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MESON-GLUONIUM MIXING FROM QCD SUM RULES*)

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ABSTRACT

We evaluate the off-diagonal two-point correlation function responsible for the gluonium-meson mixing, including the leading non-perturbative lowest dimension vacuum condensate contributions. Then, using spectral function sum rules approach, we deduce a small meson-gluonium mixing angle. We also derive upper bounds for the η' and for the strange quark masses.

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1. - INTRODUCTION

QCD sum rules à la Shifman-Vainshtein-Zakharov (SVZ)[1] have led to a considerable progress in the understanding of meson masses, couplings and their mixing. In fact, concerning the latter, it has been shown[2] that the strength of the ρ-ω mixing can be related to the typical SU(2) isospin violation order parameters which are the u,d "current" quark mass difference $m_d - m_u$ and the quark vacuum condensate $\langle \bar{u}u - \bar{d}d \rangle$. On the other hand, the relative smallness of the ω-ϕ mixing has been related[1a] to the dominance of the four-quark operator $\alpha_s^3 \langle \bar{\psi}\Gamma_1 \psi \bar{\psi}\Gamma_2 \psi \rangle$, which is suppressed by a higher power in $1/Q^2$ and by a higher order in α_s compared to the former. In this note, we would like to discuss the quarkonium-gluonium mixing along the lines adopted previously, namely by assuming that the mixing is dominated by the lowest dimension operators entering into the operator product expansion (OPE) of the off-diagonal correlation function

$$\psi_{gq}(q^2) = i \int d^4x \, e^{iqx} \langle 0 | \mathbb{T} \, J_q(x) \left(J_g(0) \right)^{\dagger} | 0 \rangle , \quad (1)$$

where for definiteness, the currents $J_i, i = q,g$ entering in Eq. (1) are scalar or pseudoscalar. So the quark operators J_q, expressed in terms of the quark fields, read:

$$J_q^- = 2im : \bar{\psi} \gamma_5 \psi : \quad \text{or} \quad J_q^+ = 2m : \bar{\psi} \psi : , \quad (2a)$$

while the gluonic currents are defined as

$$J_g^- = : \alpha_s \vec{F} \vec{\tilde{F}} : \quad \text{or} \quad J_g^+ = : \alpha_s \vec{F} \cdot \vec{F} : , \quad (2b)$$

where $F^a_{\mu\nu}$ is the gluon field strength tensor and $\tilde{F}^a_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{a\rho\sigma}$.

In what follows, we present the calculation of the correlator $\psi_{gq}(q^2)$ for large $Q^2 = -q^2 >> \Lambda^2_{QCD}$, and then we apply the familiar procedure of Ref. 1) to obtain sum rules. We use these sum rules to discuss some phenomenology concerning the η' mass, and the meson-gluonium mixing angle. We also briefly compare the results obtained within this formalism to those obtained from other approaches, such as the saturation of chiral Ward identities[3], bag models[4] and intermediate gluon exchange mechanisms[5].

2. - EVALUATION OF $\psi_{gq}(q^2)$

The evaluation of the off-diagonal two-point function needs the introduction of the renormalized gluonic current. In the minimal subtraction scheme[6] and to one loop in α_s, one has[7]:

$$(F\tilde{F})_R = (F\tilde{F})_B - \left(\frac{3}{\epsilon}\right)\left(\frac{\alpha_s}{\pi}\right)\left(\frac{N_c^2-1}{2N_c}\right)\partial^\mu(\bar{\psi}\gamma_\mu\gamma_5\psi)_B \tag{3a}$$

$$(FF)_R = (FF)_B + \left(\frac{3}{\epsilon}\right)\left(\frac{\alpha_s}{\pi}\right)\left(\frac{N_c^2-1}{2N_c}\right)\left(2m\bar{\psi}\psi\right)_B , \tag{3b}$$

which states that the gluonic currents mix with the quark currents also under renormalization. In Eqs. (3), the indices B and R refer to the bare and to the renormalized operators; $n = 4-\varepsilon$ is the dimension of space-time; $\gamma_1 = {}^3/_2\ (N_c^2-1)/2N_c$ is the anomalous dimension of the quark mass. Thus, the evaluation of $\psi_{gq}(q^2)$ from the lowest order perturbative diagram of Fig. 1a requires also the consideration of the diagram in Fig. 1b, which is induced by the second term in Eqs. (3). Such a contribution is necessary for the cancellation of terms like $(1/\varepsilon)\log -q^2/\nu^2$ (ν is the subtraction scale of the MS-scheme) which appears in the bare two-loop diagram of Fig. 1a.

a) For the 0^{+-} channel, the contribution of the diagram in Fig. 1a, for $m_u = m_d = 0$ and $m_s \neq 0$, is [1*]

$$\psi_{gg}(q^2)\Big|_{Fig\,1a} = -\alpha_s N_c \left(\frac{N_c^2-1}{2N_c}\right) 32 g^2 (n-3)(n-2) \cdot \cdot\, m_s^2 \left\{ q^2 I_4 - \frac{1}{2} I_5 \right\}, \tag{4}$$

where we have expressed ψ_{gq} in terms of two-loop integrals tabulated in the Appendix D of Ref. 6). The contribution of Fig. 1b induced by the second terms of Eq. (3a) is:

$$\psi_{gg}(q^2)\Big|_{Fig\,1b} = -\alpha_s N_c \left(\frac{N_c^2-1}{2N_c}\right)\left(\frac{3}{2\pi^2}\right) m_s^2 \left\{ \frac{2}{\epsilon^2} - \frac{1}{\epsilon} \log \frac{-q^2}{\nu^2} \right\}. \tag{5}$$

Then, after renormalization, the contribution of Fig. 1a is:

$$\psi^R_{gg}(q^2)\Big|_{Fig\,1a} = \alpha_s \cdot \left(\frac{\alpha_s}{\pi}\right) \frac{3}{\pi^2} \cdot m_s^2 \cdot q^2 \log \frac{-q^2}{\nu^2} \cdot \cdot \left\{ \log \frac{-q^2}{\nu^2} - \frac{2}{3}\left(\frac{11}{4} - 3\gamma_E\right) \right\}. \tag{6}$$

The leading (in $1/Q^2$) non-perturbative contribution is given by the lowest dimension quark vacuum condensate depicted in Fig. 2a:

(1*)
We use the following conventions in n-dimensions: $\gamma_5^2 = 1$; $\gamma_\mu\gamma_5 = -\gamma_5\gamma_\mu$; $\varepsilon_{\mu\nu\rho\sigma}\varepsilon_{\mu\nu\rho'\sigma'} = -(n-3)(g_{\rho\rho'}g_{\sigma\sigma'} - g_{\rho\sigma'}g_{\rho'\sigma})$; $g^{\alpha\beta}g_{\alpha\beta} = n$. The Feynman rule for the effective gluon vertex is $(-i)p^\rho q^\sigma \varepsilon_{\mu\nu\rho\sigma}$ for two incoming gluons with momenta p^μ and q^ν. We have checked the algebra using the Schoonship algebraic programme written by M. Veltman. We thank A. Douiri for discussions on the use of such a programme.

$$\psi^{N.P}_{gg}(q^2)\Big|_{Fig\,2a} = -8\alpha_s\left(\frac{\alpha_s}{\pi}\right) m_s \langle \bar{\psi}_s \psi_s \rangle \log \frac{-q^2}{\nu^2} \,. \quad (7)$$

The next-to-leading non-perturbative contributions are due to the diagrams in Figs. 2b and 2c. It is easy to see that the contribution of the mixed vacuum condensate $\langle\bar{\psi}\sigma^{\mu\nu}(\lambda_a/2)\psi F^a_{\mu\nu}\rangle$, shown in Fig. 2b, vanishes to leading order in the chiral symmetry breaking parameter and in α_s, by using, e.g., the routine given in Ref. 8). The leading gluon condensate is given by the diagram in Fig. 2c. Its contribution is

$$\psi_{gg}(q^2)\Big|_{Fig\,2c} = 2\left(\frac{\alpha_s}{\pi}\right) . \alpha_s \langle F^2 \rangle \left(\frac{m_s^2}{q^2}\right) \log \frac{-q^2}{m_s^2} \,. \quad (8)$$

One can see that the contribution of the four-fermion operator in Fig. 3a vanishes because of the trace of γ-matrices. The one in Fig. 3b is about $m_s^2\alpha_s^2\langle\bar{\psi}\psi\rangle^2$ and can be safely neglected. The triple gluon contribution like the one in Fig. 3c is about $m_s^2\alpha_s^2 g\langle F^3\rangle$ and we neglect it, as the contributions of the dimension eight operators are not taken into account here. The results in Eqs. (6) to (8) show that the strength of the meson-gluonium mixing is about $m_s^2\alpha_s^2$ and is expected to be small as we shall see later.

b) For the 0^{++} channel, a similar analysis can be done. As there is not a strong phenomenological motivation for a meticulous analysis of the correlation function including non-perturbative terms, we shall limit ourselves to the evaluation of the "ordinary" QCD two-loop contribution given by the analogue of Figs. 1. The result is:

$$\psi_{gg}(q^2)\Big|_{Fig\,1a} = -m_s^2\,\alpha_s\left(\frac{\alpha_s}{\pi}\right) N_c\left(\frac{N_c^2-1}{2N_c}\right).\,16\,.$$

$$\cdot\left\{-2\left(1-\frac{\epsilon}{2}\right)I_5 - 3I_2 + \left(6I_4 - I_3\right)q^2\right\}, \tag{9a}$$

which again is expressed in terms of the two-loop integrals listed in Ref. 6). The analogue of Fig. 1b induces terms similar to that of Eq. (5). The renormalized contribution of Fig. 1a is then:

$$\psi^R_{gg}(q^2)\Big|_{Fig\,1a} = \alpha_s\cdot\left(\frac{\alpha_s}{\pi}\right)\left(\frac{3}{\pi^2}\right) m_s^2\cdot q^2 \log\frac{-q^2}{\nu^2}\,.$$

$$\cdot\left\{\log\frac{-q^2}{\nu^2} - \frac{2}{3}\left(4-3\gamma\right)\right\}. \tag{9b}$$

We see that the non-leading-log contributions differ in the 0^{++} and 0^{-+} channels, which may indicate that the γ_5-invariance is not expected to hold in the presence of mass terms[1*].

3. - UPPER BOUND ON THE η' MASS AND THE VALUE OF m_s

By combining the calculations of Section 2 with previous ones[10),11)], we can get a two-loop expression for the two-point correlation $\psi_-(q^2)$ of the SU(3) singlet axial divergence

$$\partial_\mu A^\mu = 2i\,m_s\,\bar\psi_s\gamma_5\psi_s - \left(\frac{3\alpha_s}{4\pi}\right)\vec F\,\tilde{\vec F}\;. \tag{10}$$

(1*)
For more details on the γ_5-invariance in massless QCD, see e.g., Ref. 9).

(as usual we take $m_u = m_d = 0$). We shall work with the "moment" sum rule

$$R(\tau) = -\frac{d}{d\tau} \log \int_0^\infty dt\, e^{-t\tau} \frac{1}{\pi} \mathrm{Im}\, \psi_-(t) , \qquad (11)$$

where the variable τ is the Laplace variable defined by applying to $\psi_-(q^2)$ the familiar Laplace operator[1)]

$$\hat{\mathbb{L}} = \lim_{\substack{Q^2, N \to \infty \\ N/Q^2 \equiv \tau}} \frac{(-1)^N}{(N-1)!} (Q^2)^N \frac{\partial^N}{(\partial Q^2)^N} . \qquad (12)$$

The advantage of $R(\tau)$ is its sensitivity to the meson mass and its lesser sensitivity to the QCD radiative corrections in the unit operator. In the non-relativistic case[1b),12)], the τ-variable plays the role of an imaginary time variable while the minimum of R represents the optimal upper bound on the square of the ground state mass. It was conjectured[13)] that this non-relativistic result can be extended to the relativistic case provided that the non-perturbative contributions at the minimum of R are not too important in order to justify the validity of the approximation used for the QCD estimate of R at the minimum.

Using for the diagonal quark-quark and gluon-gluon parts of $\psi_-(q^2)$ the results of Refs. 10) and 11) to include higher loop corrections, we obtain, to two-loops, the sum rule[1*]

$$R(\tau) = 3\tau^{-1} \Big\{ 1 - \frac{2}{3L} - \frac{9}{2} \hat{m}_s^2 \tau \left(\frac{1}{2} L\right)^{10/9} \cdot \Big[1 +$$

(1*)
An attempt to explain the $SU(3)_F$ breaking contribution to the η' mass, using QCD sum rules, is given in Ref. 14).

$$+\frac{1}{L}\left(4.66-0.7\log L\right)+\pi\alpha_s\langle F^2\rangle\tau^2\Big]+$$

$$+2\pi^2 g_s f^{abc}\langle F^3_{abc}\rangle\tau^3\Big\}, \tag{13}$$

where $L=-\log\tau\Lambda^2$ and $\hat{m}_s$ is the renormalization group invariant mass of the strange quark defined to two-loops in the $\overline{MS}$ scheme[6]. We estimate the uncertainty on R as mainly due to the three-loop[15] and to the four-quark condensates contributions to the quark-quark part of $\psi_-(q^2)$[1a], and also to the off-diagonal piece $\psi_{gq}(q^2)$ obtained previously. Then

$$\Delta R \simeq 3\hat{m}_s^2\left(\frac{1}{2}L\right)^{10/9}\Big\{\frac{9}{2L^2}\left(9.73-3.4\log L+\right.$$

$$\left.+0.5\log^2 L\right)+\frac{2816}{9}\pi^3\alpha_s\langle\bar{\psi}_s\psi_s\rangle^2\tau^3\Big\}. \tag{14}$$

We give one example of the behaviour of R in Fig. 4a where we have used $\langle\alpha_s F^2\rangle = 0.04$ GeV^4[8),16], $g^3\langle F^3\rangle = (1.1\pm0.2)$ GeV^2 $\alpha_s\langle F^2\rangle$[17] and $\Lambda = 150$ MeV. The curve has a minimum for τ about 0.6 - 0.5 GeV^{-2}, where the corrections due to the non-perturbative contribution are rather small, making the information from the minimum of R reliable. Interpreting this minimum of R as an upper bound to the η' mass squared, we give the variation of the bound versus the range of the values of the invariant mass $\hat{m}_s$ where we expect that the approximation used for the derivation of Eq. (13) makes sense. We can see that the observed value of the η' mass can be obtained for the values of $\hat{m}_s$ in the range smaller than 0.37 GeV. If we combine this upper bound to the lower bound obtained from other QCD sum rules analysis[1c),11),15),18], we obtain for $\Lambda \simeq 150$ MeV the range:

$$0.21 \leq \hat{m}_s \leq 0.37 \; GeV . \tag{15}$$

The above results emphasize the role of the $SU(3)_F$ breaking parameter m_s^2 in the physical value of the η' mass. A similar conclusion has been reached in Ref. 13) where the m_s^2-effect on the η'-mass relation from $U(1)_A$ current algebra has been taken into account.

4. - MESON-GLUONIUM MIXING ANGLE

For definiteness, we discuss explicitly only the case of the 0^{-+} mesons. By applying the Laplace operator defined in Eq. (12) to the second derivative of $\psi_{gq}(q^2)$, which is superficially convergent, and following familiar renormalization group arguments[6)], one obtains the sum rule:

$$\int_0^\infty dt\, e^{-t\tau} \frac{1}{\pi} \operatorname{Im} \psi_{gq}(t) = \left(\frac{3}{4\pi}\right) \cdot \frac{3}{\pi} \left(\frac{\alpha_s}{\pi}\right)^2 \tau^{-2} \Big\{ \left(\frac{1}{6} + 2\gamma\right) \bar{m}_s^2 - \frac{8\pi^2}{3} m_s \langle \bar{\psi}_s \psi_s \rangle \tau + \frac{2\pi^2}{3} \left(\bar{m}_s^2 \log(\tau \bar{m}_s^2)\right) \langle F^2 \rangle \tau^2 \Big\} , \tag{16}$$

where $\alpha_s/\pi = -\dfrac{4}{9 \log \tau \Lambda^2}$ for $SU(3)_c \times SU(3)_F$;

$$\bar{m}_s \equiv \hat{m}_s \cdot / \left(-\frac{1}{2} \log \tau \Lambda^2\right)^{4/9}$$

is the running quark mass expressed in terms of the renormalization group invariant mass introduced in Ref. 19). We plot, in Fig. 5 , the relative strengths of each term within brackets of Eq. (16) normalized to

$(\bar{\alpha}_s/\pi)^2 \cdot 9/4\pi^2$. The lowest order term starts to dominate the non-perturbative terms for τ smaller than 0.5 GeV^{-2}. We have used $m_s\langle\bar{\psi}_s\psi_s\rangle \simeq$ (−)0.5 $M_K^2 f_K^2$ which takes into account the effect of $SU(3)_F$ breaking parameters to kaon PCAC[20]. We have taken the invariant mass $\hat{m}_s$ to be 0.3 GeV which is an average of various estimates[1c),15),18)]; we have used $\alpha_s\langle F^2\rangle$ = 0.04 GeV^4[8),16)] and Λ = 0.15 GeV.

For the discussion of the mixing problem, we follow the standard procedure and parametrize the spectral function Im $\psi_{gq}(t)$ using a two-component mixing formalism:

$$|G\rangle = \cos\theta\, |gg\rangle + \sin\theta\, |qq\rangle$$

$$|P\rangle = -\sin\theta\, |gg\rangle + \cos\theta\, |qq\rangle \quad , \tag{17}$$

where $|G\rangle$ and $|P\rangle$ denote the physically observed meson states, $|qq\rangle$ and $|gg\rangle$ are the pure meson and the pure gluonium states, and θ is the mixing angle. Then,

$$\frac{1}{\pi}\, \mathrm{Im}\, \psi_{gq}(t) \simeq \sin 2\theta\, M_{qq}^2 f_{qq} M_{gg}^2 f_{gg} \cdot \left\{\delta(t - M_G^2) - \delta(t - M_P^2)\right\} , \tag{18a}$$

where the QCD continuum is the one given by the discontinuity of the lowest order diagram in Fig. 1. We have defined the decay constants f_{qq} and f_{gg} by analogy with the pion decay amplitude, $f_\pi \simeq$ 93 MeV:

$$\langle 0 |\, J_q^- | qq\rangle = \sqrt{2}\, M_{qq}^2 f_{qq} \;, \tag{18b}$$

$$\langle 0 | \left(\frac{3}{4\pi}\right) J_g^- | gg\rangle = \sqrt{2}\, M_{gg}^2 f_{gg} \;. \tag{18c}$$

Then[1*]:

$$\int_0^\infty dt\, e^{-t\tau}\, \frac{1}{\pi}\, \mathrm{Im}\, \psi_{gq}(t) = \sin 2\theta\, M_{qq}^2 f_{qq} \cdot$$
$$\cdot\, M_{gg}^2 f_{gg} \left(e^{-M^2\tau} - e^{-M_G^2\tau}\right) + \left(\frac{3}{4\pi}\right)\left(\frac{\bar\alpha_s}{\pi}\right)^2 \frac{3}{\pi}\, \tau^{-2} \cdot$$
$$\cdot\, e^{-t_c\tau}\Big\{\left(1 + t_c\tau\right)\left(\frac{1}{6} + 2\delta\right)\bar m_s^2 -$$
$$- \frac{8\pi^2}{3} m_s \langle \bar\psi_s \psi_s\rangle \tau - \frac{2\pi^2}{3}\bar m_s^2 \langle F^2\rangle \tau^2 e^{-t_c\tau}\Big\}. \tag{19}$$

In confronting Eqs. (16) and (19), we do a numerical analysis based on the FUMILI χ^2-minimization programme used in Ref. 13)[2*], by demanding a coincidence of both equations for τ smaller than τ_{max} where we hope that the QCD expression in Eq. (16) makes sense. For the fitting procedure, we use $M_G \equiv M_\iota \simeq 1.44$ GeV and $M_P \equiv M_{\eta'} \simeq 0.96$ GeV. We also introduce the parameter:

$$k \equiv \sin 2\theta\, M_{qq}^2 f_{qq}\, M_{gg}^2 f_{gg}\,, \tag{20}$$

so that the free parameters in the fitting procedure will be k and t_c. For definiteness, we fix Λ to be 0.15 GeV and we do a two-parameter fit for two characteristic values of $\hat m_s$ [1c),15),18)]. The results of the fitting procedure are shown in Fig. 6a. The arrow indicates the value of τ where previous gluonium sum rules [Eq. (13)] and the one in Ref. 13) present an extremum. In our case, we cannot have any extremum as the leading theoretical contributions add (perturbative plus fermion condensate) and increase for τ going to zero. However, we expect that the leading order expansion in Eq. (16) can be a good

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(1*)
The contribution of the last term in Eq. (19) has been deduced from the result in Ref. 21).
(2*)
If one uses the finite-energy sum rule discussed in Ref. 21), one does not obtain any useful information within the approximation within which Eq. (16) is computed.

approximation of the full theoretical expression for τ smaller than 0.5 GeV^{-2}, where known terms in Eq. (16) are less than 50% of the lowest order one. In this way, we deduce for $\Lambda = 0.15$ GeV:

$$\theta \geq (2.5 - 5)^{\circ} \tag{21a}$$

$$\sqrt{t_c} \geq 2.8 \text{ GeV}, \tag{21b}$$

where we have used the values of $f_{gg} \simeq 30$ MeV[13)], $M_{gg} \simeq 1.4$ GeV[13),22)], $M_{qq} \simeq 1$ GeV[13)1*] and $f_{qq} \simeq \sqrt{3}\, f_\pi$, as given by the quark model.
It may be difficult to have here a clear statement on the optimal estimate of the mixing angle and the correlated value of the continuum threshold. However, one can notice from Fig. 6a that $\sqrt{t_c}$ is almost stable for τ larger than 0.3 GeV^{-2}. So, we would expect the following range of values:

$$\theta \simeq (2.5 \sim 11.1)^{\circ} \tag{22a}$$

$$\sqrt{t_c} \simeq (2.8 \sim 3.2) \text{ GeV}, \tag{22b}$$

for $\hat{m}_s \simeq 0.2$–0.3 GeV and $\Lambda \simeq 0.15$ GeV. We analyze in Fig. 6b the dependence of the result on the value of Λ. $\sqrt{t_c}$ is almost insensitive to the variations of Λ whereas θ is sensitive, because of the α -dependence of the QCD side of the sum rule. For Λ between 0.1 and 0.2 GeV, and taking into account the result in Eq. (22a), we would expect a range of values of θ:

$$\theta \simeq (1.6 \sim 16)^{\circ} . \tag{22c}$$

(1*) One should remember that the reality of the eigenvalues of the gluonium-meson mass matrix imposes $M_G^2 + M_P^2 = M_{gg}^2 + M_{qq}^2$ and $M_G^2 M_P^2 \leq M_{gg}^2 M_{qq}^2$ which implies $M_{qq} \geq M_P$ and $M_{gg} \lesssim M_G$. (G and P are the physically observed states.)

One should notice that despite the large uncertainty of the estimate of θ, one might still conclude that the gluonium-meson mixing angle should be small. An immediate consequence of the result in Eq. (22) is the fact that the gluonium mass obtained within pure $SU(3)_C$ Yang-Mills theory[13),23)] should not be far from the observed values of the gluonium masses. Other applications concern the predictions of the radiative and of the two photon decays of the $\iota(1.44)$ (if it is a gluonium !). The radiative decay of the ι normalized to the η'-one is:

$$\frac{\Gamma(\iota \to \rho\gamma)}{\Gamma(\eta' \to \rho\gamma)} \simeq tg^2\theta\left(\frac{k_\iota}{k_{\eta'}}\right)^3 \lesssim 2.2 \tag{23a}$$

where $$k_j \equiv \frac{M_j^2 - M_\rho^2}{2M_j} .$$

This leads to

$$\Gamma(\iota \to \rho\gamma) \lesssim (185 \pm 66)\ keV \tag{23b}$$

The two-photon decay of the ι normalized to the η'-one is:

$$\frac{\Gamma(\iota \to \gamma\gamma)}{\Gamma(\eta' \to \gamma\gamma)} \simeq tg^2\theta\left(\frac{M_\iota}{M_{\eta'}}\right)^3 \lesssim 0.28 \tag{24a}$$

Then:

$$\Gamma(\iota \to \gamma\gamma) \lesssim (1.5 \pm 0.5)\ keV \tag{24b}$$

It seems premature for the moment to compare theoretical expectations for these decays with experimental data[24)], as there is some conflict between various measurements. Furthermore, the spin parity analysis of the $\rho\gamma$-signal in ψ radiative decays is yet to be done. Concerning the various theoretical predictions, the value of the mixing angle in Eq. (22) is smaller than the one obtained from a bag model-like calculations[4),5)] or the one obtained from the lowest meson saturation of the $U(1)_A$ Ward identities[3)]. We would also note that the phenomenology of the 0^{++} channel could be done in a similar way. However, it seems at the present time[25)] that the analysis of the decays of the G (1.6) into two pseudoscalar mesons is of more immediate interest.

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FIGURE CAPTIONS

Fig. 1: a) Lowest order contribution to $\psi_{gq}(q^2)$. ● denotes the gluonic current, ⊗ denotes the quark current, × is the quark mass insertion.

b) Lowest order contribution to $\psi_{gq}(q^2)$ induced by the second term in Eqs. (3).

Fig. 2: a) $<\bar{\psi}\psi>$ contribution to $\psi_{gq}(q^2)$.

b) $<\bar{\psi}\sigma^{\mu\nu}(\lambda a/2)\ \psi\ F^{a}_{\mu\nu}>$ contribution.

c) $<F^2>$ contribution.

Fig. 3): a),b) $<\bar{\psi}\ \Gamma_1\psi\bar{\psi}\Gamma_2\psi>$ contribution.

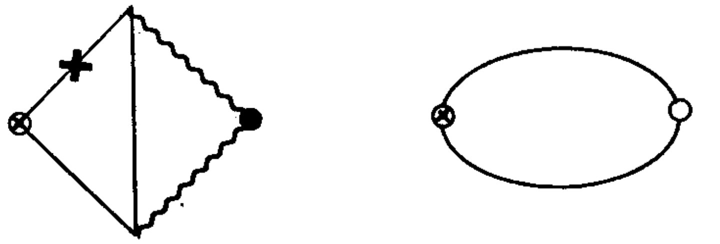
c) $<F^3>$ contribution.

Fig. 4: a) Variation of $R(\tau)$ versus τ.

b) Upper bound on the η'-mass versus the invariant mass $\hat{m}_s$.

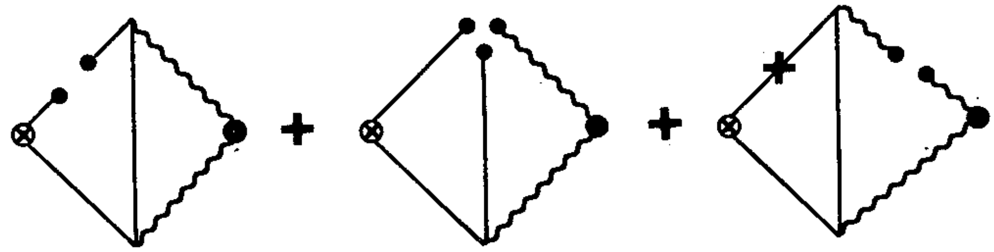
Fig. 5: Variation of the absolute values of various terms of Eq. (16) normalized to $(\bar{\alpha}_s/\pi)^2(9/4\pi^2)$; —— m_s^2-term; —•—•—• $<\bar{\psi}_s\psi_s>$; --- $<F^2>$.

Fig. 6: We have used $\hat{m}_s = 0.3$ GeV, $\Lambda \simeq 0.15$ GeV and $\alpha_s<F^2>\simeq 0.04$ GeV^4. Variation of k defined in Eq. (20) versus different values of the parameters.



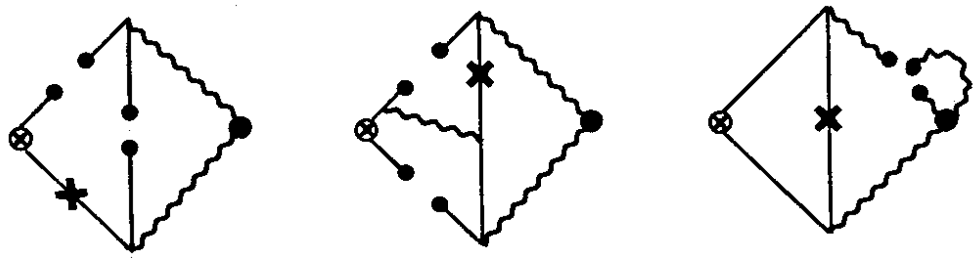
a) b)

Fig. 1



a) b) c)

Fig. 2



a) b) c)

Fig. 3

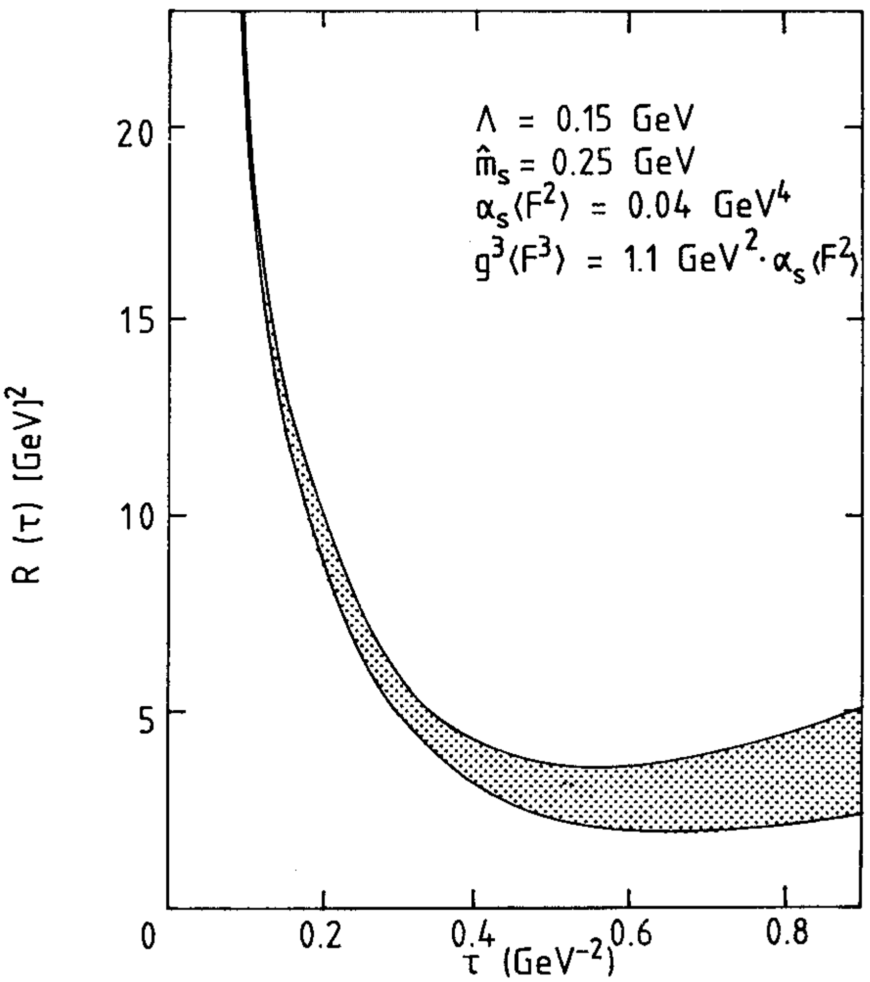

Λ = 0.15 GeV
$\hat{m}_s$ = 0.25 GeV
$\alpha_s \langle F^2 \rangle$ = 0.04 GeV^4
$g^3 \langle F^3 \rangle$ = 1.1 GeV$^2 \cdot \alpha_s \langle F^2 \rangle$
R (τ) [GeV]^2
20
15
10
5
0
0.2
0.4
0.6
0.8
τ (GeV^{-2})

Fig. 4a

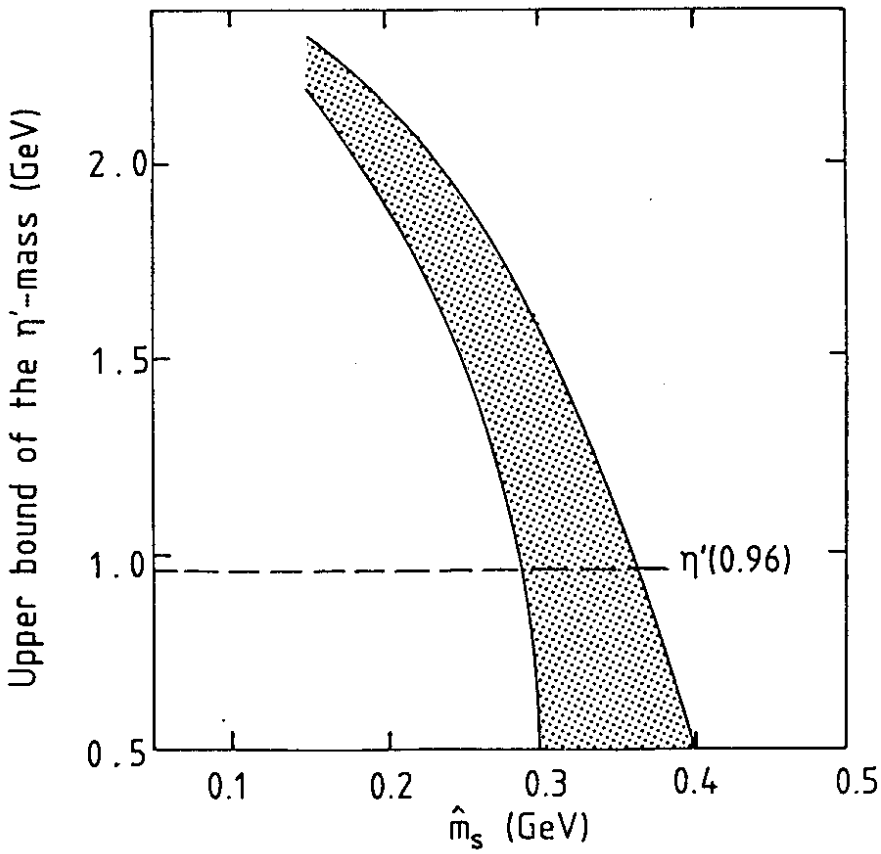
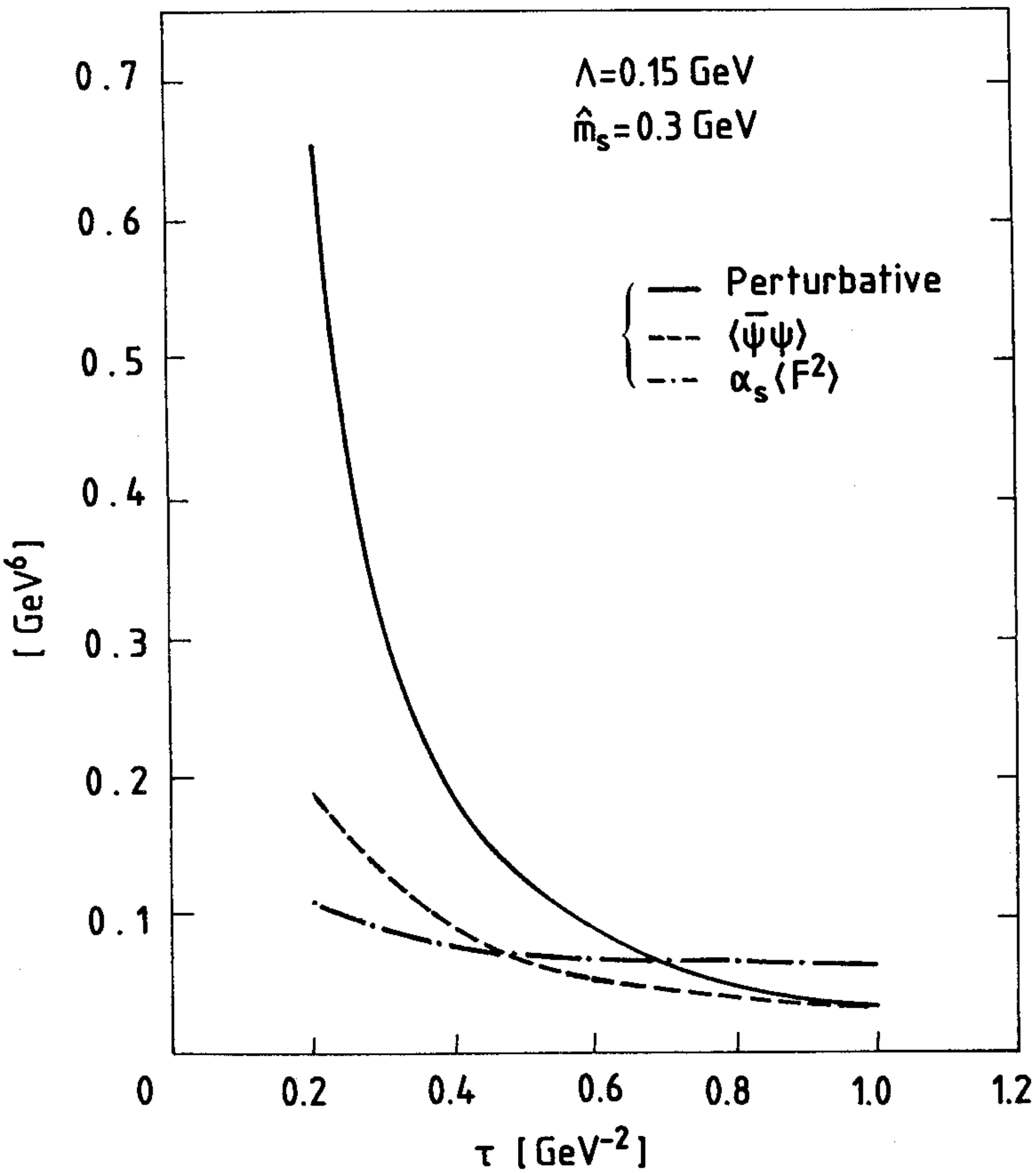
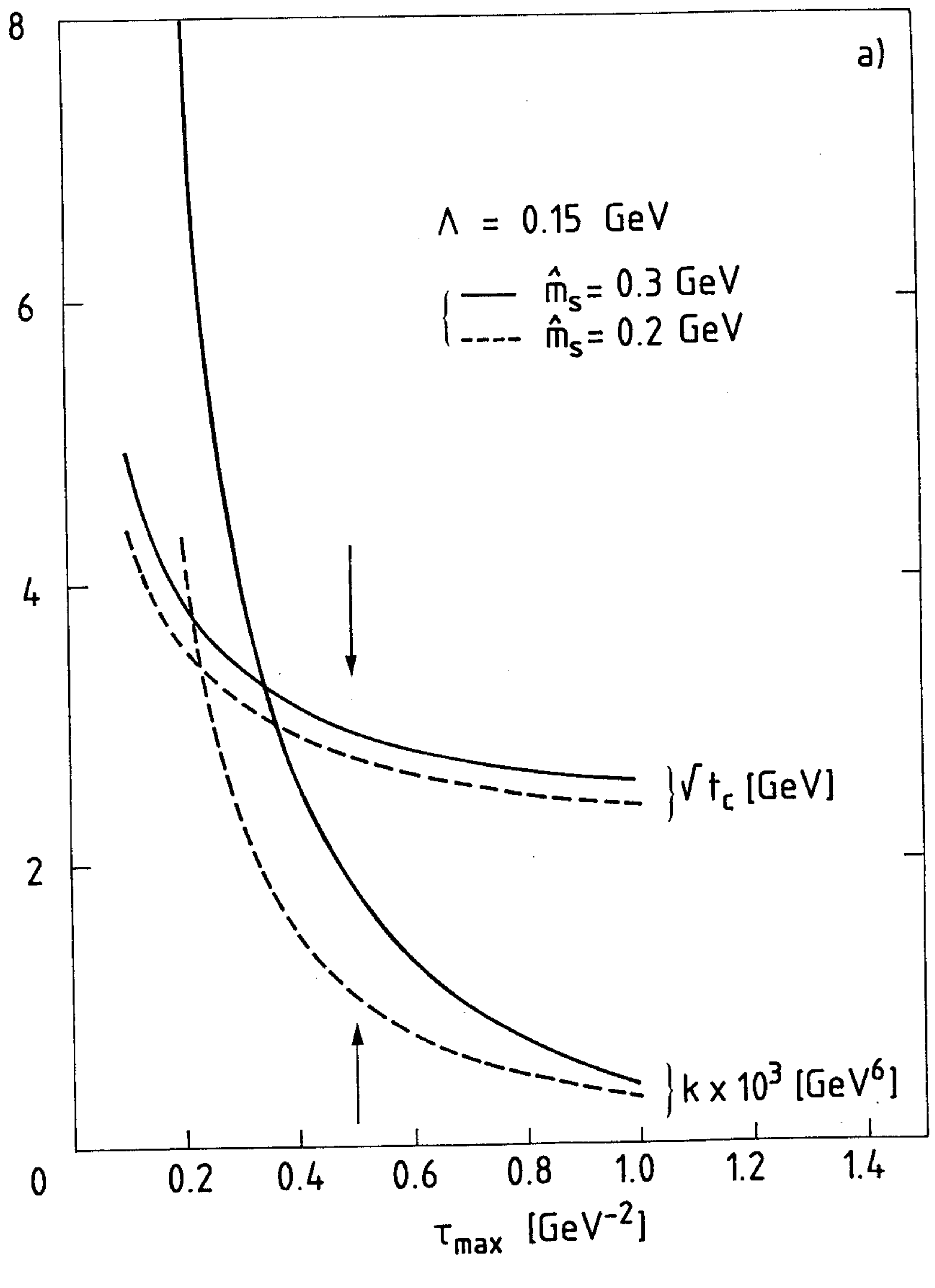

Upper bound of the η'–mass (GeV)
2.0
1.5
1.0
0.5
η'(0.96)
0.1
0.2
0.3
0.4
0.5
$\hat{m}_s$ (GeV)

Fig. 4b



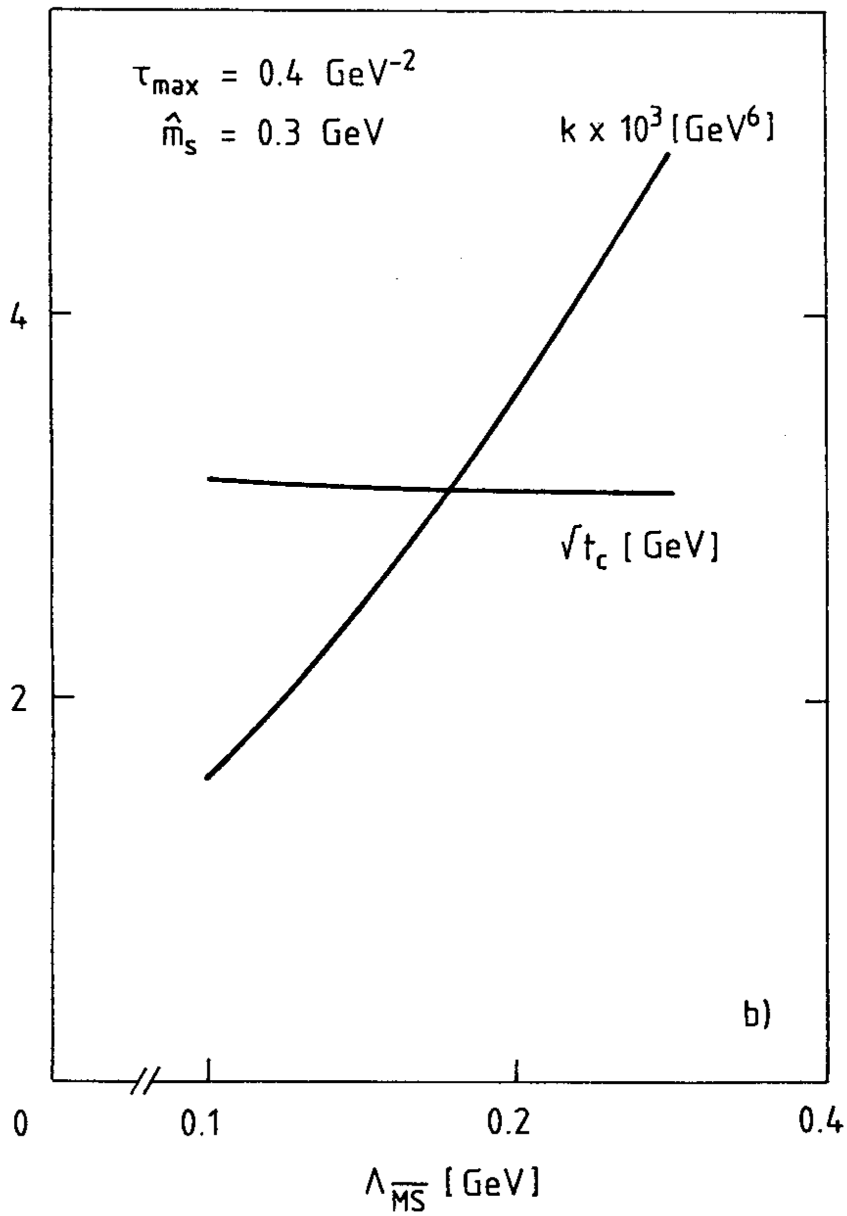
Λ=0.15 GeV
$\hat{m}_s$=0.3 GeV
Perturbative
$\langle\bar{\psi}\psi\rangle$
$\alpha_s\langle F^2\rangle$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
[GeV^6]
0
0.2
0.4
0.6
0.8
1.0
1.2
τ [GeV^{-2}]

Fig. 5



8
6
4
2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
a)
Λ = 0.15 GeV
$\hat{m}_s$ = 0.3 GeV
$\hat{m}_s$ = 0.2 GeV
$\sqrt{t_c}$ [GeV]
$k \times 10^3$ [GeV^6]
τ_{max} [GeV^{-2}]

Fig. 6a



τ_{max} = 0.4 GeV^{-2}
$\hat{m}_s$ = 0.3 GeV
k x 10^3 [GeV^6]
$\sqrt{t_c}$ [GeV]
4
2
b)
0
0.1
0.2
0.4
$\Lambda_{\overline{MS}}$ [GeV]

Fig. 6b