Eikonal reaction theory for neutron removal reactions

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Abstract
We present an accurate method of treating neutron removal reactions and it's applications. According to the method, the nuclear and Coulomb breakup processes are consistently treated by the method of the continuum discretized coupled channels. This method is referred to as the eikonal reaction theory (ERT). We analyze the two types of removal reactions of ^{31}Ne and ^{6}He with ERT.

1 Introduction

Unstable nuclei have exotic properties such as the halo structure [1–3] and the change of magicity for nuclei in the region called "island of inversion" [4]. One of the important experimental tools for exploring such exotic properties is the nucleon removal reaction; see for example Ref. [5]. Very recently, a halo structure of ^{31}Ne has been reported following the experiment on the one-neutron removal reaction σ_{-n} at 230 MeV/nucleon not only for a ^{12}C target but also for a ^{208}Pb target [6]. This is the heaviest halo nucleus at the present stage confirmed experimentally, which also resides within the region of "island of inversion".

The nucleon removal reaction is composed of the exclusive elastic breakup component and the inclusive nucleon-stripping component. For analyses of such exclusive and inclusive reactions, Glauber model [7] is often used. This model, however, becomes breakdown for Coulomb breakup reactions because of the adiabatic approximation. Meanwhile, the method of continuum discretized coupled channels (CDCC) [8, 9] is highly reliable for describing exclusive reactions but not applicable to inclusive reactions. Both method have different demerits.

In this paper, we introduce an accurate method of treating the one-neutron removal reaction at intermediate incident energies induced by both nuclear and Coulomb interactions. In the method, the nuclear and Coulomb breakup processes are accurately treated using CDCC without making the adiabatic approximation to the latter, so that the calculated cross section is reliable even in the presence of the Coulomb interaction. Thus, this method called the eikonal reaction theory (ERT) [10] is an essential extension of the Glauber model and CDCC. ERT is applied to the one-neutron removal from ^{31}Ne and the two-neutron removal from ^{6}He for both light (^{12}C) and heavy (^{208}Pb) targets and we show that ERT is useful for describing neutron removal reactions.

2 Eikonal reaction theory (ERT)

We consider as the scattering of a two-body projectile (P) composed of a core nucleus (c) and a valence neutron (n). Including a target (A), we take the three-body (c+n+A) system shown as Fig. 1

The starting point is the three-body Schrödinger equation,

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2_{\boldsymbol{R}} + h_{\rm P} + U_{\rm c}^{\rm (Nucl)}(r_{\rm c}) + U_{\rm c}^{\rm (Coul)}(r_{\rm c}) + U_{n}^{\rm (Nucl)}(r_{n}) - E\right]\Psi(\boldsymbol{R},\boldsymbol{r}) = 0\,, \tag{1}$$

where μ is the reduced mass between P and A. The three-dimensional vector $\boldsymbol{R} = (\boldsymbol{b}, Z)$ stands for the coordinate between P and A, whereas $\boldsymbol{r}_x$ ($x = n$ or c) is the coordinate between x and A and $\boldsymbol{r}$ means the coordinate between c and n. The operator $h_{\rm P}$ is the internal Hamiltonian of the projectile. $U_n^{\rm (Nucl)}$ is the nuclear part of the optical potential between n and A, and $U_{\rm c}^{\rm (Nucl)}$ and $U_{\rm c}^{\rm (Coul)}$ are, respectively, the nuclear and Coulomb parts of the optical potential between c and A.

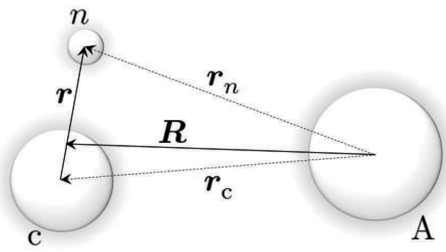

Fig. 1: The three-body model for a two-body projectile

First we make a product assumption for the total wave function so that it is devided into the plane wave part $\hat{O}$ and the remainder ψ,

$$\Psi = \hat{O}\psi(\boldsymbol{R}, \boldsymbol{r}) \ , \tag{2}$$

$$\hat{O} \equiv \frac{1}{\sqrt{\hbar\hat{v}}} e^{i\hat{K}\cdot Z} \ , \ \ \hat{K} \equiv \frac{\sqrt{2\mu(E - h_{\rm P})}}{\hbar} \ , \ \ \hat{v} \equiv \frac{\hbar\hat{K}}{\mu} \ . \tag{3}$$

we apply the eikonal approximation to the product form (2), namely, $\nabla^2_{\boldsymbol{R}}\psi$ is neglected in the kinetic energy term. It leads to the following equation,

$$i\frac{d\psi}{dZ} = \hat{O}^\dagger U \hat{O}\psi. \tag{4}$$

The scattering matrix as a formal solution to Eq.(4) is

$$S = \exp\Big[-i\mathcal{P}\int_{-\infty}^{\infty} dZ \hat{O}^\dagger \Big(U_c^{\rm (Nucl)} + U_c^{\rm (Coul)} + U_n^{\rm (Nucl)} \Big) \hat{O} \Big]. \tag{5}$$

Here, $\mathcal{P}$ is the "time" ordering (Z ordering) operator which describes the multistep scattering processes accurately.

In the Glauber model, the adiabatic approximation is made, in which $h_{\rm P}$ is replaced with the ground-state energy of the projectile, and hence $\hat{O}^\dagger U \hat{O}$ and $\mathcal{P}$ in Eq. 5 are reduced to $U/(\hbar v_0)$ and 1, respectively, where v_0 is the velocity of P in the ground state relative to A. At intermediate energies, this treatment is known to be valid for short-range nuclear interactions, but not for the long-range Coulomb interactions. Therefore, we make the adiabatic approximation in the evaluation of only $\hat{O}^\dagger U_n^{\rm (Nucl)} \hat{O}$, i.e., we use

$$\hat{O}^\dagger U_n^{\rm (Nucl)} \hat{O} \to U_n^{\rm (Nucl)}/(\hbar v_0) \ . \tag{6}$$

$U_n^{\rm (Nucl)}/(\hbar v_0)$ is just a number so that this term is commutable with the operators $\hat{O}$ and $\mathcal{P}$. As a result, S can be separated into the core part $S_{\rm c}$ and the neutron part S_n,

$$S \approx S_{\rm c} S_n \tag{7}$$

with

$$S_{\rm c} \ \equiv \ \exp\Big[-i\mathcal{P}\int_{-\infty}^{\infty} dZ \hat{O}^\dagger \Big(U_{\rm c}^{\rm (Nucl)} + U_{\rm c}^{\rm (Coul)} \Big) \hat{O} \Big] \ , \tag{8}$$

$$S_n \ \equiv \ \exp\Big[-\frac{i}{\hbar v_0}\int_{-\infty}^{\infty} dZ \ U_n^{\rm (Nucl)} \Big] \ . \tag{9}$$

This separation of S is the essence of ERT. It should be noted that one cannot evaluate S_c directly with Eq. (8), since it includes the operators $\hat{O}$ and $\mathcal{P}$.

The one-neutron removal cross section is composed of stripping ($\sigma_{n:\text{str}}$) and elastic breakup (σ_{bu}) cross sections.

$$\sigma_{-n} = \sigma_{n:\text{str}} + \sigma_{\text{bu}} \tag{10}$$

$\sigma_{n:\text{str}}$ and are written by S_c, S_n and the projectile ground-state wave function φ_0,

$$\begin{aligned} \sigma_{n:\text{str}} &= \int d^2\boldsymbol{b}\langle\varphi_0||S_c|^2(1-|S_n|^2)|\varphi_0\rangle \\ &= [\sigma_{\text{R}} - \sigma_{\text{bu}}] - [\sigma_{\text{R}}(-n) - \sigma_{\text{bu}}(-n)], \end{aligned} \tag{11}$$

where $\sigma_{\text{R}}, \sigma_{\text{bu}}$ are the total reaction and elastic breakup cross sections, respectively,

$$\sigma_{\text{R}} = \int d^2\boldsymbol{b}\big[1 - |\langle\varphi_0|S_cS_n|\varphi_0\rangle|^2\big] , \tag{12}$$

$$\sigma_{\text{bu}} = \int d^2\boldsymbol{b}\big[\langle\varphi_0||S_cS_n|^2|\varphi_0\rangle - |\langle\varphi_0|S_cS_n|\varphi_0\rangle|^2\big] , \tag{13}$$

and $\sigma_{\text{R}}(-n), \sigma_{\text{bu}}(-n)$ correspond to the total reaction and elastic breakup, respectively, in which S_cS_n is replaced with S_c. They are solution to the following equation,

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2_{\boldsymbol{R}} + h_{\text{P}} + U_{\text{c}}^{(\text{Nucl})}(r_{\text{c}}) + U_{\text{c}}^{(\text{Coul})}(r_{\text{c}}) - E\right]\Psi(\boldsymbol{R},\boldsymbol{r}) = 0 . \tag{14}$$

Eqs.(1) and (14) can be solved with CDCC. This means that ERT makes CDCC applicable to inclusive reactions.

3 One-neutron removal from ^{31}Ne

We apply ERT to the one-neutron removal reactions for the ^{31}Ne+^{12}C scattering at 230 MeV/nucleon and the ^{31}Ne+^{208}Pb scattering at 234 MeV/nucleon with a three-body (^{30}Ne+n+A) model. The optical potentials for the n-target and ^{30}Ne-target subsystems are obtained by folding the effective nucleon-nucleon interaction [11] with one-body nuclear densities. The densities of P and A are constructed by the same method as in Ref. [12]. We assume the ground state of ^{31}Ne to be either the $^{30}\text{Ne}(0^+) \otimes 1\text{p}3/2$ or the $^{30}\text{Ne}(0^+) \otimes 0\text{f}7/2$, with the one-neutron separation energy $S_n = 0.33$ MeV [13]. As for the breakup states, we include s-, p-, d-, f-, and g-waves up to the relative momentum between ^{30}Ne and n of 0.8 fm^{-1}. For more detailed numerical inputs, see Ref. [10].

Table 1 shows the elastic-breakup cross section σ_{bu}, the one-neutron stripping cross section $\sigma_{n:\text{str}}$, the one-neutron removal cross section σ_{-n}, and the spectroscopic factor $\mathcal{S} = \sigma_{-n}^{\text{exp}}/\sigma_{-n}^{\text{th}}$ for ^{12}C and ^{208}Pb targets. $\mathcal{S}$ calculated with the 1p3/2 ground-state neutron configuration little depends on the target and less than unity, but that with the 0f7/2 configuration does not satisfy these conditions. Therefore, we can infer that the major component of the ground state of ^{31}Ne is $^{30}\text{Ne}(0^+) \otimes 1\text{p}3/2$ with $\mathcal{S} \sim 0.69$. We adopt this configuration in the following.

Since the potential between ^{30}Ne and n is not well known, we change each of the potential parameters by 30% and see how this ambiguity of the potential affects the resulting value of $\mathcal{S}$. We obtain $\mathcal{S} = 0.693 \pm 0.133 \pm 0.061$ for a ^{12}C target and $\mathcal{S} = 0.682 \pm 0.133 \pm 0.062$ for a ^{208}Pb target; the second and third numbers following the mean value stand for the theoretical and experimental uncertainties, respectively. Thus, $\mathcal{S}$ includes a sizable theoretical uncertainty. This situation completely changes if we look at the asymptotic normalization coefficient C_{ANC}, i.e., $C_{\text{ANC}} = 0.320 \pm 0.010 \pm 0.028$ fm$^{-1/2}$ for a ^{12}C target and $C_{\text{ANC}} = 0.318 \pm 0.008 \pm 0.029$ fm$^{-1/2}$ for a ^{208}Pb target. Thus, C_{ANC} has a

Table 1: Integrated cross sections and the spectroscopic factor for the ^{31}Ne-^{12}C scattering at 230 MeV/nucleon and the ^{31}Ne-^{208}Pb scattering at 234 MeV/nucleon. The cross sections are presented in unit of mb and the data are taken from Ref. 6.

	^{12}C target			^{208}Pb target		
	$p_{3/2}$	$f_{7/2}$	Exp.	$p_{3/2}$	$f_{7/2}$	Exp.
$\sigma_{\rm EB}$	23.3	3.3		799.5	73.0	(540)
$\sigma_{n:{\rm str}}$	90	29		244	53	
σ_{-n}	114	32	79	1044	126	712
$\mathcal{S}$	0.693	2.47		0.682	5.65	

much smaller theoretical uncertainty than $\mathcal{S}$. This means that the one-nucleon removal reaction is quite peripheral.

The experimental value of S_n has a large error, $S_n = 0.29 \pm 1.64$ MeV [13], so we also see the S_n dependence of $C_{\rm ANC}$ and $\mathcal{S}$ When $S_n = 0.1$ MeV, $C_{\rm ANC} = 0.128 \pm 0.003 \pm 0.011$ fm$^{-1/2}$ and $\mathcal{S} = 0.530 \pm 0.084 \pm 0.047$ for a ^{12}C target, and $C_{\rm ANC} = 0.105 \pm 0.004 \pm 0.010$ fm$^{-1/2}$ and $\mathcal{S} = 0.358 \pm 0.057 \pm 0.033$ for a ^{208}Pb target. These values are plotted in Fig. 2. $C_{\rm ANC}$ and $\mathcal{S}$ are sensitive to the value of S_n. We can see from the S_n dependence of $\mathcal{S}$ for a ^{208}Pb target that $\mathcal{S} < 1$ when $S_n < 0.6$ MeV. It is thus necessary to determine S_n experimentally in the future to evaluate $C_{\rm ANC}$ and $\mathcal{S}$ properly. However, we can say at least that $C_{\rm ANC}$ has a smaller theoretical error and weaker target dependence than $\mathcal{S}$ for any value of S_n.

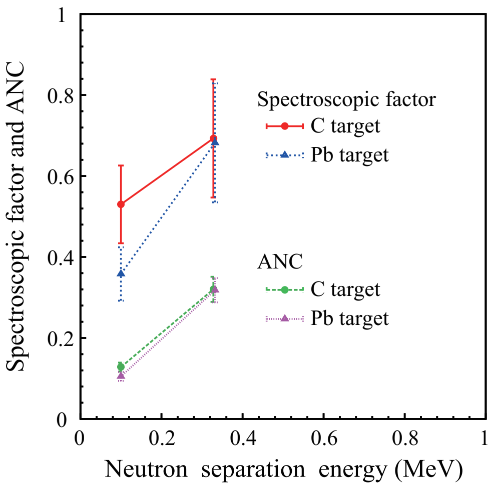

Fig. 2:

4 Two-neutron removal from ^{6}He

ERT could be easily extended to three-body projectile. Combining this four-body ERT with four-body CDCC [14, 15], we can calculate two-neutron removal cross sections. ERT is applied to two-neutron removal reactions of ^{6}He on ^{12}C and ^{208}Pb targets at 240 MeV/nucleon. In this case, the projectile is treated as a three-body $(\alpha + n + n)$ system and hence four-body CDCC is used.

We use the microscopic folding potentials obtained by folding the Melbourne nucleon-nucleon

g-matrix interaction [16] with the densities obtained by the spherical Hartree-Fock calculation with the Gogny D1S interaction. [17, 18] The present framework has no adjustable parameters. the three-body calculation of ^{6}He and the model space of the reaction calculation is the same as in Ref. [15], with which good convergence is achieved.

Table 2: Integrated cross sections for two-neutron removal reaction of ^{6}He on ^{12}C and ^{208}Pb targets at 240 MeV/nucleon. The cross sections are presented in unit of mb and the experimental data are taken from Ref. [19].

	^{12}C target		^{208}Pb target	
	Calc.	Exp.	Calc.	Exp.
$\sigma_{n:\mathrm{str}}$	153.4	127 ± 14	353.6	320 ± 90
$\sigma_{2n:\mathrm{str}}$	29.0	33 ± 23	148.9	180 ± 100
σ_{-2n}	198.5	190 ± 18	1016.6	1150 ± 90

Table 2 shows the one- and two-neutron stripping cross sections, $\sigma_{n:\mathrm{str}}$ and $\sigma_{2n:\mathrm{str}}$, respectively, and the two-neutron removal cross section σ_{-2n}. The present framework well reproduces the experimental data [19] with no adjustable parameters. Thus, we can clearly see the reliability of ERT for two-neutron removal reactions on both light and heavy targets.

5 Summary

We have presented an accurate method, which called the eikonal reaction theory (ERT), of treating the neutron removal reaction at intermediate energies. According to the method, the nuclear and Coulomb breakup processes are accurately and consistently treated by the framework of CDCC. ERT is an extention of the Glauber model and CDCC.

C_{ANC} and $\mathcal{S}$ of the last neutron in ^{31}Ne are evaluated from the measured one-neutron removal reaction. For the $1p3/2$ orbit, $\mathcal{S}$ and C_{ANC} have weak target dependence and $\mathcal{S} < 1$. On the other hand, for the $1f7/2$ orbit, $\mathcal{S}$ and C_{ANC} have strong target dependence and $\mathcal{S} > 1$. These results indicate that the last neutron mainly occupy the $1p3/2$ orbit. C_{ANC} has a smaller theoretical error and weaker target-dependence than $\mathcal{S}$. Thus, C_{ANC} is determined more accurately than $\mathcal{S}$. This means that the one-neutron removal reaction is quite peripheral. We could understand the one-neutron removal from ^{31}Ne within the naive shell model.

The application of ERT to two-neutron removal reactions of ^{6}He is also shown. The present framework well reproduces the experimental data with no adjustable parameters. It was clearly shown that ERT is useful for describing neutron-removal reactions.

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