Measurement of the Mass Difference Between Top and Anti-top Quarks

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We present a measurement of the mass difference between top (t) and anti-top ($\bar{t}$) quarks using $t\bar{t}$ candidate events reconstructed in the final state with one lepton and multiple jets. We use the full data set of Tevatron $\sqrt{s} = 1.96$ TeV proton-antiproton collisions recorded by the CDF II detector, corresponding to an integrated luminosity of 8.7 fb^{-1}. We estimate event-by-event the mass difference to construct templates for top-quark signal events and background events. The resulting mass difference distribution of data compared to signal and background templates using a likelihood fit yields $\Delta M_{\text{top}} = M_t - M_{\bar{t}} = -1.95 \pm 1.11$ (stat) ± 0.59 (syst) GeV/c^2 and is in agreement with the standard model prediction of no mass difference.

The laws of the standard model of particle physics (SM) are invariant under the simultaneous trans-

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formations of charge conjugation, parity, and time reversal (CPT). Conservation of CPT is, therefore, fundamental and provides one of the most important constraints on the SM. However, examining any possibility of CPT violation is important, as there are well-motivated extensions of the SM allowing for CPT symmetry breaking [1]. In CPT-conserving models, particles and their anti-particles must have identical masses and widths. Thus, any mass difference between a particle and its anti-particle would indicate a violation of CPT. CPT invariance has been tested for many elementary particles such as leptons and hadrons [2, 3], but not in the quark sector except for the top quark [4]. For all quarks except the top quark, direct mass measurements are nearly impossible because the quark hadronization time scale is approximately an order of magnitude less than the quark decay time. After hadronization occurs, only the masses of hadrons are observable and give, at best, only an approximate estimate of the constituent quark's masses. On the other hand, as the lifetime of the top quark is of the order of 10^{-24} seconds, it decays before hadronizing and a precision measurement of its mass and of the difference between the quark and anti-quark masses can be made.

Since the top quark discovery, close to three thousand of $t\bar{t}$ events has been collected at the Tevatron $p\bar{p}$ collider. This sample makes measuring the top-quark mass (M_{top}) possible to an accuracy of approximately 0.5% ($M_{\text{top}} = 173.2 \pm 0.9$ GeV/c^2) [5] and the mass difference ($\Delta M_{\text{top}} =$

$M_t - M_{\bar{t}}$) between t and $\bar{t}$ quarks to a comparable precision. The D0 collaboration performed a few measurements of $\Delta M_{\rm top}$ using matrix element analyses [6, 7]. The most recent D0 result, based on a 3.6 fb^{-1} data sample, reports $\Delta M_{\rm top} = 0.8 \pm 1.9$ GeV/c^2, consistent with zero as predicted in the SM. The CDF collaboration performed a measurement using a 5.6 fb^{-1} data sample [8] and found $\Delta M_{\rm top} = -3.3 \pm 1.7$ GeV/c^2 which is also consistent with zero to within two standard deviations. To date, the most precise measurement is performed by the CMS collaboration, $\Delta M_{\rm top} = -0.44 \pm 0.53$ GeV/c^2 [9].

This paper reports on the final CDF measurement of $\Delta M_{\rm top}$ based on the full Run II data set corresponding to an integrated luminosity of 8.7 fb^{-1}. We reconstruct the mass difference between t and $\bar{t}$ quarks in each data event and compare its distribution with template distributions derived from Monte Carlo (MC) model simulations to estimate $\Delta M_{\rm top}$. This is an update of a previous measurement that used a subset of the present data [8]. In addition to the larger data sample, we improve the jet energy calibration by applying an artificial neural network to achieve better jet energy resolution [10], as in a recent measurement of $M_{\rm top}$ [11]. We also re-examine the systematic uncertainties resulting from the larger size of the control samples.

In the SM, t and $\bar{t}$ quarks decay almost exclusively into a W boson and a bottom quark ($t \to bW^+$ and $\bar{t} \to \bar{b}W^-$) [12]. The case where one W boson decays to a charged lepton (electron or muon) and a neutrino ($W^+ \to \ell^+\nu$ or $W^- \to \ell^-\bar{\nu}$ including the cascade decay of $W \to \tau\nu$ and $\tau \to \ell\nu$) and the other to a pair of jets, defines the lepton+jets channel. To select $t\bar{t}$ candidate events in this channel, we require one electron (muon) with $E_T > 20$ GeV ($p_T > 20$ GeV/c) and pseudorapidity $|\eta| < 1.1$ [13]. We also require large missing transverse energy [14] ($\not{E}_T > 20$ GeV) and at least four jets. Jets are reconstructed applying a cone algorithm with radius $\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} = 0.4$ [15]. Besides the standard jet energy scale corrections [16], we use an artificial neural network that includes additional information to the calorimeter one, such as jet momentum from the charged particles inside the jet [10]. This additional information improves the resolution on the reconstructed jet variables, resulting in approximately a 10% improvement in statistical precision. Jets originating from b quarks are identified (tagged) using a secondary vertex tagging algorithm [17]. In order to optimize the background reduction and to improve the statistical power of the measurement, we divide the sample of $t\bar{t}$ candidates into subsamples with zero (0-tag), one (1-tag), and two or more (2-tag) b-tagged jets.

For the 0-tag events, we require exactly four *tight* jets (transverse energy $E_T > 20$ GeV and $|\eta| < 2.0$). In case of the 1-tag and 2-tag events, three tight jets and one or more *loose* jets ($E_T > 12$ GeV and $|\eta| < 2.4$) are required. To reduce background contributions in the 0-tag or 1-tag events, we require the scalar sum of transverse energies in the event, $H_T = E_T^{\rm lepton} + \not{E}_T + \sum_{\rm four\ jets} E_T^{\rm jet}$,

to exceed 250 GeV. The H_T requirement is not applied to the 2-tag events because of the small background contribution in this subsample. We divide the 1-tag and 2-tag samples into subsamples based on the number of tight jets. We denote as *tight subsample* the sample requiring exactly four tight jets and *loose subsample* the sample consisting of the remaining events. This results in five subsamples: 0-tag, 1-tagL, 1-tagT, 2-tagL, and 2-tagT, where T and L denote tight and loose subsamples, respectively.

The primary sources of background events are W+jets and QCD multijet processes. To estimate the contribution of each process, we use a combination of data- and MC-based techniques described in Ref. [18]. For the Z+jets, diboson, single top quark, and $t\bar{t}$ events we normalize the number of simulated events using their theoretical cross sections [19–21]. We use the data-driven techniques described in Ref. [22] to estimate the QCD multijet background. The W+jets background shape is modeled using MC generated samples but the number of events is derived from the data sample by subtracting all other contributions, including the $t\bar{t}$ signal, from the data events. Table I summarizes the data sample composition.

We assume that all selected events are lepton+jets $t\bar{t}$ events and reconstruct $\Delta M_{\rm top}$, event-by-event, using a special-purpose kinematic fitter [8]. Measured four-vectors of the lepton and jets are corrected for known effects as described in Ref. [16], and appropriate resolutions are assigned. The unclustered transverse energy (U_T) is estimated as a sum of all transverse energy in the calorimeters that is not associated with the primary lepton or with one of the leading four jets. It is used to calculate the neutrino transverse momentum. The longitudinal momentum of the neutrino is a free parameter which is effectively determined by the constraint on the invariant mass of the leptonically-decaying W boson. To estimate $\Delta M_{\rm top}$, we define a kinematic χ^2 function,

$$\begin{aligned}
\chi^2 =\ & \Sigma_{i=\ell,4{\rm jets}}(p_T^{i,{\rm fit}} - p_T^{i,{\rm meas}})^2/\sigma_i^2 \\
& + \Sigma_{k=x,y}(U_{T_k}^{\rm fit} - U_{T_k}^{\rm meas})^2/\sigma_k^2 \\
& + (M_{jj} - M_W)^2/\Gamma_W^2 + (M_{\ell\nu} - M_W)^2/\Gamma_W^2 \\
& + \{M_{bjj} - (M_{\rm top}^{\rm ave.} + dm_{\rm reco}/2)\}^2/\Gamma_t^2 \\
& + \{M_{b\ell\nu} - (M_{\rm top}^{\rm ave.} - dm_{\rm reco}/2)\}^2/\Gamma_t^2, \qquad (1)
\end{aligned}$$

where $dm_{\rm reco}$ is obtained at the lowest χ^2 and represents the reconstructed mass difference between the hadronically- and leptonically-decaying top quarks, $M_{bjj} - M_{b\ell\nu}$. In Eq. (1), we constrain the lepton p_T and the four leading jets p_T to their measured values and uncertainties (σ_i). We also constrain U_T in the second term of Eq. 1. In the remaining terms, we constrain the W boson mass (M_W) to M_W=80.4 GeV/c^2 and the average of t and $\bar{t}$ masses to $M_{\rm top}^{\rm ave.}$=172.5 GeV/c^2. The quantities $M_{jj}, M_{\ell\nu}, M_{bjj}$, and $M_{b\ell\nu}$ refer to the invariant masses of the particles denoted in the subscripts. The total widths of the W boson, $\Gamma_W = 2.1$ GeV, and of the

TABLE I: Expected and observed numbers of signal and background events assuming a $t\bar{t}$ production cross section $\sigma_{t\bar{t}} = 7.45$ pb and $M_{\rm top} = 172.5$ GeV/c^2.

	0-tag	1-tagL	1-tagT	2-tagL	2-tagT
W+jets	778±219	197±69	114±42	11.4±4.9	8.0±3.4
Z+jets	55.7±4.9	10.3±1.2	6.7±0.8	0.8±0.2	0.5±0.1
Single top	5.1±0.4	11.7±1.0	7.2±0.6	2.2±0.2	1.7±0.2
Diboson	63.9±5.9	11.7±1.5	9.0±1.2	0.9±0.2	0.9±0.2
QCD multijet	133±107	31.7±1.2	20.9±16.9	4.3±4.3	2.9±3.5
Total background	1038±244	262±70	158±45	19.5±6.5	14.0±5.0
$t\bar{t}$ signal	620±83	694±87	847±105	188±29	294±45
Expected	1658±257	957±111	1005±114	208±30	308±45
Observed	1712	919	1018	214	286

top quark, $\Gamma_t = 1.5$ GeV, are taken from Ref. [12]. We assume that the total widths of the t and $\bar{t}$ quarks are equal. Determining the reconstructed mass difference of t and $\bar{t}$, $\Delta m_t^{\rm reco}$, requires the identification of the particle type (t or $\bar{t}$), which is achieved using the electric charge of the lepton ($Q_{\rm lepton}$), $\Delta m_t^{\rm reco} = -Q_{\rm lepton} \cdot dm_{\rm reco}$. In the events with a positive (negative) lepton, t ($\bar{t}$) decays leptonically and $\bar{t}$ (t) decays hadronically. Because of the different resolutions of the jets, lepton, and unclustered energy, the $\Delta m_t^{\rm reco}$ distribution from the positive lepton events is different from that of the negative lepton events. To improve the resolution of the $\Delta m_t^{\rm reco}$, and allow using the appropriate distribution in the hadronic-to-leptonic and in the leptonic-to-hadronic mass difference, we divide each subsample into the two new subsamples based on the lepton charge. We then have ten subsamples in total.

Assuming that the leading four jets in any event come from the four final quarks at the hard scattering level, there are 12, 6, and 2 possible jet-to-quark assignments for 0-tag, 1-tag, and 2-tag samples, respectively. The χ^2 minimization is performed for each jet-to-quark assignment, and $\Delta m_t^{\rm reco}$ is taken from the assignment that yields the lowest χ^2 ($\chi^2_{\rm min}$). The b-tagged (zero b-tag) events with $\chi^2_{\rm min} > 9.0$ ($\chi^2_{\rm min} > 3.0$) are rejected due to the poorly reconstructed kinematical properties. To increase the statistical power of the measurement, we employ an additional observable, $\Delta m_t^{\rm reco(2)}$, which corresponds to the 2nd lowest χ^2 in the jet-to-quark combinatorics. Although it has a poorer sensitivity, $\Delta m_t^{\rm reco(2)}$ provides additional information on $\Delta M_{\rm top}$ and reduces the statistical uncertainty by approximately 10%. We use two observables ($\Delta m_t^{\rm reco}$ and $\Delta m_t^{\rm reco(2)}$) simultaneously for the measurement.

Using MADGRAPH [24], we generate $t\bar{t}$ signal samples with $\Delta M_{\rm top}$ between -20 GeV/c^2 and 20 GeV/c^2 in 2 GeV/c^2 intervals. Parton showering of the signal events is simulated with PYTHIA [25], and the CDF II detector is simulated using a GEANT-based software package [26].

We estimate the probability density functions (PDFs) of signal and background using the kernel density es-

TABLE II: Summary of systematic uncertainties on $\Delta M_{\rm top}$.

Source	Uncertainty (GeV/c^2)
Signal modeling	0.14
Parton showering	0.17
b and $\bar{b}$ jets asymmetry	0.38
Higher-Order effect	0.16
Jet energy scale	0.07
Parton distribution functions	0.12
b-jet energy scale	0.05
Background shape	0.20
Gluon fusion fraction	0.05
Initial and final state radiation	0.10
Finite Monte Carlo samples	0.07
Lepton energy scale	0.06
Multiple hadron interaction	0.05
Color reconnection	0.23
Total systematic uncertainty	0.59

timation [27, 28]. We construct the two dimensional PDFs that account for the correlation between $\Delta m_t^{\rm reco}$ and $\Delta m_t^{\rm reco(2)}$. First, at discrete values of $\Delta M_{\rm top}$ from -20 GeV/c^2 to 20 GeV/c^2, we estimate the PDFs for the observables from the above-mentioned MADGRAPH $t\bar{t}$ samples. We interpolate the MC distributions to find PDFs for arbitrary values of $\Delta M_{\rm top}$ using the local polynomial smoothing method [29]. Then, we fit the signal and background PDFs to the unbinned distributions observed in the data using a maximum likelihood fit [30]. Separate likelihoods are built for the ten subsamples, and the overall likelihood is obtained by multiplying them together.

We test the fitting procedure generating 3000 simulated experiments for each of eleven equally-spaced $\Delta M_{\rm top}$ values ranging from -10 GeV/c^2 to 10 GeV/c^2. The fit estimates and their uncertainties in the simulated experiments are found to be unbiased.

We examine a variety of systematic effects that could affect the $\Delta M_{\rm top}$ measurement. To estimate the system-

atic uncertainties, we compare the results from simulated experiments in which we vary relevant parameters within one standard deviation. All systematic uncertainties are summarized in Table II. The dominant source of systematic uncertainty is attributed to a possible difference in the detector response between b and $\bar{b}$ jets. To estimate this effect, we select a $b\bar{b}$ sample by requiring exactly two b-tagged jets per event using a sample triggered on jet ($E_T > 20$ GeV). In addition, one b-tagged jet is required to contain a soft muon from leptonic decay so that the charge of the b quark associated with the jet can be determined. The energy scale of b and $\bar{b}$ jets in the data is compared with di-jet MC events to estimate independently the difference of the b and $\bar{b}$ jet energy scale. The result ($-0.44 \pm 0.40\%$) is consistent between data and MC events. We perform simulated experiments by varying the b and $\bar{b}$ energy separately. We estimate the signal modeling uncertainty by using simulated experiments with events generated with MADGRAPH and PYTHIA. We also estimate a parton showering uncertainty by applying different showering models (PYTHIA and HERWIG [31]) to a sample generated with ALPGEN [32]. Higher-order effects are estimated using MC@NLO generator [33]. The background shape systematic uncertainty accounts for the variation of the background composition. The color reconnection systematic uncertainty [34] is evaluated using samples simulated with and without color reconnection effects in PYTHIA tunes [35]. We vary the parameters of parton distribution functions to account for systematic effects. The jet energy scale uncertainty, the dominant uncertainty in most of the M_{top} measurements, is partially canceled in the t and $\bar{t}$ mass difference. Other sources of systematic effects, including uncertainties in gluon radiation, multiple hadron interaction, finite size of MC samples, b-jet energy scale, and lepton energy scale, give small contributions. The total systematic uncertainty of 0.59 GeV/c^2 is calculated as a quadrature

sum of the listed uncertainties.

The resulting mass difference is

$$\Delta M_{\text{top}} = -1.95 \pm 1.11 \text{ (stat)} \pm 0.59 \text{ (syst) GeV}/c^2. \quad (2)$$

Figure 1 shows the observed distributions of the observables used for the ΔM_{top} measurement. The density estimates for $t\bar{t}$ signal events with $\Delta M_{\text{top}} = 0$ GeV/c^2 and for background events are overlaid.

In conclusion, we examine the mass difference between t and $\bar{t}$ quarks in the lepton+jets channel using CDF II data corresponding to an integrated luminosity of 8.7 fb^{-1} from $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV. We measure the mass difference to be $\Delta M_{\text{top}} = M_t - M_{\bar{t}} = -1.95 \pm 1.11$ (stat) ± 0.59 (syst) GeV/$c^2 = -1.95 \pm 1.26$ GeV/c^2. This result is consistent with $\Delta M_{\text{top}} = 0$ GeV/c^2 and conservation of CPT symmetry.

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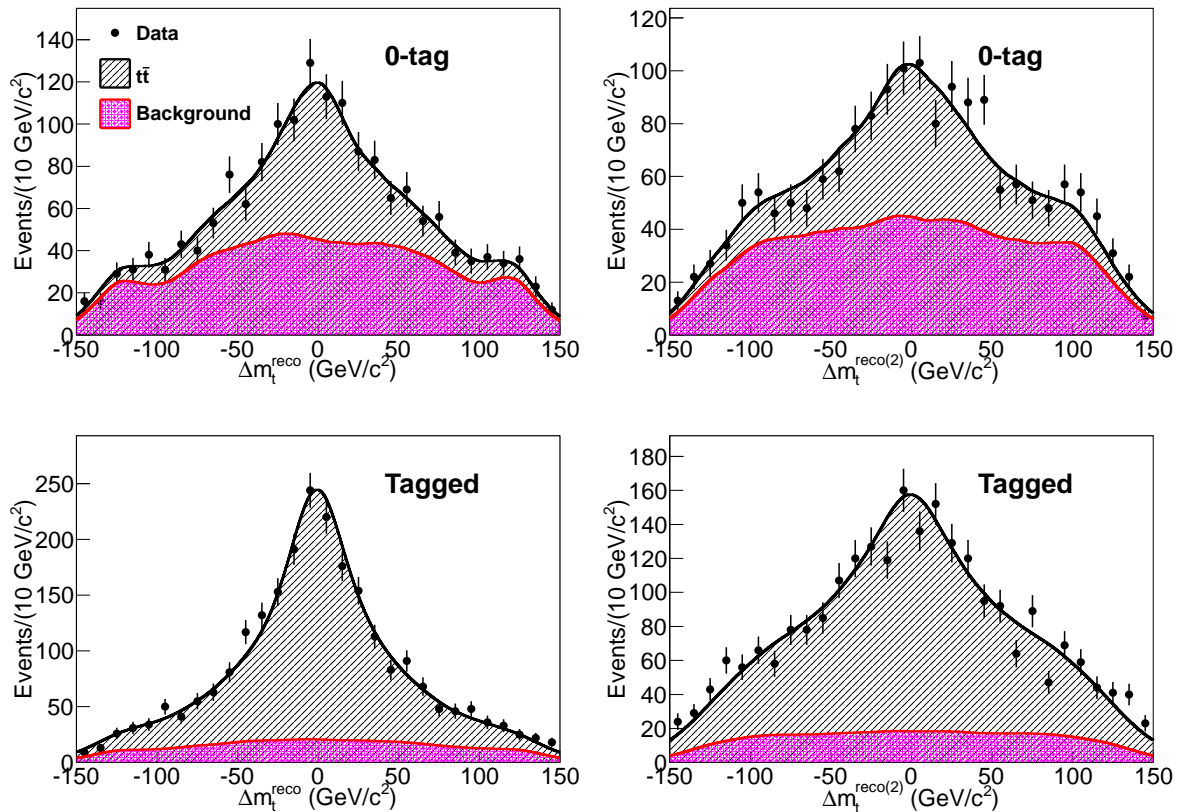[14] The missing transverse energy, an imbalance of energy in

FIG. 1: Distributions of Δm_t^{reco} and $\Delta m_t^{\text{reco}(2)}$ used to extract ΔM_{top} for zero b-tagged (0-tag) events and one or more b-tagged (Tagged) events. The data are overlaid with predictions from the kernel density estimation probability distributions assuming $\Delta M_{\text{top}} = 0$ GeV/c^2. The fitted number of signal and background events are used.

the transverse plane of the detector, is defined by $\not{E}_T = -|\sum_{towers} E_T \hat{n}_T|$, where $\hat{n}_T$ is the unit vector normal to the beam and pointing to a given calorimeter tower and E_T is the transverse energy measured in that tower.

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