Importance of final-state fluctuations in radiative capture reactions and applications to surrogate reaction measurements

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Abstract
Fluctuation effects in the final state of a direct reaction leading to unbound states were studied by Kerman and McVoy (KM). A simplified form of the KM theory has provided the key to the interpretation of a $^{89}\mathrm{Y}(\vec{p},\gamma)^{90}\mathrm{Zr}^*$ measurement in which the residual $^{90}\mathrm{Zr}^*$ nucleus was formed at excitation energies up to ≈28 MeV, well above the proton separation energy in $^{90}\mathrm{Zr}$. The same modified KM theory can be applied to other processes, such as the use of the (d,p) reaction to insert a neutron into a target to form a compound nucleus, as an alternative to direct formation by neutron bombardment. This is an example of the surrogate reaction mechanism, currently being developed for the indirect measurement of reactions on unstable targets. Leakage of the final-state neutron into the continuum invalidates the mechanism and thus its magnitude must be estimated. We use the modified KM theory to estimate this effect.

1 Introduction

A treatment of fluctuation effects in the final state of a direct reaction leading to highly-excited states in the residual nucleus was given by Kerman and McVoy (KM) [1], using an extension of the reaction framework developed by Kawai, Kerman and McVoy (KKM) [2]. The KM theory provides a basis for understanding the formation and subsequent decay of a compound nucleus B^* resulting from direct reactions such as $\mathrm{A}(d,p)\mathrm{B}^*$.

We will show how a simplified form of the KM theory provided the key to the interpretation of the $^{89}\mathrm{Y}(\vec{p},\gamma)^{90}\mathrm{Zr}^*$ reaction measurement [3] with 19.6-MeV polarized protons, in which the residual $^{90}\mathrm{Zr}^*$ nucleus was formed at excitation energies up to ≈28 MeV, well above the proton separation energy in $^{90}\mathrm{Zr}$. A straightforward extension of the direct-semidirect capture theory to unbound final states completely failed to explain the observed gamma spectra and angular distributions, but the addition of an absorptive term for the final-state proton obtained from the modified KM theory solved the problem and yielded an excellent reproduction of the observed gamma spectra, angular distributions, and analyzing powers [3].

The same modified KM theory can be applied to other direct reactions forming an unstable final state. A case of contemporary interest is the use of the (d,p) reaction to insert a neutron into a target to form an unstable compound nucleus, as an alternative to direct formation of the compound system by neutron bombardment. This is an example of the surrogate reaction mechanism, which is being developed for the indirect measurement of statistical reactions on rare or unstable targets. This topic has been covered in a recent review article [4]. Since it is assumed in applications of the surrogate reaction technique that the final-state neutron damps into a compound nucleus, leakage of the captured neutron into the continuum invalidates the surrogate mechanism, and thus its magnitude must be estimated. The modified KM theory (as well as closely related approaches [5,6]) can estimate the leakage fraction, and preliminary estimates have been made [4]. Since the direct-semidirect (n,γ) radiative-capture reaction deposits a neutron into a nucleus in a manner similar to the (d,p) stripping reaction, we can use the capture reaction to get an estimate of the leakage. We show estimates of the leakage probability as a function of the orbital angular momentum of the deposited neutron, and conclude that it is significant (of the order of 50% for low angular momenta).

It has so far been assumed that the surrogate compound nucleus decays according to simple Hauser-Feshbach branching ratios, but this ignores possible correlations between the decay channels and the direct-reaction formation process. This part of the problem will require application of the full KM theory.

We now show a few of the key results from the KKM, KM, and direct-semidirect capture theories that will be relevant to the following discussion.

In KKM [2], the S matrix element connecting entrance channel c and exit channel c' is written as an optical-potential background term plus a sum over resonances identified by q,

$$S_{cc'}(E) = \overline{S}_{cc'}(E) - i\sum_q \frac{g_{qc}g_{qc'}}{E-\mathscr{E}_q}, \tag{1}$$

where g_{qc} is an amplitude for decay from the state q into the channel c, and $\mathscr{E}_q$ is the (complex) energy of q. By construction, the energy average over an interval containing many states q is zero. The fluctuation (compound) cross section in the large width-to-spacing limit, $\Gamma/D \gg 1$, is defined in terms of certain averages over the resonance parameters,

$$X_{cc'} = \left(\frac{2\pi}{D\Gamma}\right)^{1/2} \langle g_{qc}g^*_{qc'}\rangle_q. \tag{2}$$

In KM [1], it was recognized that population of resonances q in a 2-body entrance channel c could be accomplished via a direct transfer reaction as well. An example would be the replacement of the absorption reaction $n + \mathrm{A} \rightarrow \mathrm{B}^*$ by the stripping reaction $\mathrm{A}(d,p)\mathrm{B}^*$. The expression in KM analogous to Eq. 1 is for the T matrix,

$$T_{Rc} = \overline{T}_{Rc} + \sum_q \frac{M_{Rq}g_{qc}}{E-\mathscr{E}_q}, \tag{3}$$

where $\overline{T}_{Rc}$ is the usual direct amplitude (calculated, e.g., in DWBA) and M_{Rq} is the replacement for the KKM amplitude g_{qc}. The factor M_{Rq} is defined in terms of an amplitude $m_{Rc_1}(r)$ for finding the deposited particle at position r in channel c_1,

$$m_{Rc_1}(r) = \frac{1}{2\pi}\sum_{c_0}\int dr' M^R_{c_0}(r')\mathscr{G}^{(+)}_{c_0c_1}(r',r), \tag{4}$$

where $M^R_{c_0}(r')$ is the direct-reaction amplitude for depositing the particle at spatial position r' in channel c_0, which is then propagated to position r in channel c_1 by the Green's function $\mathscr{G}^{(+)}$. Then M_{Rq} is

$$M_{Rq} = \sum_{c_1}\int dr V_{qc_1}(r) m_{Rc_1}(r), \tag{5}$$

where $V_{qc_1}(r)$ is the interaction that captures the particle at r in channel c_1 into the resonant state q. The main result from KM that is relevant to the present work was obtained by calculating the inclusive cross section, i.e., the sum over all final channels c. After several approximations, KM found the expression

$$\sum_c \langle\sigma^{fl}_{Rc}\rangle \approx -4\pi\sum_{c_1}\int dr\,|m_{Rc_1}(r)|^2\,W_{c_1}(r), \tag{6}$$

where $W_{c_1}(r)$ is the imaginary part of the optical potential acting on the captured particle after the transfer reaction.

Direct-semidirect capture (DSD) is a well-known process that may be regarded as the DWBA theory for radiative capture, mainly useful for nucleons. One calculates matrix elements of an effective radial operator, which for electric radiation of multipolarity L is

$$Q_L = q_L r^L + \left(\frac{1}{E_\gamma - E_{res} + i\Gamma/2} - \frac{1}{E_\gamma + E_{res}}\right) h'_L(r), \tag{7}$$

where the first term represents direct capture and q_L is a kinematic effective charge. The second (semidirect) term describes capture through excitation and subsequent gamma emission of a giant resonance at excitation energy E_{res} with width Γ; $h'_L(r)$ is a radial form factor describing the excitation. The second part of the semidirect term represents excitation by the particle in the final state (core polarization). Nearly all calculations preceding the work described here [3], such as that described in Ref. [7], were for capture of a continuum nucleon into a bound final state.

2 Radiative capture to unbound states

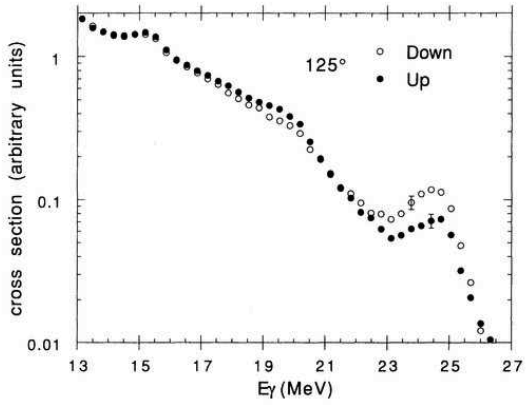

Fig. 1: Gamma spectra at 125^o from the ^{89}Y($\vec{p}$,$\vec{\gamma}$) reaction with protons protons polarized up and down along an axis perpendicular to the reaction plane.

We describe the work on radiative capture in chronological order to emphasize the important contributions of Prof. Arthur Kerman to this project. Before this work, several candidate mechanisms were proposed to explain the spectra of nucleon-induced gamma spectra populating both bound and unbound final states. These included equilibrium statistical emission, preequilibrium or multistep reactions (e.g., intermediate nucleon emission preceding the gamma), and DSD (although this had been implemented only for direct capture in light nuclei [8]).

To clarify this problem, we carried out measurements of the angular distributions and analyzing power of gammas emitted in the ^{89}Y($\vec{p}$,$\vec{\gamma}$) reaction with 19.6 MeV polarized protons [3]. Spectra were measured at 5 angles between 30^o and 150^o with both signs of the proton polarization along an axis perpendicular to the reaction plane. The spectra at 125^o, shown in Fig. 1, exhibit significant polarization effects above ≈17 MeV. Gammas above 19.6 MeV correspond to bound final states in the residual ^{90}Zr; those below, to states in which the captured proton is unbound and may be emitted into the continuum.

To explain the results, we first implemented a straightforward DSD capture calculation with a continuum (optical-model) final state wave function, similar to what was done in Ref. [8]. This calculation underestimated the magnitude of the cross section by 7 orders of magnitude. We soon realized that this discrepancy was due to the fact that the emission of the captured proton was suppressed by the Coulomb barrier, and that instead the proton was absorbed.

At this point Prof. Kerman pointed out that the KM paper could be applied to this problem. We implemented Eqs. 4 and 6 shown above, using an on-shell approximation for the Green's function in Eq. 4, and found that the discrepancy in the magnitude was reduced to a mere 2 orders of magnitude! We then calculated the full Green's function and obtained an excellent reproduction of the angular distributions

and analyzing powers as well as the spectral shapes. During this last stage we re-examined the theory and found that the inclusive cross section, which is all that is needed for the present case, can be derived more easily by applying closure to the final states, without requiring the approximations used to get the KM expression of Eq. 6. The resultant expressions are given in Ref. [3], in which the double-differential cross section for the full extended DSD theory can be expressed as a sum of two components,

$$\frac{d\sigma}{dE_\gamma d\Omega_\gamma} = \sigma_1 + \sigma_2, \tag{8}$$

where σ_1 is identical to the KM expression of Eq. 6 (in a slightly different notation) and represents compound-nucleus absorption in the final state. σ_2 is the direct escape contribution, which is the straightforward extension of DSD using a continuum final state wave function. This term is negligible in the present case as noted above, but, as will be seen below, it is significant for neutron capture since there is no Coulomb barrier.

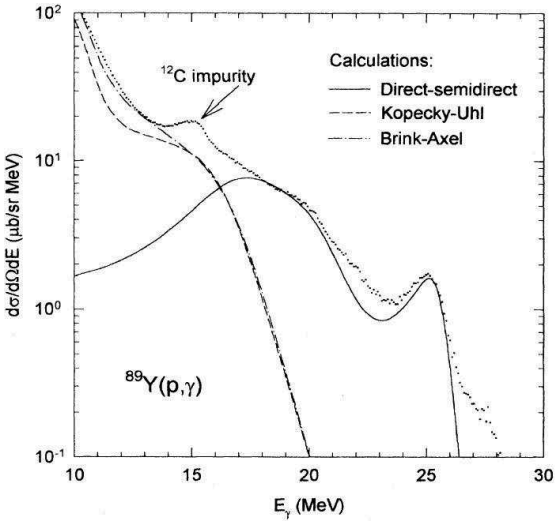

Fig. 2: 90^o spectrum of the gammas from the $^{89}\mathrm{Y}(p,\gamma)$ reaction, together with the full DSD and Hauser-Feshbach calculations.

Figure 2 shows the gamma spectrum at 90^o, together with the extended DSD calculations and equilibrium statistical (Hauser-Feshbach) calculations of the spectra using two commonly used models for the gamma strength function. We see that there is no apparent need for reaction mechanisms other than those shown in the figure, at least up to ≈20 MeV incident energy. In carrying out this calculation, we have included direct E1, E2, and E3 radiation as well as semidirect E1.

3 Application to surrogate reactions

The compound nuclear reaction, illustrated in the top portion of Fig. 3, may be difficult or impossible to measure if the target A is rare or unstable. An alternative approach, the surrogate reaction technique [4], involves forming the *same* compound nucleus in a direct reaction on a different target, as shown in the bottom portion of the figure. Corrections using nuclear reaction theory are required, since the spin-parity distribution of the compound system is different in the two cases, and the final state of the direct reaction may emit particles before an equilibrium compound nucleus is formed (i.e., incomplete or partial

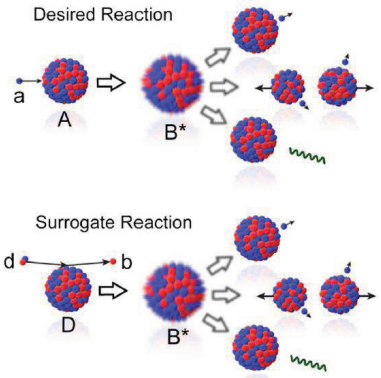

Fig. 3: Schematic picture of the formation of a compound nucleus B^* via either an absorption reaction $a + A \rightarrow B^*$, or a direct interaction $D(d,b)B^*$. In both cases, the compound nucleus subsequently decays into the various open channels.

fusion). Note that the relation between the desired and surrogate reactions is exactly the same as between the KKM and KM theories.

A particularly interesting case is the use of a (d,p) reaction as a surrogate for neutron absorption on an unstable target, since it may be useful for the determination of astrophysical neutron capture reactions. Since the (n,γ) process described by the extended DSD theory and the (d,p) reaction deposit a neutron onto a target in a similar manner, we can use a DSD calculation to give relevant estimates of both the spin distributions and the compound formation probability. The compound formation probability is easily obtainable since the calculation separately identifies the compound formation and the neutron escape (σ_1 and σ_2, respectively, in Eq. 8).

In Fig. 4 we show calculations of the cross sections and compound formation probabilities for the $^{89}\mathrm{Y}(n,\gamma)$ reaction at 19.6 MeV incident energy. These quantities are shown as a function of the orbital angular momentum L of the captured neutron, for three values of the energy available for neutron escape, 1, 5, and 11 MeV.

The upper panels of the graph show a striking odd-even effect in the dependence of the cross section on L. This is a consequence of the single-particle spectroscopy of the captured neutron in the potential well of the $^{89}\mathrm{Y}$ target, and can be associated with the alternation of even and odd parities in the major shells in a harmonic oscillator potential. The lower panels show that for low L and low escape energies the compound formation probability is rather low, of the order of 0.5. For increasing L, the angular momentum barrier increases and eventually becomes large enough to inhibit escape, so that the formation probability approaches unity. Both of these effects are large enough that they will need to be carefully taken into account in the analysis of (d,p) surrogate reactions. Some preliminary escape calculations for (d,p) surrogate reactions using a similar reaction theory [5,6] have been reported in Ref. [4].

4 Conclusions

The extended DSD theory, supplemented by Hauser-Feshbach, describes capture to both bound and unbound regions. Together with further work not shown here, there is no evident need for multistep contributions up to approximately 33 MeV. The theoretical result for inclusive reactions agrees with the expression in KM, but obtaining it does not require detailed treatment of resonance structure as in KM.

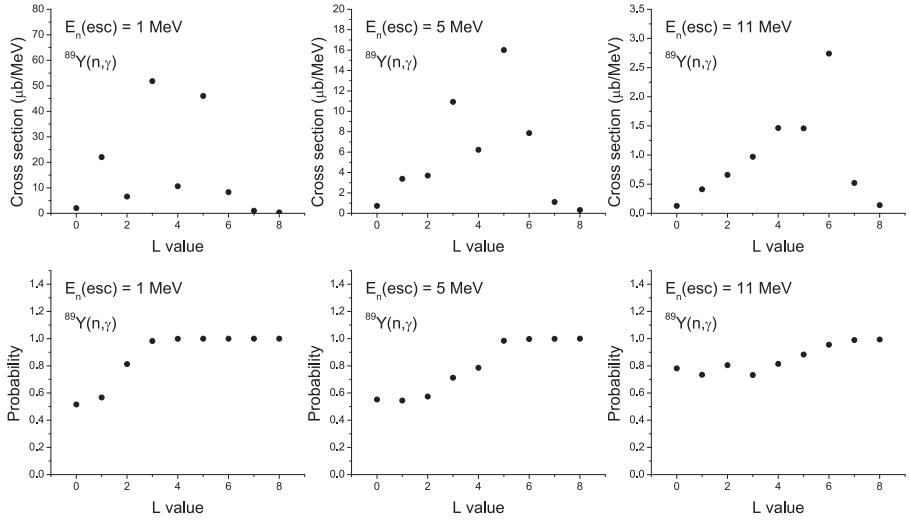

Fig. 4: Cross section and compound-nucleus formation probability for radiative capture to unbound final states in the $^{89}\mathrm{Y}(n,\gamma)$ reaction at 19.6 MeV incident energy, as a function of the orbital angular momentum of the neutron following capture. Results are shown for final-state neutron escape energies of 1, 5, and 11 MeV. The upper graphs show the cross sections, which are the angle-integrated values calculated from the extended DSD theory. The lower graphs show the probability that a compound nucleus is formed.

The theory predicts the ratio of compound formation to direct escape of the particle after capture. In comparing $^{89}\mathrm{Y}(p,\gamma)$ and $^{89}\mathrm{Y}(n,\gamma)$, we find that for protons the compound formation dominates hugely because of the Coulomb barrier. However, for neutrons, the neutron escapes a significant fraction of the time.

The calculation of escape vs. compound formation for neutron capture has been useful in understanding and quantifying the challenges in using (d,p) as a surrogate reaction to form the same compound nucleus as in neutron absorption.

It is important to realize that the treatment of inclusive reactions discussed here is not sufficient to understand possible correlations between compound decay channels and the formation of the compound nucleus by a direct reaction. If these are important, the statistical properties of the full KM theory will be required. As a pertinent example, the cross sections for formation of the compound resonances in the (d,p) reaction may very well be correlated with the neutron decay widths of these resonances.

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