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Pion-Oxygen Scattering in the Four-α-Particle Model

Li Qing-run*
TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A3

Abstract

The pion-^{16}O elastic scattering is studied via a parameter-free theoretically deduced optical potential based on a four-α-particle model for ^{16}O. The differential and total cross sections are calculated from the (3,3) resonance to the low energy region. The results are compared with the experimental data and a satisfactory agreement is obtained.

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*On leave from the Institute of High Energy Physics, Academy of Sciences Peking, China.

Considerable theoretical efforts have been devoted to the investigation of the pion-nucleus scattering over the past decade. Some studies used the first-order π-nucleus optical potential defined in the multiple scattering theory of KMT.[1,2,3] Some others included also the second-order effects, which could only be achieved either phenomenologically or through the approximations.[4,5] It has been found that the various effects, for example the Fermi motion, the binding energy, the angle transformation and the true π-absorption etc. are quite important and not negligible in the low energy region.

For nuclei such as ^{12}C and ^{16}O which can be regarded as being made up of α-clusters, one can treat the α-particles in the nucleus as the scatterers and utilize the π-α amplitude, obtained directly from fitting the experimental data, as the basic input to construct a theoretical π-nucleus optical potential. With this approach, the various effects mentioned above would be "automatically" included to a certain extent in the π-α formalism. Therefore, it is anticipated that the optical potential constructed should yield better results than those obtained from using the π-N amplitudes as the input, particularly in the low energy region where the scattering is not dominated by the strong absorption.

In an earlier work,[6] we have applied this model to the π-^{12}C scattering and obtained quite satisfactory agreement with the experimental data. In this work we extend the analysis to the study of π-^{16}O scattering (i.e. four α-particles system).

In the multiple scattering theory of KMT,[7] the transition matrix T for the scattering of an incident pion from a nucleus is given by

$$T = \frac{N}{N-1} T' \ , \tag{1}$$

where T' satisfies

$$T' = U + U \frac{A}{E - K_0 - H_N + i\varepsilon} P_0 T' \ . \tag{2}$$

Here N is the number of target-particles, A is the antisymmetrization operator, P_0 is the projection operator onto the ground state of the nucleus, K_0 is the kinetic energy of the incident pion, H_N is the nuclear Hamiltonian, and U represents the optical potential which can be expanded into a multiple scattering series.

After making multiple scattering, impulse and factorization approximations, one obtains the first-order optical potential

$$\langle \vec{k}'|U|\vec{k}\rangle \cong (N-1)\langle \vec{k}'|t|\vec{k}\rangle \eta(\vec{q}) \ . \tag{3}$$

Here $\vec{k}$ is the momentum of the incoming pion, $\vec{k}'$ the momentum of the outgoing pion, $\langle \vec{k}'|t|\vec{k}\rangle$ the transition matrix element of the incident pion from a free target-particle, $\eta(\vec{q})$ the form factor representing the target-particle distribution in the nucleus, and $\vec{q}$ is the momentum transfer.

In expression (3), if $\langle \vec{k}'|t|\vec{k}\rangle$ is taken to be the π-N amplitude, then one has the usual first-order optical potential which is used in many previous calculations. With the α-cluster model, $\langle \vec{k}'|t|\vec{k}\rangle$ should then be taken as the π-α amplitude and $\eta(\vec{q})$ as the form factor of α-particle distribution in the nucleus.

For the π-α amplitude, Binon et al. have used a parameterized form

$$f_{\pi\alpha}(q) = f_{\pi\alpha}(0)\left(1 - \frac{q^2}{t_1}\right)\left(1 - \frac{q^2}{t_2}\right)e^{-R_s'^2 q^2/6} \tag{4}$$

which fit the experimental data very well.[8] Here $f_{\pi\alpha}(0)$ is the amplitude

at $q = 0$, t_1 and t_2 are two complex parameters and R_s' is a real parameter. In this work, the values for $f_{\pi\alpha}(0)$, t_1, t_2 and R_s' are taken from Ref. 8.

In addition to $f_{\pi\alpha}(q)$, another input form factor $\eta_\alpha(\vec{q})$ is also needed to construct the π-nucleus optical potential. We have earlier proposed an α-particle model for light nuclei.[9] In this model, the form factor of the α-particle distribution in the ^{16}O nucleus is given by

$$\eta_\alpha(q) = (1 - 0.534\ q^2 + 0.00432\ q^4)e^{-0.27q^2} \tag{5}$$

By multiplying the internal charge form factor of the α-particle to $\eta_\alpha(q)$ given above, one obtains the total charge form factor for ^{16}O and can fit the electron scattering data for ^{16}O very well.[9]

Combining equations (3), (4) and (5), we then have an explicit expression of the π-nucleus optical potential in the momentum space. Since both $f_{\pi\alpha}(q)$ and $\eta_\alpha(q)$ depend only on the momentum transfer $\vec{q} = \vec{k}' - \vec{k}$, by carrying out a transformation from the momentum space to the coordinate space, one then obtains a local optical potential which can be substituted into the Schrödinger equation to solve for the amplitudes in π-^{16}O scattering. The Coulomb interactions are also taken into account in the present calculations.

We have calculated the differential cross section for π^{+}-^{16}O elastic scattering at incident energies T_π = 163, 114, 79 and 50 MeV. The calculated results together with the experimental data[10,11] are given in Figs. 1 and 2. Although there exists some experimental data at T_π = 240 MeV, the π-α amplitude $f_{\pi\alpha}(q)$ is however not available. So the comparison with data is not made at this energy.

From Fig. 1, one can see that the calculated results agree very well with the experimental data before the first minimum in the (3,3) resonance region (for 163 and 114 MeV). In addition, the positions of the "peaks" and "deeps" are correctly predicted. Also in the large-angle region ($\theta > 90°$), the magnitude and the variation tendency of the calculated cross sections are also in good agreement with the data.

As is well known, the first-order optical potentials constructed using the π-N amplitude as the input fail badly in the low energy region. It has been shown by many calculations that if the various higher-order effects, in particular the true π-absorption, are taken into account, then the discrepancy between theory and experiment can be significantly reduced. For comparison, the first-order calculations by Liu and Shakin[4] which include the effects due to the Fermi motion, the binding energy and the angle transformation are also plotted in Fig. 2 (dashed curve). It is obvious that their calculations deviate considerably from the experimental data in the forward angle region. In contrast, the present calculations yield quite significant improvements in the same forward angle region. This is not surprising since, as mentioned earlier, the various effects, in particular that due to the true π-absorption, have been "automatically" included to some extent in the present calculations. Also, as already discussed in an earlier paper,[6] the approximations (i.e. multiple scattering, impulse and factorization approximations) used to deduce the first-order optical potential are more valid for the present α-particle model than for the usual nucleon model.

From Fig. 1 and 2 it can be seen that all our calculated results underestimate the values of the second maximum in the differential cross sections. Similar results are also observed in the π-^{12}C scattering.[6] The cause of this underestimation is not clear to us at present.

We have also calculated total cross sections for the π-^{16}O scattering. Again, a good agreement with the experimental data is obtained, as is shown in Fig. 3.

In summary, our theoretical (parameter-free) optical potential provides a quite successful description for the π-^{16}O scattering data over a wide energy region. This success certainly provides a strong support for the four-α-particle model for the ^{16}O nucleus.[9]

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Figure Captions

1. The differential cross sections for π^+ elastic scattering on ^{16}O at 163, 114, and 79 MeV. The curves are the results obtained with the optical potential of our α-particle model. The experimental data are from Ref. 10.
2. The differential cross sections for π^+ elastic scattering on ^{16}O at 50 MeV. The solid curve is the result obtained with the optical potential of our α-particle model. The dashed line is the result calculated with the first-order optical potential using π-N amplitude as input.[4] The data is from Ref. 11.
3. The total cross sections for π-^{16}O scattering. The curve is the result obtained with the optical potential of the α-particle model. The points are the experimental data from Ref. 12.

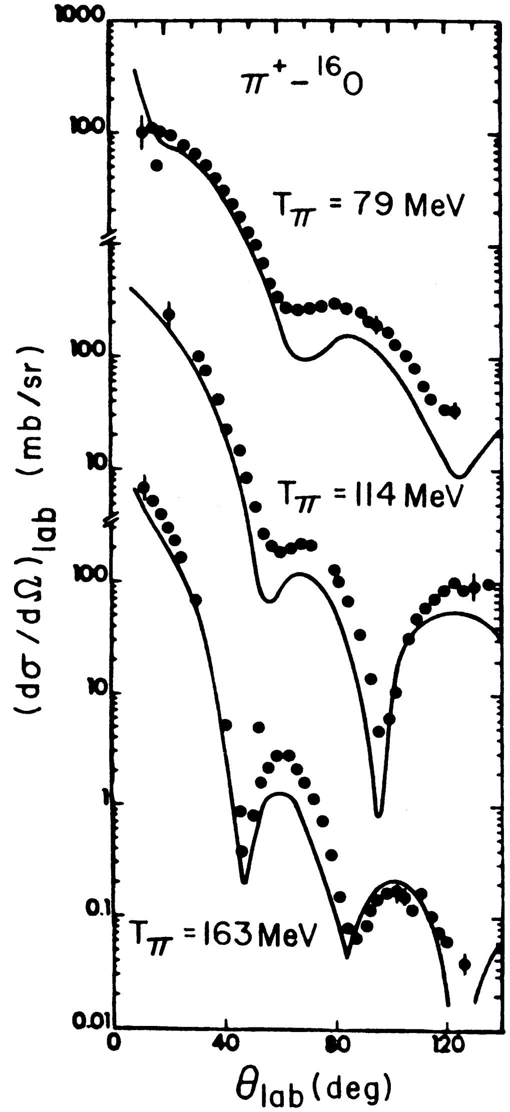

Fig. 1

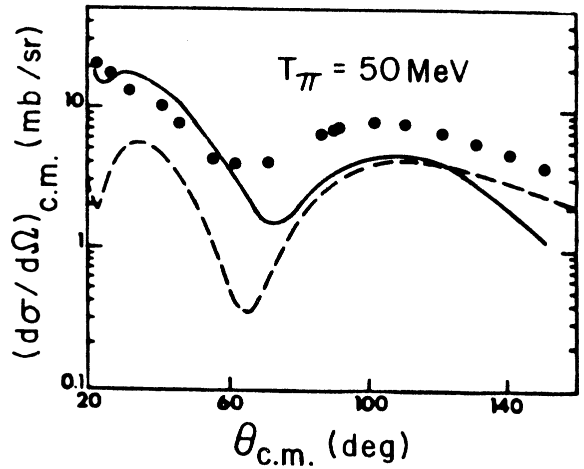

Fig. 2

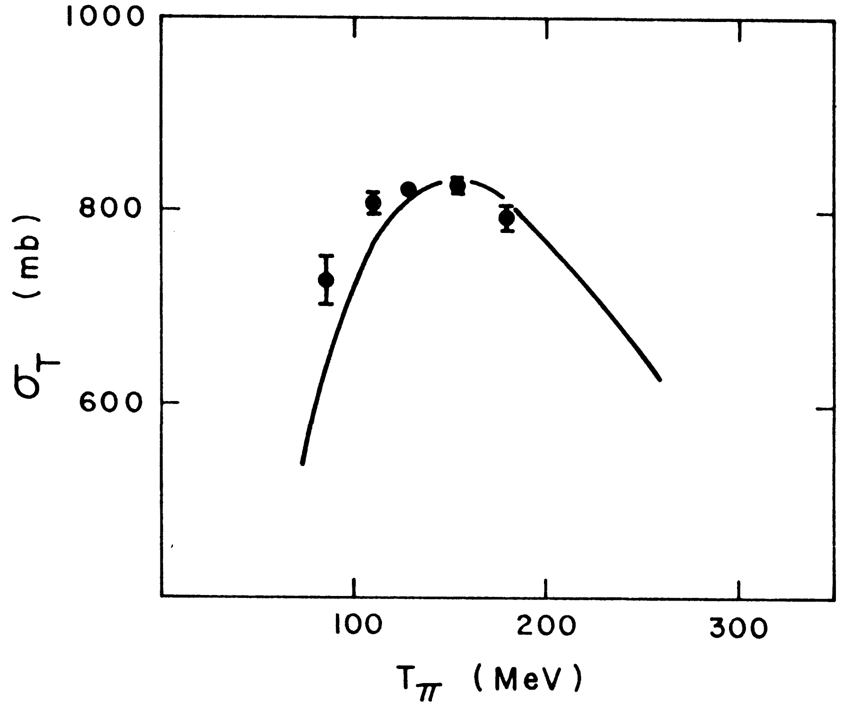

Fig. 3