, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \mathbf{R}) \right\rangle, \tag{2}$$

where the wavefunction $|\chi_\beta(\mathbf{R})\phi_b(\xi_b)\psi_B(\xi_A, \mathbf{r}_{A1}, \mathbf{r}_{A2})\rangle$ corresponds to the final channel, in the sense that, when $\mathbf{R} \to \infty$ so that the residual nucleus can be collected in the detector, it describes a state in which the two transferred neutrons are bounded to the core A to form the nucleus B. The distorted wave $|\chi_\beta(\mathbf{R})\rangle$ is the solution of the Scrhödinger equation with the optical potential $U(\mathbf{R})$, and we schematically write the structure part as

$$|\psi_B(\xi_A, \mathbf{r}_{A1}, \mathbf{r}_{A2}))\rangle = |\phi_A(\xi_A)\rangle \sum_n S_n(B) |\varphi_n^A(\mathbf{r}_{A1})\varphi_n^A(\mathbf{r}_{A2})\rangle. \tag{3}$$

Similarly, the asymptotic form of the wavefunction of the entrance channel is $|\chi_\alpha(\mathbf{R})\psi_a(\xi_b, \mathbf{r}_{b1}, \mathbf{r}_{b2})\phi_A(\xi_A)\rangle$, with

$$|\psi_a(\xi_b, \mathbf{r}_{b1}, \mathbf{r}_{b2}))\rangle = |\phi_b(\xi_b)\rangle \sum_m S_m(a) |\varphi_m^b(\mathbf{r}_{b1})\varphi_m^b(\mathbf{r}_{b2})\rangle. \tag{4}$$

We could of course be more general in our description by coupling the two–neutron states to different configurations of the cores b and A, but this would not change the conclusions of the present letter and we can safely avoid the extra complication.

The 2–step DWBA consists in approximating $\left|\Psi^{(+)}(\xi_b, \xi_A, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \mathbf{R})\right\rangle$ by a state containing the entrance channel and the one–neutron transfer channels,

$$\begin{aligned} \left|\Psi^{(+)}\right\rangle \approx\ & |\chi_\alpha(\mathbf{R})\psi_a(\xi_b, \mathbf{r}_{b1}, \mathbf{r}_{b2})\psi_A(\xi_A)\rangle \\ & + \sum_n |\chi_n(\mathbf{R})\psi_{fn}(\xi_b, \mathbf{r}_{b1})\psi_{Fn}(\xi_A, \mathbf{r}_{A2})\rangle, \end{aligned} \tag{5}$$

with

$$\begin{aligned} |\psi_{fn}(\xi_b, \mathbf{r}_{b1})\rangle &= |\phi_b(\xi_b)\rangle|\varphi_n^b(\mathbf{r}_{b1})\rangle, \\ |\psi_{Fn}(\xi_A, \mathbf{r}_{A2})\rangle &= |\phi_A(\xi_A)\rangle|\varphi_n^A(\mathbf{r}_{A2})\rangle. \end{aligned} \tag{6}$$

We can split in four terms the interaction V defined in (1) and write it as

$$V = V_{bA}(\xi_b, \xi_A, \mathbf{R}) + V_1(\mathbf{r}_{A1}) + V_2(\mathbf{r}_{A2}) + V_{res}(\xi_b, \xi_A, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \mathbf{R}), \tag{7}$$

where we expect the residual term $V_{res}(\xi_b, \xi_A, \mathbf{r}_{b1}, \mathbf{r}_{b2}, \mathbf{R})$ to be small. If, in addition, we define the optical potential such as

$$U(\mathbf{R}) = \langle \phi_b(\xi_b)\phi_A(\xi_A)|V_{bA}(\xi_b, \xi_A, \mathbf{R})|\phi_b(\xi_b)\phi_A(\xi_A)\rangle, \tag{8}$$

we are left with just the single–particle term $V_1(\mathbf{r}_{A1}) + V_2(\mathbf{r}_{A2})$ as the interaction potential responsible for the transfer. The substitution of (5) in (2) gives rise to three terms, corresponding to the simultaneous, non–orthogonality and successive contributions [5, 10]. The simultaneous and non–orthogonality terms arise because of the finite overlap between the wavefunctions $\varphi^b(\mathbf{r})$ and $\varphi^A(\mathbf{r})$. In fact, as a two–particle transfer reaction is a process in which two nucleon change state, it is of (at least) second order in the single–particle interaction potential $V_1(\mathbf{r}_{A1}) + V_2(\mathbf{r}_{A2})$. It is then not surprising that the non–orthogonal amplitude tend to cancel the simultaneous transfer contribution, which is only a spurious consequence of the fact that the initial and final states are described with non–orthogonal wavefunctions. This cancellation is exact if the number of intermediate states form a complete basis of the two–particle Hilbert space, and the two–neutron transfer reaction is a pure successive, two–step process. In Fig. 1 we show an actual numerical realization of this cancellation. To further emphasize this important point, let us consider the following complete set of orthogonal wavefunctions:

$$
\begin{aligned}
&|\widetilde{\varphi}_0^b(\mathbf{r}_b)\rangle = |\varphi_0^b(\mathbf{r}_b)\rangle,\\
&|\widetilde{\varphi}_0^A(\mathbf{r}_A)\rangle = |\varphi_0^A(\mathbf{r}_A)\rangle - \langle\widetilde{\varphi}_0^b(\mathbf{r}_b)|\varphi_0^A(\mathbf{r}_A)\rangle\, |\widetilde{\varphi}_0^b(\mathbf{r}_b)\rangle,\\
&|\widetilde{\varphi}_1^b(\mathbf{r}_b)\rangle = |\varphi_1^b(\mathbf{r}_b)\rangle - \langle\widetilde{\varphi}_0^b(\mathbf{r}_b)|\varphi_1^b(\mathbf{r}_b)\rangle\, |\widetilde{\varphi}_0^b(\mathbf{r}_b)\rangle - \langle\widetilde{\varphi}_0^A(\mathbf{r}_A)|\varphi_1^b(\mathbf{r}_b)\rangle\, |\widetilde{\varphi}_0^A(\mathbf{r}_A)\rangle,\\
&|\widetilde{\varphi}_1^A(\mathbf{r}_A)\rangle = |\varphi_1^A(\mathbf{r}_A)\rangle - \langle\widetilde{\varphi}_0^b(\mathbf{r}_b)|\varphi_1^A(\mathbf{r}_A)\rangle\, |\widetilde{\varphi}_0^b(\mathbf{r}_b)\rangle\\
&\qquad - \langle\widetilde{\varphi}_0^A(\mathbf{r}_A)|\varphi_1^A(\mathbf{r}_A)\rangle\, |\widetilde{\varphi}_0^A(\mathbf{r}_A)\rangle - \langle\widetilde{\varphi}_1^b(\mathbf{r}_b)|\varphi_1^A(\mathbf{r}_A)\rangle\, |\widetilde{\varphi}_1^b(\mathbf{r}_b)\rangle,\\
&\vdots\\
&|\widetilde{\varphi}_k^b(\mathbf{r}_b)\rangle = |\varphi_k^b(\mathbf{r}_b)\rangle - \sum_{n=0}^{k-1}\langle\widetilde{\varphi}_n^b(\mathbf{r}_b)|\varphi_k^b(\mathbf{r}_b)\rangle\, |\widetilde{\varphi}_n^b(\mathbf{r}_b)\rangle\\
&\qquad - \sum_{m=0}^{k-1}\langle\widetilde{\varphi}_m^A(\mathbf{r}_A)|\varphi_k^b(\mathbf{r}_b)\rangle\, |\widetilde{\varphi}_m^A(\mathbf{r}_A)\rangle,\\
&|\widetilde{\varphi}_k^A(\mathbf{r}_A)\rangle = |\varphi_k^A(\mathbf{r}_A)\rangle - \sum_{n=0}^{k}\langle\widetilde{\varphi}_n^b(\mathbf{r}_b)|\varphi_k^A(\mathbf{r}_A)\rangle\, |\widetilde{\varphi}_n^b(\mathbf{r}_b)\rangle\\
&\qquad - \sum_{m=0}^{k-1}\langle\widetilde{\varphi}_m^A(\mathbf{r}_A)|\varphi_k^A(\mathbf{r}_A)\rangle\, |\widetilde{\varphi}_m^A(\mathbf{r}_A)\rangle.
\end{aligned}
\tag{9}
$$

Noting that all the overlaps in the above expressions tend to zero as $\mathbf{R}\to\infty$, it is clear that $|\widetilde{\varphi}_n(\mathbf{r})\rangle \to |\varphi_n(\mathbf{r})\rangle$ when $\mathbf{R}\to\infty$, and we can use the channel states

$$|\psi_a(\xi_b,\mathbf{r}_{b1},\mathbf{r}_{b2}))\rangle = |\phi_b(\xi_b)\rangle \sum_m S_m(a)|\widetilde{\varphi}_m^b(\mathbf{r}_{b1})\widetilde{\varphi}_m^b(\mathbf{r}_{b2})\rangle, \tag{10}$$

$$|\psi_B(\xi_A,\mathbf{r}_{A1},\mathbf{r}_{A2}))\rangle = |\phi_A(\xi_A)\rangle \sum_n S_n(B)|\widetilde{\varphi}_n^A(\mathbf{r}_{A1})\widetilde{\varphi}_n^A(\mathbf{r}_{A2})\rangle \tag{11}$$

instead of (3) and (4), as they are asymptotically identical. When we express the transition amplitude in terms of the new set, the first term of (5) gives no contribution. We turn our attention to the contribution to the transition amplitude (2) of the second term, for which we need the distorted waves in the intermediate channels $\chi_n(\mathbf{R})$, that is

$$\chi_n(\mathbf{R}) = -\frac{2\mu}{\hbar^2}\int d\mathbf{R}' G_n(\mathbf{R},\mathbf{R}')\langle\widetilde{\varphi}_n^A(\mathbf{r}_{A2})|V_2(\mathbf{r}_{A2})|\widetilde{\varphi}_n^b(\mathbf{r}_{b2})\rangle, \tag{12}$$

where the Green function $G_n(\mathbf{R},\mathbf{R}')$ is the solution of

$$\left(-\nabla_{\mathbf{R}}^2 - k_n^2 + \frac{2\mu}{\hbar^2}U(\mathbf{R})\right) G_n(\mathbf{R},\mathbf{R}') = \delta(\mathbf{R}-\mathbf{R}'). \tag{13}$$

The Green function $G_n(\mathbf{R},\mathbf{R}')$ propagates each virtual intermediate state with a kinetic energy

$$\frac{\hbar^2 k_n^2}{2\mu} = E - \varepsilon_{fn} - \varepsilon_{Fn}, \tag{14}$$

where $\varepsilon_{fn}, \varepsilon_{Fn}$ are the internal energies of nuclei f, F in the intermediate channel n. We thus obtain an expression for the transition amplitude

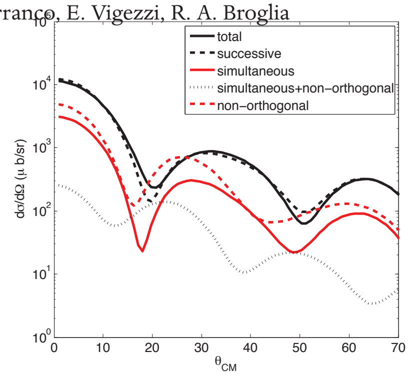

Fig. 1: Contributions to the total two–neutron transfer cross section of the different amplitudes (simultaneous, successive and non–orthogonal), for the ^{112}Sn$(p,t)^{110}$Sn reaction at a laboratory energy of 26 MeV. Note that the simultaneous and non–orthogonal contributions are in anti–phase, so that the contribution corresponding to the coherent superposition of these two amplitudes tend to cancel. The calculated total cross section thus essentially coincides with the successive contribution.

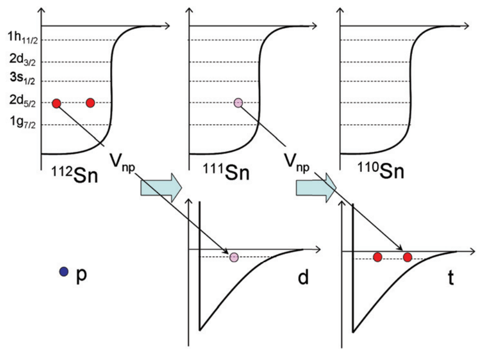

Fig. 2: Depiction of one of the successive single–particle orbital transfer processes contributing to the total successive amplitude in the ^{112}Sn$(p,t)^{110}$Sn reaction. All five contributions (arising from the $(1g_{7/2})^2, (2d_{5/2})^2, (3s_{1/2})^2, (2d_{3/2})^2, (1h_{11/2})^2$ configurations) contribute coherently to the total cross section.

$$T_{2NT} = T_{succ} = -\frac{4\mu}{\hbar^2}\sum_n \langle \widetilde{\varphi}_n^A(\mathbf{r}_{A1})|V_1(\mathbf{r}_{A1})|\widetilde{\varphi}_n^b(\mathbf{r}_{b1})\rangle \times \int d\mathbf{R}' G_n(\mathbf{R},\mathbf{R}')\langle \widetilde{\varphi}_n^A(\mathbf{r}_{A2})|V_2(\mathbf{r}_{A2})|\widetilde{\varphi}_n^b(\mathbf{r}_{b2})\rangle \tag{15}$$

which only contains a successive, two–step term. This is clearly a direct consequence of neglecting the residual interaction V_{res} in (7), which should be much smaller than the mean field single particle potential $V_1(\mathbf{r}), V_2(\mathbf{r})$. This is in general the case, but the validity of this approximation can break down in particular cases. For example, if some relevant intermediate states are strongly off shell (i.e. the kinetic energy (14) becomes negative), their contribution is significantly quenched. An interesting case can arise when this situation becomes operative for all possible intermediate states, in which case they can only be virtually populated, thus emphasizing the role of simultaneous transfer through the residual interaction V_{res}.

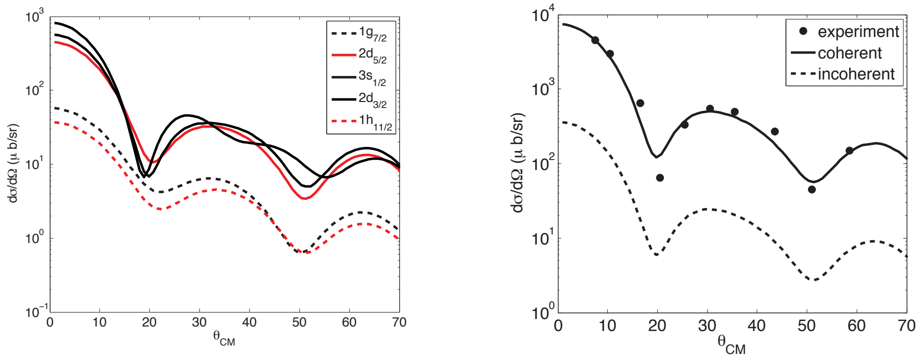

Fig. 3: In the left figure we show the contributions to the total ^{112}Sn(p,t)^{110}Sn cross section of each $(nl_j)^2$ configuration. The figure in the right–hand side compares the coherent (σ) with the incoherent (σ_{inc}, see text) cross sections for the same reaction, together with the experimental data [6].

3 Coherence of the successive transfer

We wish to emphasize that the fact that the transfer process arises through the successive migration of the neutrons from one core to the other by no means imply any correlation loss. The two nucleons are correlated over a distance $\xi = \hbar v_F / E_{corr}$, where v_F is the Fermi velocity and E_{corr} plays the role of the pairing gap for open shell, super- fluid, nuclei. In the case of, e.g., ^{210}Pb, $E_{corr} \approx$ 1.2 MeV. Thus $\xi = 25$ fm. Of course, if the two nucleons are subject to an external field (the central potential generated by, e.g., the ^{208}Pb core), they cannot move away from each other more than 14 fm, in keeping with the fact that the radius of ^{208}Pb is $\approx$ 7 fm. On the other hand, in a heavy ion reaction with e.g. impact parameter 17 fm, the central single–particle potential acting on one of the two nucleons to be transferred is much stronger than typical values of the pairing field V_{res}. It will thus be this potential responsible for the transfer of one partner of the Cooper pair, and this two–step process will take place without loose of (pairing) correlation between the two nucleons, because the Cooper pair is equally well formed in the intermediate states, where the relative distance between the two neutrons is always less than 15 fm.

To illustrate this point, we present the results of the 2–step DWBA formalism applied to the ^{212}Sn$(p,t)^{210}$Sn reaction with a proton beam of energy $E_p = 26$ MeV. The ^{212}Sn is a superfluid nucleus, and in its ground state the valence neutrons form a Cooper pair condensate. This state of correlated pairs of neutrons can be described by mixing single-particle configurations corresponding to the outer shell, namely the $1g_{7/2}, 2d_{5/2}, 3s_{1/2}, 2d_{3/2}, 1h_{11/2}$ orbitals. Being a collective mode, this state is characterized by an enhanced absolute value of the two-nucleon differential cross section, measured in terms of the average pure two-particle units [2–4]. As we have exemplified in Fig.1, each single–particle orbital contribution $T(nl_j)$ to the total transition amplitude (see Fig. 2) arise essentially from a successive process. Despite that, they all contribute coherently to the total cross section σ, so

$$\sigma \sim |T(1g_{7/2}) + T(2d_{5/2}) + T(3s_{1/2}) + T(2d_{3/2}) + T(1h_{11/2})|^2. \tag{16}$$

In Fig.3 we compare this two–neutron transfer cross section, together with the experimental points, with the uncorrelated result σ_{unc} obtained by combining incoherently the transition amplitudes, schematically

$$\sigma_{inc} \sim |T(1g_{7/2})|^2 + |T(2d_{5/2})|^2 + |T(3s_{1/2})|^2 + |T(2d_{3/2})|^2 + |T(1h_{11/2})|^2. \tag{17}$$

That the uncorrelated cross section fall well below the data while the correlated cross section reproduce the experimental findings testifies to the fact that the pairing correlations among the two transferred neutrons is not lost during the two–step process.

4 Conclusions

It is well established that single Cooper pair transfer is the specific tool to probe pairing correlations in nuclei. The reaction formalism of 2–step DWBA have proved to be successful in predicting the absolute values of the differential transfer cross sections in a number of scenarios [1, 7–9], thus allowing to quantitatively assess the nature of such correlations through two–neutron transfer reaction experiments. In this paper we emphasize that, under most circumstances, these reactions consist in the successive transfer of the pair of nucleons. This is a consequence of neglecting the residual interaction V_{res} which, as a rule, is considerably smaller than the mean field potential. However, we also point out that, due to Q–value effects, the intermediate channels could be closed in some cases, a situation in which the successive transfer would be significantly quenched. Financial support from the Ministry of Science and Innovation of Spain grants FPA2009–07653 and ACI2009–1056 are acknowledged by FB and GP and by FB respectively.

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