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NON-PERTURBATIVE QCD VACUUM FROM $e^+e^- \to I = 1$ HADRON DATA

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ABSTRACT

We estimate the gluon vacuum condensate $\alpha_s \langle F^2 \rangle$ from the $e^+e^- \to I = 1$ hadron cross-section known below 2 GeV using moment sum rules ratios. We obtain $\alpha_s \langle F^2 \rangle = (3.9 \pm 0.4)\ 10^{-2}\ \mathrm{GeV}^4$. We also re-evaluate the contribution of the dimension-six vacuum condensates to the above sum rule and test the factorization hypothesis of the four-quark operator.

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1. INTRODUCTION

Substantial progress has been made in extending the applicability domain of QCD to the understanding of the hadron properties. The driving force behind this progress comes from the work of Shifman, Vainshtein and Zakharov (SVZ) [1], where the authors start from a semi-phenomenological perturbative expansion of the hadronic two-point correlation functions. In this expansion, one incorporates the non-vanishing contribution of the local operator vacuum expectation values which, to lowest dimension (d = 4) can be parametrized by the gluon condensate, $\alpha_s\langle F^2\rangle$, and by the quark condensate, $m\langle\bar{\Psi}\Psi\rangle$. The next-to-leading dimension (d = 6) operators are the triple gluon condensate $g\, f_{abc}\,\langle F^a F^b F^c\rangle$, the four-fermion condensate $\langle\bar{\Psi}\Gamma_1\Psi\bar{\Psi}\Gamma_2\Psi\rangle$ and the quark-gluon condensate $m\langle\bar{\Psi}\sigma^{\mu\nu}(\lambda_a/2)\Psi F^a_{\mu\nu}\rangle$. Using such an operator product expansion (OPE) parametrization of the hadronic correlation functions and with the help of certain improved dispersion relations, the role of the lowest hadron contribution to the spectral function becomes more important as compared to the usual dispersion relation. One thus gets a link between the hadron parameters (masses and couplings) and the quark and gluon vacuum condensates. The question of a precise estimate of these unknown non-perturbative vacuum condensates becomes then important for a better understanding of the hadron parameters. The quantity $m\langle\bar{\Psi}\Psi\rangle$ is accurately determined from pion PCAC, while an estimate of such a quantity taking into account $SU(3)_F$ breaking effects has been performed recently [2]. To our knowledge, the estimate of $\alpha_s\langle F^2\rangle$ from the analysis of heavy quark systems can only be considered as an order of magnitude determination [1),3)]. In fact, the latter analysis depends strongly on the choice of uncertain parameter values (heavy quark mass, continuum threshold and the QCD scale Λ). Actually, a little change in this system of parameter values may lead to a totally different output value for the $\alpha_s\langle F^2\rangle$ gluon condensate [4]. Also, to our knowledge, a reliable estimate of the gluon condensate from the heavy quark systems needs a careful treatment of the so-called Coulomb-like effects, the role of which is expected to be more important for the bottomium than for the charmonium system.

In this paper, we estimate the QCD vacuum condensates using a method which is insensitive to the above set of parameters. It is based on the moment sum rules ratio [1),3d)]

$$R\left(\frac{1}{M^2}\right) \equiv -\frac{d}{d\left(\frac{1}{M^2}\right)} \log \int_0^\infty dt\, e^{-t/M^2}\, \frac{1}{\pi}\, \mathrm{Im}\,\Pi(t) = \frac{\int_0^\infty dt\, t\, e^{-t/M^2}\, \frac{1}{\pi}\,\mathrm{Im}\,\Pi(t)}{\int_0^\infty dt\, e^{-t/M^2}\, \frac{1}{\pi}\,\mathrm{Im}\,\Pi(t)} \qquad (1)$$

where M^2 is the usual Laplace operator variable and $\pi(Q^2)$ will be, in this paper, the two-point correlation function associated to the hadronic current $J^\mu_H \equiv \frac{1}{2}(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d)$ having the quantum numbers of the ρ meson

$$\Pi^{\mu\nu}(q) \equiv -(g^{\mu\nu}q^2 - q^\mu q^\nu)\Pi(Q^2 \equiv -q^2 > 0) \equiv i\int d^4x\, e^{iqx} \langle 0|\mathrm{T}\, J^\mu_H(x)\left(J^\nu_H(0)\right)^\dagger|0\rangle \tag{2}$$

We shall see later on that the main advantages of the sum rule in Eq.(1), compared to the Laplace transform sum rules *), are the following :

- The moment sum rule of Eq.(1) is less sensitive to the QCD radiative contribution to the unit operator than the Laplace sum rule one, as the leading-log contribution cancels in the moment ratios of Eq.(1).
- The relative strength of the non-perturbative vacuum is more important in the moment sum rule than in the Laplace one.
- The phenomenological form of the moment sum rule is more accurate than the Laplace one for given data of the hadronic spectral function, as the uncertainty appearing in the Laplace sum rule tends to be reduced in the ratio of moments of Eq.(1).

According to the above remarks, we thus expect to extract with a good accuracy the values of the leading non-perturbative vacuum condensates from the analysis of the moments of Eq.(1). In the following analysis, we shall neglect non-dominant quark mass contributions, i.e., $m^2_{u,d}$ will be taken to be zero, where the u and d quarks are the only relevant quark flavours here. Actually, the u and d quark masses are known to have a value of a few MeV [6].

2. THE ρ MESON SUM RULE

The theoretical side of the ρ meson sum rule reads

$$R^\rho\left(\tau \equiv \frac{1}{M^2}\right) \simeq M^2\left\{1 + \frac{C_4\langle O_4\rangle}{M^4} + \frac{C_6\langle O_6\rangle}{M^6} + \ldots\right\} \tag{3a}$$

where we have retained only to a first approximation the contributions coming from the first two operators of lowest dimension (d = 4, 6) in the OPE of the two-point correlation function +). The contribution of the unit term comes from

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*) Actually, the Laplace transform sum rules applied to the ρ meson current have been used in Ref. 5), in order to estimate the QCD scale Λ and the strength of the dimension-four and -six vacuum condensates.

+) Such an approximation is expected to be good if one recognizes the estimate of $C_8\langle O_8\rangle$ done in Ref. 5) which shows that its contribution to the sum rule is about a few per cent of the $C_6\langle O_6\rangle$ one even at M equal to M_ρ. We shall check this result later on.

the diagrams in Fig. 1. Notice that the α_s/π correction cancels in $R^\rho(1/M^2)$. The gluon condensate contribution comes from Fig. 2a, while the quark condensate is given by Fig. 2b. The result is

$$C_4\langle O_4\rangle = -\frac{2\pi}{3}\alpha_s\langle F^2\rangle\left(1+12\pi\,\frac{m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle}{\alpha_s\langle F^2\rangle}\right). \tag{3b}$$

The contributions of the d = 6 condensates come from the diagrams shown in Figs. 3 and 4, which we shall discuss in more detail in section 4.

Notice that the relative strength of the $1/M^4$ contribution is two times larger in Eq.(3) than in the Laplace transform sum rule and the coefficient of the $1/M^6$ term is three times larger.

The experimental side of the sum rule is computed using the relation between the ρ meson spectral function and the total cross-section $\sigma_H(t)$ of the $e^+e^- \to I = 1$ hadron data :

$$\frac{1}{\pi}\,\mathrm{Im}\,\Pi(t) = t\,\sigma_H(t)\,\frac{1}{16\pi^3\alpha^2}\,, \tag{4}$$

which we know up to $\sqrt{t_c} \simeq 2$ GeV. Beyond $\sqrt{t_c}$ we parametrize the spectral function by the QCD continuum expression. Then the sum rule reads

$$R^\rho_{exp} \simeq \frac{\frac{1}{2\pi\alpha^2}\int_0^{t_c} dt\, e^{-t/M^2}\, t^2\sigma_H(t) + M^4 e^{-t_c/M^2}\left(1+\frac{t_c}{M^2}\right)}{\frac{1}{2\pi\alpha^2}\int_0^{t_c} dt\, e^{-t/M^2}\, t\,\sigma_H(t) + M^2 e^{-t_c/M^2}}\,. \tag{5}$$

Requiring that the two sides of the sum rule [Eqs.(3) and (5)] coincide both for leading M^2 and subleading M^0 terms for $M^2 \to \infty$, one obtains the constraint

$$t_c \simeq \frac{1}{2\pi\alpha^2}\int_0^{t_c} dt\, t\,\sigma_H(t)\,. \tag{6}$$

We introduce the data of $\sigma_H(t)$ as follows : for $\sqrt{t}$ between $2m_\pi$ and 1 GeV, we use the $\pi^+\pi^-$ data of DCI [7] parametrized by a Breit-Wigner form factor. For $1 \lesssim \sqrt{t} \lesssim 1.4$ GeV, we use a simple fit of the $\pi^+\pi^-$ Novosibirsk data [8] and we re-use the Breit-Wigner form factor given by DCI [7] for $\sqrt{t} \gtrsim 1.4$ GeV. For $1 \lesssim \sqrt{t} \lesssim 2$ GeV we take the $2(\pi^+\pi^-)$ DCI data [9]. The events containing n (even) neutral π^0's have been taken from VEPP [10] for $1 \lesssim \sqrt{t} \lesssim 1.4$ GeV and from DCI [11] for $1.4 \lesssim \sqrt{t} \lesssim 2$ GeV. We have neglected the events containing K and η mesons because of their small production cross-section in the range of energies which we are considering here. The fit of the data is shown in Fig. 5. We then solve Eq.(6) and find

$$t_c = (6.0 \pm 2.7)\ GeV^2 \tag{7}$$

which, due to the large error, shows that the value of $\sqrt{t_c} = 2$ GeV, up to which data are available and where we take the continuum threshold, is consistent with the large M^2 coincidence of the two sides of the sum rule. The minimal sensitivity of R^{ρ}_{exp} to the value of the continuum threshold is obtained by requiring $dR^{\rho}_{exp} / dt_c = 0$. This leads to :

$$t_c\, \sigma_H(t_c) \simeq 2\pi\alpha^2 \tag{8}$$

which is well satisfied for $\sqrt{t_c} = 2$ GeV. We therefore fix $\sqrt{t_c} = 2$ GeV for the following analysis, as in any case no accurate data are available beyond this value *). The behaviour of $R^{\rho}_{exp}(\tau)$ obtained from the data of Fig. 5 is shown in Fig. 6. As expected, using for example potential model arguments [3d)], the sum rule gets saturated by the ground state as $\tau \to \infty$.

3. ESTIMATE OF THE VACUUM CONDENSATES

We require that both sides of the sum rule [Eqs.(3) and (5)] coincide for a range of values $\tau_{min} \lesssim \tau \lesssim \tau_{max}$. We take τ_{min} to be 0 GeV^{-2} as asymptotically the sum rule is satisfied. The value of τ_{max} is the largest one, where one expects that the approximation in retaining the dimension $d \leq 6$ operator contribution to the sum rule is still good. A usual expectation for the validity of the approximations used indicates a value of τ_{max} in the range from 0.8 GeV^{-2} to M_ρ^{-2}. In order to select τ_{max} in this range of values, we fit the $1/M^4$ and $1/M^6$ contributions in Eq.(3), using the "MINUIT" program of the CERN library for τ_{max} varying in this range. We find that the value of χ^2 per degree of freedom varies slowly with τ_{max} in the region between 0.7 and 1.5 GeV^{-2} but presents a maximum of about 0.34 at $\tau_{max} \simeq 1$ GeV^{-2} as shown in Fig. 7. For lower values of τ_{max}, the χ^2 value diminishes because there are less constraints on the fit

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*) In fact, if we naively extrapolate our fit of the data up to $t_c = 6$ GeV^2, we can see that the data are under the QCD continuum at this point, i.e., the criterion in Eq.(8) is no longer satisfied. So it becomes uncertain to estimate $\langle C_4 O_4 \rangle$ and $\langle C_6 O_6 \rangle$ at this value of t_c.

as the region where one demands coincidence is smaller. For slightly larger values of τ_{max}, χ^2 diminishes because the experimental data are less accurate. We therefore choose $\tau_{max} = 1\ \mathrm{GeV}^{-2}$ as a compromise between quantity and quality. In this way we get with a very high confidence level

$$C_4 \langle O_4 \rangle = -(6.9 \pm 0.8)\,10^{-2}\ \mathrm{GeV}^4 \tag{9a}$$

$$C_6 \langle O_6 \rangle = (0.18 \pm 0.02)\ \mathrm{GeV}^6 . \tag{9b}$$

Using pion PCAC for the estimate of $m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle$, we deduce

$$\alpha_s \langle F^2 \rangle = (3.9 \pm 0.4)\,10^{-2}\ \mathrm{GeV}^4 , \tag{10}$$

which is precisely the standard value [1),3),12)]. Eq.(10) indicates that the breaking of asymptotic freedom due to the inverse power term of the gluon vacuum condensate at energies of about 1 GeV is rather modest. Now, we improve the estimate of $C_6\langle O_6\rangle$ using the value in Eq.(10) as input in Eq.(3a) and adding to this latter equation the $C_8\langle O_8\rangle$ contribution. Using the same condition as previously, we deduce :

$$C_6 \langle O_6 \rangle = (0.19 \pm 0.02)\ \mathrm{GeV}^6 \tag{11a}$$

$$C_8 \langle O_8 \rangle = -(1.9 \pm 1.6)\,10^{-2}\ \mathrm{GeV}^8 \tag{11b}$$

Our result shows that $C_8\langle O_8\rangle$ is indeed small [5)] and the inclusion of such a term in the series does not change the estimate of $C_6\langle O_6\rangle$.

The error bars appearing in Eq.(9) are related to the uncertainty of the measurement of the $e^+e^- \to I = 1$ total cross-section which is quite inaccurate in the region between 1 and 2 GeV. We give in Fig. 6 (curve) the fit of R^ρ using the central values in Eq.(9). One can notice that the minimum of the theoretical curve corresponding to the values in Eq.(9) is about $0.98\ \mathrm{GeV}^2$. One can also notice that, if one had used the expression of R^ρ as only given by asymptotic freedom, i.e., with no non-perturbative effects, one can see that the theoretical curve is lower than the data for τ larger than $0.5\ \mathrm{GeV}^{-2}$ (dotted curve of Fig. 6). This indicates that the theoretical expression of R^ρ is, in that case, a bad approximation of the exact form of R^ρ. In fact, the positivity of the spectral function requires the theoretical curve to lie above the experimental one. Saturating the sum rule by a ρ meson and using the positivity of the other states and continuum contribution to the sum rule, we can deduce at the minimum :

$$M_\rho \lesssim 0.97\ GeV. \tag{12}$$

If one compares this bound to the observed mass, one can deduce that a complete coincidence of the data and the theory at the minimum requires an appreciable contribution of the other states and "continuum" of about 37% of the total contribution. This shows that the sum rule in Eq.(1) is governed by the higher states contributions. So, it seems difficult to extract from the sum rule the lowest-ground state mass with a good precision. Accordingly, the difference of sum rules used in Ref. 13), which is less sensitive to the contribution of higher states than the one used here, can be a good direction in the understanding of the meson masses or mass-difference within the context of QCD sum rules.

4. d = 6 VACUUM CONDENSATE AND FACTORIZATION HYPOTHESIS

The fitted value of $C_6\langle O_6\rangle$ given in Eq.(9b) will be compared to the one obtained from the evaluation of the diagrams of Figs. 3 and 4 which lead to the $d = 6$ condensates. The main difficulty which one encounters in the computation of such diagrams comes from the fact that when one applies Wick's theorem to the time ordered product of two gauge invariant currents, the normal ordered composite operators which appear are neither local nor gauge invariant, whereas the condensates are both. The most convenient technique for solving this problem is the use of the Schwinger, fixed point or co-ordinate gauge [14)]

$$x^\mu B_\mu^a(x) = 0 \quad . \tag{13}$$

In this gauge the ordinary derivatives of the quark and the gluon field strength can be conveniently replaced by covariant derivatives and the Taylor expansions of the quark and gluon fields are explicitly gauge covariant. Non-local, non-gauge-invariant normal ordered products become thus local gauge invariant normal operators.

We shall not repeat the discussion of the evaluation of the triple gluon condensate $\langle g f_{abc} F^a F^b F^c\rangle$ coming from Fig. 3a as it has been shown independently [15)], that it does not contribute to the two-point function of a vector current in the chiral limit. The contribution of the diagrams in Fig. 3b corresponds to the mixed quark-gluon condensate $m\langle\bar\psi\sigma^{\mu\nu}(\lambda_a/2)\psi F^a_{\mu\nu}\rangle$. The evaluation of the first diagram in Fig. 3b can be done with the help of

$$\langle \bar{\psi}_i(x)\lambda^a\psi_\ell(0)B^a_\omega(y)\rangle \equiv \frac{1}{48}\langle\bar{\psi}\sigma^{\mu\nu}\frac{\lambda_a}{2}\psi F^a_{\mu\nu}\rangle \cdot \left[y^\rho\sigma_{\rho\omega} + \frac{m}{2}x^\alpha y^\rho\left(g_{\alpha\rho}\gamma_\omega - g_{\alpha\omega}\gamma_\rho + i\gamma_\alpha\sigma_{\rho\omega}\right)\right]_{\ell i} \tag{14}$$

where the equality has been obtained using Lorentz covariance and the quark equations of motion. A straightforward computation gives, at leading order of chiral symmetry breaking, a non-transverse contribution, due to the last term of Eq.(14)

$$\tilde{\Pi}_{3b} = 0 \tag{15a}$$

$$q_\mu q_\nu \tilde{\Pi}^{\mu\nu} = -\frac{mg}{4Q^2}\langle\bar{\psi}\frac{\lambda_a}{2}\sigma^{\mu\nu}\psi F^a_{\mu\nu}\rangle . \tag{15b}$$

The computation of the last two diagrams of Fig. 3b is formally different from the first one. One cannot start from a normal product of the type of Eq.(14) because that implies a zero momentum propagator. Instead, one starts from a normal ordered product of the type $\langle\bar{\psi}_i(x)\psi_j(0)\rangle$ which, after retaining the contribution of the mixed condensate and with the help of the equations of motion, reads

$$\langle\bar{\psi}_i(x)\psi_j(0)\rangle \equiv \frac{g}{64}\langle\bar{\psi}\frac{\lambda_a}{2}\sigma^{\mu\nu}\psi F^a_{\mu\nu}\rangle x^2\left[\delta_{ji} + \frac{i}{6}m x^\mu(\gamma_\mu)_{ji}\right] . \tag{16}$$

A simple computation leads, to leading order, to a non-transverse part, coming from the last term of Eq.(16), which exactly cancels the one in Eq.(15). We find this check of transversality reassuring in view of the different approaches one follows in the computation of the diagrams in Fig. 3b and as an indication of the gauge invariance of the result. Our result confirms that the mixed condensate enters the correlation function of the vector current at most to order m^2/Q^2 and is thus negligible. Fig. 4a is evaluated using similar methods. The first diagram of Fig. 4a is computed with the help of

$$\langle\bar{\psi}_\ell(x)\lambda_a B^a_\rho(z)\psi_f(0)\rangle = -\frac{g}{24}\langle\bar{\psi}\gamma^\mu\frac{\lambda_a}{2}\psi\sum_{u,d,s}\bar{q}\frac{\lambda_a}{2}\gamma_\mu q\rangle \cdot \left[\frac{1}{3}\left(z^2\gamma_\rho - z_\rho z^\alpha\gamma_\alpha\right) + \frac{i}{4}x^\beta z^\alpha\gamma_\beta\sigma_{\alpha\rho}\right]_{f\ell} , \tag{17}$$

and gives

$$\Pi_{4a} = -\frac{\pi}{9}\alpha_s \langle(\bar{u}\gamma^\mu\lambda_a u + \bar{d}\gamma^\mu\lambda_a d)\sum_{u,d,s}\bar{q}\gamma_\mu\lambda_a q\rangle \frac{1}{Q^6} , \qquad (18a)$$

$$q^\mu q^\nu \Pi_{\mu\nu} = -\frac{3}{8}(Q^2)^2\Pi_{4a} . \qquad (18b)$$

The last two diagrams in Fig. 4a can be evaluated with the help of

$$\langle\bar{\psi}_i(x)\psi_j(0)\rangle \equiv i\frac{g^2}{4608}\langle\bar{\psi}\gamma^\mu\lambda_a\psi\sum_{u,d,s}\bar{q}\gamma^\mu\lambda_a q\rangle x^2 x^\alpha(\gamma_\alpha)_{ji} \qquad (19)$$

and one easily sees that they do not contribute to π but give a contribution which cancels the non-transverse term in Eq.(18). The evaluation of the diagrams in Fig. 4b is straightforward as the gluon momentum is hard. We find the gauge invariant result

$$\Pi_{4b} = -\frac{\pi}{2}\alpha_s\langle(\bar{u}\gamma_\mu\gamma_5\lambda^a u - \bar{d}\gamma_\mu\gamma_5\lambda^a d)^2\rangle\frac{1}{Q^6} . \qquad (20)$$

Putting all this together, we find for the total leading contribution of the dimension-six vacuum condensates to the sum rule in Eq.(3a) the SVZ result [1)]

$$C_6\langle O_6\rangle = 6\pi^3\alpha_s\Big[\langle(\bar{u}\gamma^\mu\gamma_5\lambda^a u - \bar{d}\gamma^\mu\gamma_5\lambda^a d)^2\rangle + \frac{2}{9}\langle(\bar{u}\gamma^\mu\lambda_a u + \bar{d}\gamma^\mu\lambda_a d)\sum_{u,d,s}\bar{q}\gamma^\mu\lambda_a q\rangle\Big], \qquad (21)$$

which, using the factorization hypothesis of the four-quark operator [1)] and $SU(2)_F$ symmetry for the quark condensates [1),16)], gives

$$C_6\langle O_6\rangle = \frac{448}{27}\pi^3\alpha_s\langle\bar{u}u\rangle^2 . \qquad (22)$$

A renormalization group invariance analysis of the step leading from Eq.(21) to Eq.(22) has been done in detail in Ref. 17). Strictly speaking, the factorization is, in general, inconsistent with the renormalization group, as various six-dimension fermion operators mix under renormalizations. However, one can notice [*)] from the result in Ref. 17), that the renormalization group invariance of the $O_2 \equiv \bar{\psi}\psi\bar{\psi}\psi$

[*)] We thank E. de Rafael for his important remark.

operator is realized to leading order in $1/N_c$ up to order α_s so that in this approximation, the factorization hypothesis becomes compatible with the renormalization group. Then, under such approximations, the O_2 operator can be made renormalization group invariant. Eq.(23) is convenient for QCD sum rules phenomenology because it avoids the introduction of new vacuum parameters.

If one takes the values $\alpha_s(\nu = 200 \text{ MeV}) \simeq 0.7$ and $\langle\bar{u}u\rangle \simeq (-0.25 \text{ GeV})^3$ used previously by SVZ [1], one gets

$$C_6 \langle O_6 \rangle_{fact} \simeq 0.09 \text{ GeV}^6 , \tag{23}$$

which agrees whithin a factor two with the fitted value given in Eq.(9b). However, this choice of parameter values seems inconsistent with recent estimates of the u, d quark mass values [6] which is correlated with the quark condensate value through the pion PCAC relation $(m_u + m_d)\langle\bar{u}u\rangle = - m_\pi^2 f_\pi^2$. In fact, as Eq.(22) is almost independent of the subtraction point at which it is evaluated due to the cancellation of anomalous dimensions of the coupling and of the condensate; we evaluate it at $Q^2 = M_\rho^2$ with the standard expressions of the renormalization group. Then, using $\Lambda \simeq 100$ MeV and $\langle\bar{u}u\rangle(M_\rho) \simeq -(0.20 \sim 0.23)^3 \text{GeV}^3$ corresponding to $\bar{m}_u(M_\rho) \simeq$ $\simeq 7 - 10$ MeV [6], one obtains

$$C_6 \langle O_6 \rangle_{fact} \simeq (0.03 \sim 0.05) \text{ GeV}^6 , \tag{24}$$

which is smaller by a factor two to three than Eq.(23), and so differs by a factor four to six from the fitted value given in Eq.(9b). It is clear that the estimate of the four-quark condensate coming from factorization is strongly dependent on the quark mass and can be a factor four to six below its actual value.

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FIGURE CAPTIONS

Fig. 1 : The leading and next-to-leading perturbative diagrams.

Fig. 2 : a) The gluon condensate diagrams;

b) The quark condensate diagram.

Fig. 3 : a) The triple-gluon condensate diagrams;

b) The mixed condensate diagrams.

Fig. 4 : a) The soft gluon quartic quark condensate diagrams;

b) The hard gluon quartic quark condensate diagrams.

Fig. 5 : Our fit to the data : $r \equiv [\sigma(e^+e^- \to I = 1 \text{ Hadron})] / [\sigma(e^+e^- \to \mu^+\mu^-)]$ below 2 GeV.

Fig. 6 : $R_{exp}(\tau)$ as given in Eq.(5), using the e^+e^- data below 2 GeV and the QCD parametrization of the continuum beyond 2 GeV.

Fig. 7 : Behaviour of the χ^2/NDF versus various values of τ_{max}, corresponding to the fit of $\alpha_s \langle F^2 \rangle$ and $C_6 \langle O_6 \rangle$ which comes from a confrontation of R^ρ_{th} and R^ρ_{exp} for the τ values between 0 and τ_{max}.

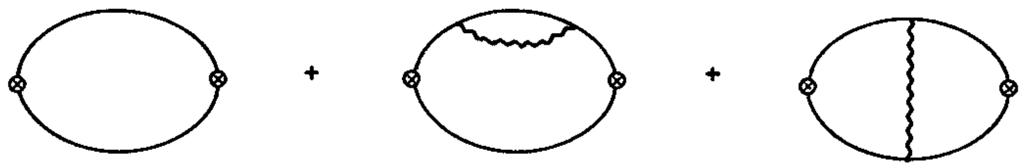


Fig. 1

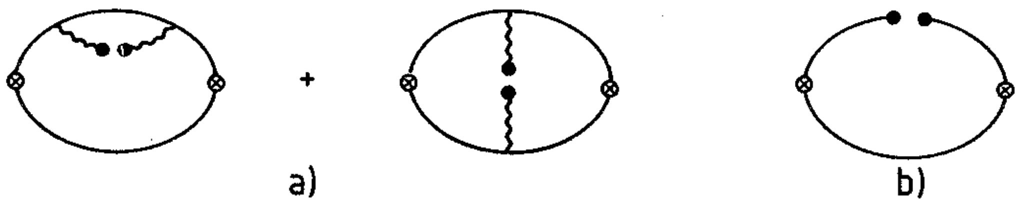


Fig. 2

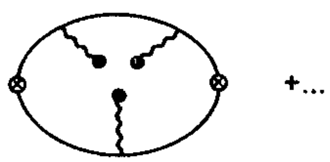


Fig. 3a

Fig. 3b

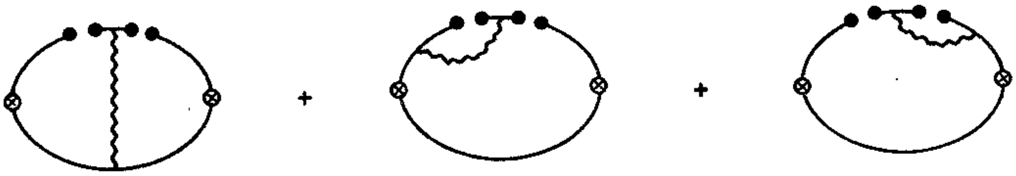


Fig. 4a

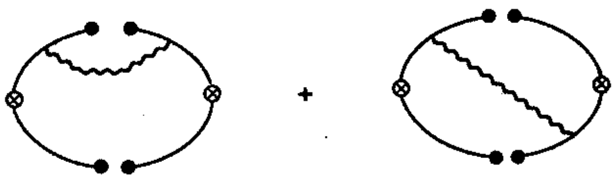


Fig. 4b

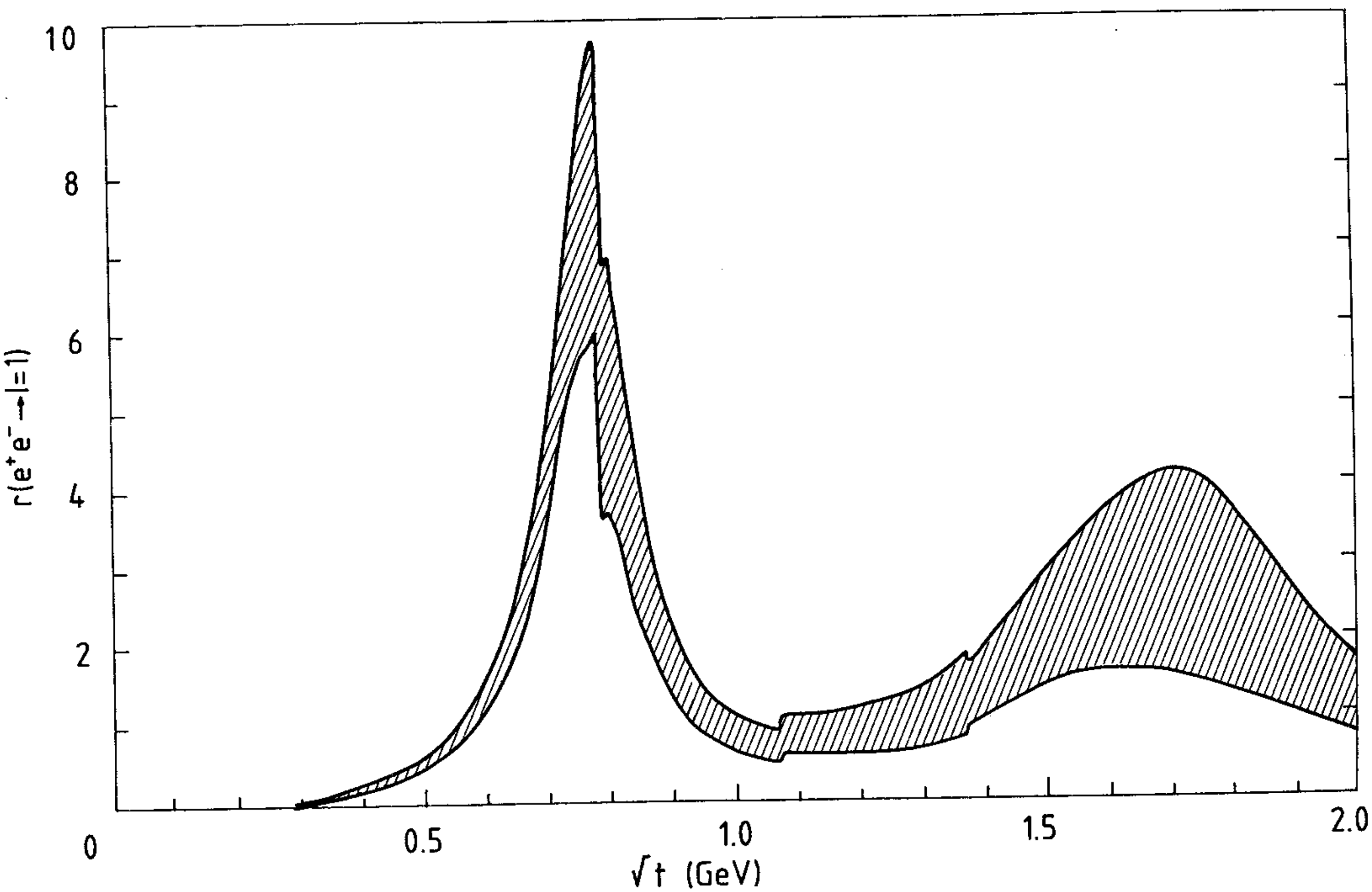
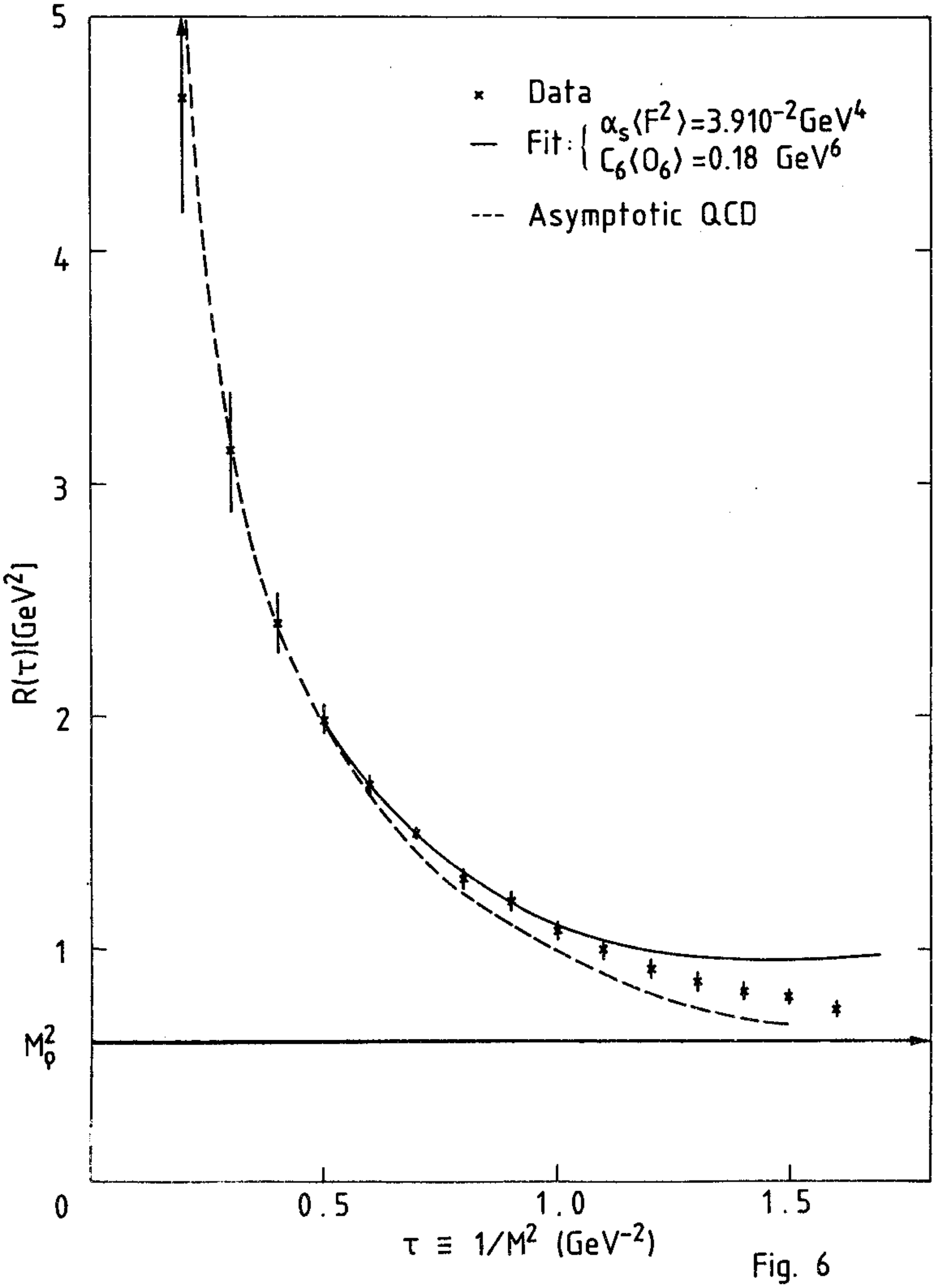


Fig. 5



Data
Fit: { $\alpha_s\langle F^2\rangle = 3.9\,10^{-2}\,GeV^4$, $C_6\langle O_6\rangle = 0.18\ GeV^6$ }
Asymptotic QCD
$R(\tau)\,[GeV^2]$
M_ρ^2
$\tau \equiv 1/M^2\ (GeV^{-2})$
0
0.5
1.0
1.5
1
2
3
4
5

Fig. 6

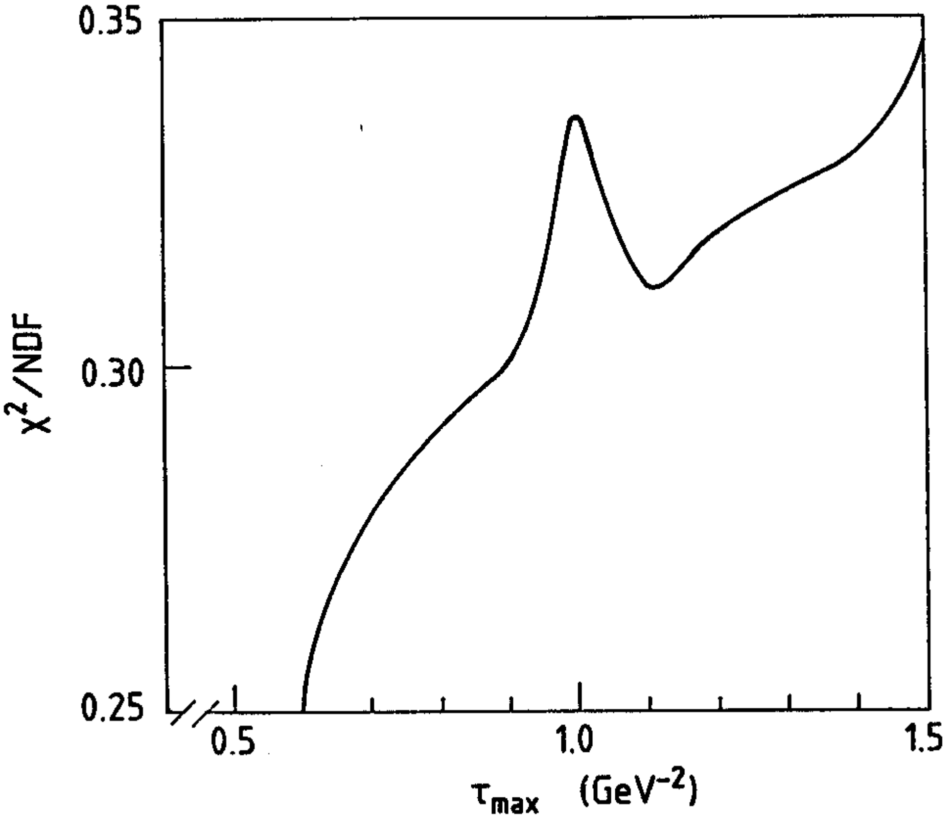


FIG. 7

Ref.TH.3712-CERN
ERRATUM

NON-PERTURBATIVE QCD VACUUM FROM $e^+e^- \to I = 1$ HADRON DATA

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and

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CH - 1211 Geneva 23

First page: - Instead of Ref. 4) read Ref. 4a)
- Instead of Ref. 3) read Refs. 3c)d)e)

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§) Boursier DGRST.

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Erratum