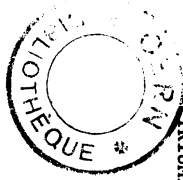


Quark models and the structure of the  $\Delta(1232)$  resonance

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Abstract

We investigate the structure of the  $\Delta(1232)$  resonance within a model which contains pion ( $\pi$ ) interacting with nucleon (N) and  $\Delta$ . With a  $\pi NN$  (or  $\pi N\Delta$ ) vertex form factor obtained from quark models, we demonstrate explicitly that the perturbation series of  $\pi$ -N scattering converges rapidly. Thus a crossing symmetric solution to the  $\pi$ -N scattering problem can be obtained reliably by perturbation expansion. We expand the  $\pi$ -N scattering series to fourth order in renormalized coupling constants, and then the (3,3) phase shift is calculated using both T- and K-matrices. The main results are: (1) The  $\Delta$ -contribution is dominant in generating the observed resonance. (2) The observed resonance mass is equal to the dressed (renormalized)  $\Delta$  mass; and it is shown that this result is independent of the order of the perturbation expansion. Our calculation and results are compared with those of the cloudy bag model.

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I. INTRODUCTION

The MIT bag model<sup>1</sup> and the constituent quark model (CQM)<sup>2</sup> are both quite successful in describing hadronic static properties. A major extension of the original bag model has been the introduction of the pion ( $\pi$ ), as a fundamental field, into the bag model Lagrangian.<sup>3-8</sup> The inclusion of the pion field is required to restore chiral symmetry, and so the resulting model is often called the chiral bag model. Recently pionic effects have also been incorporated into the CQM.<sup>9</sup> Here the inclusion of the pion field is not guided by any symmetry principle, but is simply necessitated by the fact that pions interact with baryons. In both cases, the three-quark core of a baryon is considered as a static, extended source of pions. It is assumed that point-like pions are coupled directly to u and d quarks in the core; from this one can then deduce the effective pion-baryon interaction Hamiltonian. Now if the pion is second quantized, the result is an effective field theory of  $\pi$ -baryon interactions. This model differs from the old static pion-nucleon (N) model in two respects. Firstly, the degree of freedom in the baryon sector has been extended to include excited states of the nucleon ( $\Delta$  and  $N^*$ ) explicitly. Secondly, the  $\pi NN$  vertex form factor is no longer arbitrary; it can be obtained from the quark wave function which is in turn determined by the static properties of the nucleon.

The notion of coupling pions directly to quarks is taken more seriously by some physicists, who tend to regard the  $\pi$ -quark coupling as fundamental. Then, to be consistent, one has to sum over all possible virtual quark states in a  $\pi$ -loop such as the one shown in Fig. 1. This, however, leads to divergent baryon self-energies in the second quantized version of the chiral bag model - the cloudy bag model (CBM).<sup>10</sup> We shall

not address ourself to this problem here, but shall instead regard the introduction of direct  $\pi$ -quark coupling as merely a convenient means of deriving an effective theory of  $\pi$ -baryon-interactions.

We distinguish three different descriptions of the  $\Delta$ . A bare  $\Delta$  or a three-quark  $\Delta$  is defined as a purely three-quark bound state. A dressed  $\Delta$  consists of a three-quark core surrounded by a pion cloud. Finally, a physical  $\Delta$  or  $\Delta(1232)$  resonance is what one experimentally observes in laboratories. The purpose of this paper is to investigate the structure of the  $\Delta(1232)$  resonance within the model sketched above. On the level of an effective field theory, the underlying hadronic model (CBM or CQM) turns out to be irrelevant for our purpose. The important thing is that there exists a 3-quark  $\Delta$  state, which is predicted by all quark models. Moreover, as will be shown in the following section, form factors determined by hadronic static properties in the CQM and CBM turn out to be similar.

The formation of the  $\Delta(1232)$  resonance is the most prominent feature in intermediate-energy  $\pi$ -N scattering. The  $\Delta(1232)$  also plays an important role in many other nuclear reactions at intermediate energies. These include  $\pi$ -nucleus scattering,  $(\gamma, \pi)$ ,  $(\pi, p)$  and  $(\gamma, p)$  reactions, etc. In all these reactions the  $\Delta(1232)$  formation takes place within a nucleus. In a nuclear medium the properties of the  $\Delta(1232)$  are expected to be modified; and this medium effect has been the subject of many discussions in recent years.<sup>11</sup> It is clear that a knowledge of the different components of the  $\Delta(1232)$ , apart from interesting in its own right, is also necessary for a deeper understanding of nuclear medium effects on its free space properties. Furthermore, knowing the structure of the  $\Delta(1232)$  is also important in  $N\Delta$  coupled channel calculations. An

attempt is being made to incorporate the bare  $\Delta$  in a  $\pi NN$  three-body calculation.<sup>12</sup>

The  $\pi$ -baryon ( $N$  and  $\Delta$ ) interaction Hamiltonian is given and discussed in Sec. II. The  $\pi$ -N scattering amplitude is calculated perturbatively in Sec. III, where comparison is also made with previous works on describing the  $\Delta(1232)$  in the quark model. Results are presented and discussed in Sec. IV. Finally, summary and conclusions are given in Sec. V.

## II. Interaction Hamiltonian

In the subsequent calculation, we shall include only  $N$  and  $\Delta$  intermediate states. Contributions from higher excited states are expected to be small due to larger energy denominators. Also, the  $\pi NN^*$  coupling is suppressed for low-momentum pions because the quark wave functions of  $N$  and  $N^*$  are orthogonal.

In quark models, coupling pions directly to quarks results in the following effective  $\pi$ -( $N, \Delta$ ) interaction Hamiltonian<sup>6-9</sup>:

$$H_I = \int \frac{d^3k}{(2\pi)^{3/2}} \left[ \mathcal{H}_{\pi NN}(k) + \mathcal{H}_{\pi N\Delta}(k) + \mathcal{H}_{\pi\Delta\Delta}(k) \right] \quad (1)$$

with

$$\mathcal{H}_{\pi NN}(k) = \sum_{\alpha} \left( \frac{4\pi}{2\omega_k} \right)^{1/2} \frac{f_{\pi NN}^{(0)}}{m_{\pi}} u(k) N^{\dagger} \tau_{\alpha} \vec{\sigma} \cdot \vec{k} N a_{\alpha}(\vec{k}) + \text{h.c.} \quad (2)$$

$$\mathcal{H}_{\pi N\Delta}(k) = \sum_{\alpha} \left( \frac{4\pi}{2\omega_k} \right)^{1/2} \frac{f_{\pi N\Delta}^{(0)}}{m_{\pi}} u(k) \Delta^{\dagger} T_{\alpha} \vec{\sigma} \cdot \vec{k} N a_{\alpha}(\vec{k}) + \text{h.c.} \quad (3)$$

$$\mathcal{H}_{\pi\Delta\Delta}(k) = \sum_{\alpha} \left( \frac{4\pi}{2\omega_k} \right)^{1/2} \frac{f_{\pi\Delta\Delta}^{(0)}}{m_{\pi}} u(k) \Delta^{\dagger} \vec{J}_{\alpha} \cdot \vec{k} \Delta a_{\alpha}(\vec{k}) + \text{h.c.} \quad (4)$$

where  $a_{\alpha}(\vec{k})$  is the annihilation operator for a pion of charge  $\alpha$  and momentum  $\vec{k}$ .  $\vec{S}(\vec{T})$  is the 1/2 to 3/2 spin (isospin) transition operation, and  $\vec{J}(\vec{T})$  is the 3/2 spin (isospin) operator.  $f^{(0)}$ 's are the bare coupling constants. If we make the usual choice that<sup>13</sup>

$$\langle \frac{3}{2} | S | \frac{1}{2} \rangle = \langle \frac{3}{2} | T | \frac{1}{2} \rangle = 2$$

$$\langle \frac{3}{2} | \sigma | \frac{3}{2} \rangle = \langle \frac{3}{2} | \sigma | \frac{3}{2} \rangle = \sqrt{15}, \quad (5)$$

$$f_{\pi N \Delta}^{(0)} = \left( \frac{72}{25} \right)^{1/2} f_{\pi N N}^{(0)}$$

$$f_{\pi \Delta \Delta}^{(0)} = \frac{4}{5} f_{\pi N N}^{(0)} \quad (6)$$

from the zeroth order quark spin-isospin wave functions of the baryon cores. It is often assumed that the renormalized coupling constants ( $f$ 's) also obey the same relations (6).<sup>7,9</sup> However, when one splits the  $N$  and  $\Delta$  masses, relations (6) may no longer hold. Furthermore, coupling a pion to a bound system of quark would inevitably involve the notion off-shell bound states (or off-shell bags), of which we do not have any knowledge. The issue is therefore more complicated than one would naively think. In view of the above remarks, we shall regard the renormalized coupling constants  $f_{\pi N \Delta}$  and  $f_{\pi \Delta \Delta}$  as free parameters to be fixed by experimental data. The renormalized  $\pi N N$  coupling  $f_{\pi N N}$  is fixed by experiment at

$$f_{\pi N N}^2 = 0.08. \quad (7)$$

In Eqs. (2)-(4),  $u(k)$  is the vertex form factor. We have assumed  $u(k)$  to be identical for the  $\pi N N$ ,  $\pi N \Delta$  and  $\pi \Delta \Delta$  vertices. In the CQM with a harmonic oscillator inter-quark potential,  $u(k)$  is given by

$$u(k) = e^{-k^2/6\alpha^2}. \quad (8)$$

With point-like pions, the cut-off parameter  $\alpha$  is related to the root mean square radius of the baryon core by

$$\alpha^{-1} = r_{rms}. \quad (9)$$

A consistent fit to baryon masses, magnetic moments, and nucleon charges radii gives<sup>9,14</sup>

$$\alpha = 1.2 \text{ fm}^{-1}. \quad (10)$$

In the CBM,<sup>6,7</sup>

$$u(k) = 3j_1(kR)/kR, \quad (11)$$

where  $R$  is the bag radius. Equation (11) can be well approximated by the Gaussian form of Eq. (8) with<sup>15</sup>

$$\alpha^{-2} = 0.636 R^2. \quad (12)$$

In the CBM calculation  $R$  was varied, together with  $f_{\pi N N}$  and the dressed  $\Delta$  mass  $\omega_\Delta (= M_\Delta - M_N)$ , in order to obtain a fit to the total cross section in the (3,3) channel. It turned out that  $R = 0.82 \text{ fm}$  and  $\omega_\Delta = 280 \text{ MeV}$ . However, this solution is not unique. As shown in Ref. 6, another possible solution is  $R = 0.22 \text{ fm}$  and  $\omega_\Delta = \infty$  (equivalent to  $f_{\pi N \Delta} = 0$ ). We feel that  $R$  should not be considered as a free parameter in a scattering calculation. In order to have a stable bag, and avoid multiplicity of solutions,  $R$  should be determined separately by minimizing the total bag energy, including the pion cloud contribution, with respect to the bag radius:

$$\left. \frac{\partial E(R)}{\partial R} \right|_{r=R} = 0. \quad (13)$$

In a semi-classical approach,<sup>16</sup> the baryon mass spectrum and magnetic moments have been studied with pionic effects included in the bag model. The mass study also took into account the centre-of-mass correction and the gluon magnetic interaction energy. The  $N$  and  $\Delta$  bag radii were determined to be around 1 fm. Nucleon magnetic moments and charge radii have also been studied in the CBM.<sup>17,18</sup> The best fit to experimental values corresponds also to  $R \approx 1 \text{ fm}$ . Now substituting  $R = 1 \text{ fm}$  into Eq. (12) gives  $\alpha = 1.25 \text{ fm}^{-1}$ , which is very close to that obtained in the CQM [Eq. (10)]. So there seems to be an approximate agreement on the range of the pion vertex form factor in the CQM and the bag model. We shall

use Eq. (8) with  $\alpha$  given by Eq. (10) in our calculation. We would not argue about the exact value of  $\alpha$ . As long as  $\alpha \approx 1.2 \text{ fm}^{-1}$  or, equivalently,  $R \approx 1 \text{ fm}$ , our calculation and conclusions will not be affected in any essential way.

### III. THE $\Delta(1232)$ RESONANCE

For a long time since the 1950's, mainly due to the work of Chew and Low,<sup>19</sup> the  $\Delta(1232)$  resonance has been considered as a resonant state of a nucleon and pions. A more modern view is that it possesses, apart from a  $\pi$ -N part, a 3-quark  $\Delta$  part as well.<sup>7,13</sup> This 3-quark  $\Delta$  state appears naturally in the quark model of hadrons. In this work, we take the latter point of view and include the  $\Delta$  explicitly in our calculation. With the  $\pi$ -vertex form factor given in the last section, one can show that the perturbation series of the  $\pi$ -N scattering amplitude converges very fast. Consequently, a crossing symmetric solution to the (3,3) channel  $\pi$ -N scattering problem can be obtained by perturbation expansion to finite order.

We first expand the  $\pi$ -N scattering T-matrix to fourth order in renormalized coupling constants; it is diagrammatically shown in Fig. 2. To be consistent, second-order corrections to propagators and vertices should also be included in evaluating the second-order scattering diagrams. These corrections are shown in Fig. 3. Figure 3(a) renormalizes baryon masses, and Fig. 3(b) renormalizes coupling constants. The renormalization procedure can be carried out in a standard way, and is shown explicitly in Appendix A. The evaluation of diagrams shown in Fig. 2 involves straightforward manipulation of spin and isospin operators. The resulting expressions are given in Appendix B. The on-energy-shell T-matrix is related to the phase shift,  $\delta_{33}$ , by the

unitarity condition, which gives<sup>19</sup>

$$T_{33}(k', k) = -\frac{4\pi}{2\omega_k} \frac{3}{k^3} e^{i\delta_{33}} \sin\delta_{33} \mathcal{P}_{33}(k', k) \quad (14)$$

where  $\mathcal{P}_{33}$  is the (3,3) channel projection operator defined by

$$\mathcal{P}_{33}(k', k) = P_{3/2}(k', k) Q_{3/2}(\vec{k}', \vec{k}) \quad (15)$$

with  $P_{3/2}$  and  $Q_{3/2}$  given in Appendix B.

A T-matrix solution to the  $\pi$ -N scattering amplitude has also been obtained in the CBM.<sup>6,7</sup> In that calculation the Low equation was solved by including only one-pion-one-nucleon intermediate states, and neglecting crossing symmetry. This solution is equivalent to iterating Fig. 2(a) and Fig. 2(b) combined to infinite order. A fit to the experimental (3,3)-channel total cross section was then obtained by varying  $f_{\pi NN}$ ,  $\omega_\Delta$  and R. Our calculation differs from it in several respects. Firstly, we fix  $f_{\pi NN}$  at the experimental value [Eq. (7)]. In Ref. 7, the value  $f_{\pi NN}^2 = 0.064$  was used. This is not satisfactory since Eq. (7) is well established.

Secondly, we use a vertex form factor which is fixed by the static properties of the nucleon in the quark model (see the discussion on this point in Sec. II). Thirdly, our solution is crossing symmetric and is obtained through a consistent perturbation expansion of the scattering series; specifically, we include all diagrams up to 4th order. In this approach, the  $\pi NN$  and  $\pi N\Delta$  vertex corrections can be taken into account in a straightforward way. Vertex corrections have not been included correctly in Ref. 7; e.g. Eq. (3.6) of that paper holds only for the  $\pi$ -N crossed Born term.

A different but convenient approach is to calculate the reactance matrix (K or R), for which one replaces the Green's function

$$G(E) = \frac{1}{E-H_0+1\epsilon} \quad (16)$$

that appears in the T-matrix approach by its real part, i.e.

$$G(E) + G_R(E) = \frac{P}{E-H_0} \quad (17)$$

where P indicates that the Cauchy principal value is taken whenever the energy denominator vanishes. And again, by unitarity, the on-energy-shell K-matrix can be related to the phase shift:

$$K_{33}(k', k) = -\frac{4\pi}{2\omega_k} \frac{3}{k^3} \tan^2 \delta_{33}(k', k) \quad (18)$$

The K-matrix approach has been used in many past calculations of  $\pi$ -N scattering. For example in the 50's Blair and Chew<sup>20</sup> used it to calculate  $\pi$ -N phase shifts by expanding the scattering amplitude to fourth order in  $f_{\pi NN}$  in the old static  $\pi$ -N model. More recently, several groups<sup>13, 21, 22</sup> have also used the K-matrix approach in calculating  $\pi$ -N phase shifts. They have, however, included only the lowest-order diagrams. Moreover, convergence of the perturbation series has never been established explicitly.

In an exact calculation, the T-matrix and K-matrix methods should yield identical results. This, however, is not guaranteed in a perturbative calculation truncated at finite order. Therefore, a comparison of the results from both methods actually provides a measure of the deviation of our solution from the exact one.

Finally, we point out a kinematic ambiguity which is inherent to all static models. In a strictly static sense, the total energy is given by

$$E = \omega_k = (m_\pi^2 + k^2)^{1/2} \quad (19)$$

However, in order to compare calculated results with experimental data, one has to decide whether k is a laboratory or centre-of-mass quantity.

Both choices have been made in the literature.<sup>22</sup> However, it is obvious that neither choice is satisfactory, because at the resonance energy  $E_{lab}$  and  $E_{c.m.}$  differ by as much as 35 MeV due to nucleon recoil. To remedy this embarrassing situation, we identify k with the centre-of-mass pion momentum; but instead of Eq. (19) we use the exact relation

$$E = \omega_k + (k^2 + M_N^2)^{1/2} - M_N \quad (20)$$

This prescription amounts to including kinetic energies of real nucleons in our calculation.

#### IV. RESULTS AND DISCUSSION

The main results of this calculation are shown in Figs. 4 and 5. Figure 4 corresponds to the T-matrix approach, and Fig. 5 the K-matrix approach. While both methods are able to reproduce the data quite well, the K-matrix method provides a better overall fit. In obtaining the fits, we have varied only two parameters,  $f_{\pi N\Delta}$  and  $\omega_\Delta$ . As for  $f_{\pi\Delta\Delta}$ , we used the quark model relation

$$f_{\pi\Delta\Delta} = \frac{4}{5} f_{\pi NN} \quad (21)$$

The results are not very sensitive to small changes of  $f_{\pi\Delta\Delta}$  around this value. In the T-matrix approach, the best fit values of the parameters are  $f_{\pi N\Delta} = 0.624$  and  $\omega_\Delta = 301$  MeV; while in the K-matrix approach,  $f_{\pi N\Delta} = 0.523$  and  $\omega_\Delta = 293$  MeV. The fact that these two sets of parameters are close in magnitudes indicates that higher-order corrections are not very important. For comparison, the CBM<sup>7</sup> parameters are  $f_{\pi N\Delta} \approx 0.4$  and  $\omega_\Delta = 280$  MeV.

In order to examine the importance of different contributions, we have also calculated separately the Chew-low contribution and the  $\Delta$ -contribution. The former is obtained by setting  $f_{\pi N\Delta} = 0$ , and the latter

by keeping only diagrams which contain intermediate  $\Delta$ 's. The results are also shown in Fig. 4 and Fig. 5, which indicate that the  $\Delta$ -contribution is dominant. The Chew-Low contribution is small but nevertheless not negligible. A similar conclusion was also reached in the CBM calculation.<sup>6</sup> From Fig. 5, we notice that in the K-matrix approach, the resonance energy  $E_\Delta$  is unaffected by the Chew-Low contribution. This point will become clear later on. In the T-matrix approach, if we set  $f_{\pi NN} = 0$ , we find that resonance position would shift upward by about 8 MeV. However, in Ref. 6, setting  $f_{\pi NN} = 0$  would cause the resonance position to shift upward by about 50 MeV. The reason behind this substantial difference is not quite clear. Nevertheless, the small shift found in our calculation can be understood as follows:

For the complete amplitude,  $\delta_{33}$  goes through  $\pi/2$  at  $E = 293$  MeV. When we set  $f_{\pi NN} = 0$ , the dominance of the contribution from diagram 2(b) implies that now  $\delta_{33} = \pi/2$  when  $E = \omega_\Delta = 301$  MeV. Therefore, a small upward shift of about 8 MeV is expected. Furthermore, a small shift is consistent with the K-matrix result of no shift at all.

In the K-matrix approach, the observed resonance energy,  $E_\Delta = 293$  MeV, is exactly equal to the dressed (renormalized)  $\Delta$  mass,  $\omega_\Delta$ . This result is not accidental. The reason is that, from Eqs. (A3) and (B6), the K-matrix for diagram 2(b) diverges at  $E = \omega_\Delta$ , while all other contributions are finite. Therefore, from Eq. (18),  $\delta_{33}(E = \omega_\Delta) = \pi/2$ . This result is independent of the magnitude of  $f_{\pi NN}$  or  $f_{\pi NN\Delta} (\neq 0)$ . That explains why the Chew-Low contribution does not affect  $E_\Delta$ . Furthermore, the result  $E_\Delta = \omega_\Delta$  is also independent of the order of the perturbation expansion, because, by definition, Eq. (A3) is always true. In this sense, the result  $E_\Delta = \omega_\Delta$  is exact.

As mentioned earlier, the fact that the T-matrix and the K-matrix results are similar already implies that truncation of the scattering series at 4th order is a good approximation. Nevertheless, it is worthwhile to check explicitly the convergence of the perturbation series.

Figure 2(a) plus iterations with itself is usually called the "Chew-Low series". It has been argued that all terms in the series are of the same order of magnitude.<sup>25,15</sup> However, with the  $\pi NN$  form factor we use, the ratio of the first three terms of the series is actually  $\approx 1:0.14:0.02$  (real part), indicating fast convergence.

## V. SUMMARY AND CONCLUSIONS

In quark models, a nucleon consists of a three-quark core surrounded by a pion cloud. The  $\pi$ -N interaction form factor depends on the size of the core. It is interesting to note that two very different quark models (bag model and CQM) give similar form factors. Using the quark model form factor, we have shown that the perturbation series of  $\pi$ -N scattering converges very fast. The (3,3) phase shift is calculated by perturbation expansion to 4th order in renormalized coupling constants. We find that the three-quark  $\Delta$ -state is very important in producing the physically observed  $\Delta(1232)$  resonance. In fact the general features of the observed resonance (mass and width) are determined mainly by the  $\Delta$ -contribution. The contribution from the conventional Chew-Low series is small but not negligible, i.e. its inclusion is necessary to fit the data. For the above general conclusion, we agree with the CBM results. However, from the comparison made in Sec. IV, the  $\pi NN$  coupling seems to play a much more important role in reproducing the data in the CBM calculation than in ours. Finally, in the K-matrix approach, we have shown that the

observed resonance energy,  $E_{\Delta}$ , is equal to the dressed  $\Delta$  mass,  $\omega_{\Delta}$ ; and this result is independent of the order of the perturbation expansion.

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Appendix A

The renormalization procedure for a static theory has been given before in other works. 17, 26 We include it here for completeness and clarity. In the following, "iε" and "p" (Cauchy principal value) will be understood wherever needed.

A. Mass Renormalization

If we denote the self-energy correction, shown in Fig. 3(a), by  $\Sigma(E)$ , then the inverse of the modified baryon propagator is given by

$$G'(E)^{-1} = E - M^{(0)} - \Sigma(E) \quad (A1)$$

where  $M^{(0)}$  is the bare mass. Next we expand  $\Sigma(E)$  in Taylor series around  $M$ , the renormalized mass:

$$\Sigma(E) = \text{Re } \Sigma_r(M) + \text{Re } \Sigma_r'(M)(E-M) + \Sigma_r''(E) \quad (A2)$$

where  $\text{Re}$  denotes the real part, and  $\Sigma_r''(E)$  is the remainder. Note that in the K-matrix formulation,  $\Sigma(E)$  is real, and thus from Eq. (A2)

$$\Sigma_r''(M) = \Sigma_r''(M) = 0. \quad (A3)$$

Substituting Eq. (A2) into Eq. (A1) and rearranging terms, we get

$$G'(E) = Z_2 G(E) \quad (A4)$$

where the renormalization constant  $Z_2$  is defined by

$$Z_2 = [1 - \text{Re } \Sigma_r'(M)]^{-1}; \quad (A5)$$

$G(E)$  is the renormalized propagator:

$$G(E)^{-1} = E - M - Z_2 \Sigma_r''(E); \quad (A6)$$

and we have made the identification

$$M = M^{(0)} + \text{Re } \Sigma_r(M). \quad (A7)$$

B. Vertex Renormalization

Let the vertex correction, shown in Fig. 3(b), be  $\Gamma(E, E')$ . The modified vertex function is then given by

$$V(E, E') = (Z_2 Z_2')^{1/2} f^{(0)} u(k) [1 + \Gamma(E, E')] \quad (A8)$$

where the factor  $(Z_2 Z_2')^{1/2}$  comes from the propagator renormalization of the baryon lines. Rearranging terms, we get

$$V(E, E') = f u(k) v(E, E'), \quad (A9)$$

where we have identified the renormalized coupling constant  $f$  as

$$f = \frac{(Z_2 Z_2')^{1/2} f^{(0)}}{Z_1}; \quad (A10)$$

the renormalization constant  $Z_1$  is defined by

$$Z_1 = [1 + \Gamma(0, 0)]^{-1}, \quad (A11)$$

and

$$v(E, E') = Z_1 [1 + \Gamma(E, E')] \quad (A12)$$

Lowest order expressions for  $\Sigma(E)$  and  $\Gamma(E, E')$  can be found in Refs. 7 and 17.



Appendix B

In this appendix, we write down explicit expressions for the scattering diagrams shown in Fig. 2. First, we define some notations to be used later:

$$P_{3/2}(k', k) = \frac{6}{3} k' k - \frac{1}{3} \tau_k^+ \tau_k$$

$$P_{1/2}(k', k) = \frac{1}{3} \tau_k^+ \tau_k$$

$$Q_{3/2}(k', k) = k' \cdot k - \frac{1}{3} \hat{\sigma} \cdot k' \hat{\sigma} \cdot k$$

$$Q_{1/2}(k', k) = \frac{1}{3} \hat{\sigma} \cdot k' \hat{\sigma} \cdot k \quad (B1)$$

where the P's (Q's) are the usual isospin (angular momentum) projection operators. (The arguments will be omitted in later use.) Also,

$$\rho(a,b) = (aP_{3/2} + bP_{1/2})(aQ_{3/2} + bQ_{1/2}) \quad (B2)$$

$$F_2(k) = \frac{4\pi}{m^2} u^2(k) \frac{1}{2\omega_k}$$

$$F_4(k) = \frac{4}{3m^4} u^2(k) \frac{1}{2\omega_k} \quad (B3)$$

and

$$I(E_1, E_2, E_3) = \int dq q^4 [u_q(E-\omega_q-E_1)(E-\omega_q-E_2)(E-\omega_q-E_3)]^{-1} \quad (B4)$$

A. Second Order Amplitudes

Fig. 2(a):

$$A_2(N) = F_2(k) f_{\pi NN}^2 v^2(E_N, E_N - \omega_k) [E_N - \omega_k - Z_2^2(N) \Sigma_k^+(N)(E_N - \omega_k)]^{-1} \rho(2, -1). \quad (B5)$$

Fig. 2(b):

$$A_2(\Delta) = F_2(k) f_{\pi N \Delta}^2 v^2(E_N, E) [E - \omega_\Delta - Z_2^2(\Delta) \Sigma_k^+(\Delta)(E)]^{-1} \rho(1, 0) \quad (B6)$$

where  $E_N$  is the initial nucleon kinetic energy in the  $\pi$ -N centre-of-mass frame, such that  $E = E_N + \omega_k$ . See Eq. (20) and the discussion preceding it. The crossed  $\Delta$  amplitude [Fig. 2(c)] can be obtained from

$A_2(\Delta)$  by crossing the external pions, i.e.,

$$k \leftrightarrow -k' \text{ (isospin indices)} \\ (\omega_k, k) \leftrightarrow (-\omega_{k'}, -k') \quad (B7)$$

Note that  $\omega_k$ 's in  $F_2$  and  $F_4$  are from pion wave function normalizations, and therefore should not be affected by Eqs. (B7).

B. Fourth Order Amplitudes

Fig. 2(d):

$$A_4(N, N, N) \equiv A_4(B=N, B'=N, B''=N) \\ = F_4(k) f_{\pi NN}^4 I(\omega_k, 0, \omega_k) \rho(4, 1) \quad (B8)$$

$$A_4(\Delta, N, N) = A(N, N, \Delta) \\ = F_4(k) f_{\pi NN}^2 f_{\pi N \Delta}^2 I(\omega_k + \omega_{\Delta}, 0, \omega_k) \rho\left(\frac{2}{3}, -\frac{4}{3}\right) \quad (B9)$$

$$A_4(N, \Delta, N) = F_4(k) f_{\pi NN}^2 f_{\pi N \Delta}^2 I(\omega_k, \omega_{\Delta}, \omega_k) \rho\left(\frac{5}{3}, \frac{8}{3}\right) \quad (B10)$$

$$A_4(\Delta, \Delta, N) = A(N, \Delta, \Delta) \\ = F_4(k) f_{\pi NN} f_{\pi N \Delta}^2 f_{\pi \Delta \Delta}^2 I(\omega_k + \omega_{\Delta}, \omega_{\Delta}, \omega_k) \rho\left(-\frac{5}{3}, \frac{10}{9}\right) \quad (B11)$$

$$A_4(\Delta, N, \Delta) = F_4(k) f_{\pi N \Delta}^4 I(\omega_k + \omega_{\Delta}, 0, \omega_k + \omega_{\Delta}) \rho\left(\frac{1}{9}, \frac{16}{9}\right) \quad (B12)$$

$$A_4(\Delta, \Delta, \Delta) = F_4(k) f_{\pi N \Delta}^2 f_{\pi \Delta \Delta}^2 I(\omega_k + \omega_{\Delta}, \omega_{\Delta}, \omega_k + \omega_{\Delta}) \rho\left(\frac{5}{3}, \frac{25}{18}\right). \quad (B13)$$

The crossed 4th order amplitudes [Fig. 2(e)] can again be obtained easily from the corresponding  $A_4$ 's by crossing symmetry [see Eq. (B7)].

Figure Captions

1. Microscopic view of pionic contribution to baryon self-energy. The solid lines represent quarks and the dashed line represents a pion.
2. Pion-nucleon scattering amplitude to 4th order. The B's stand for N or  $\Delta$ .
3. (a) Pionic contribution to baryon self-energy. (b)  $\pi$ -baryon vertex correction. The B's stand for N or  $\Delta$  and E's for energies.
4. Results from the T-matrix approach. The solid line represents the best fit to the data. The dashed line corresponds to the  $\Delta$ -contribution and the dash-dotted line the Chew-Low contribution. The experimental data are from Ref. 24.
5. Results from the K-matrix approach. The meaning of the lines are the same as in Fig. 4.

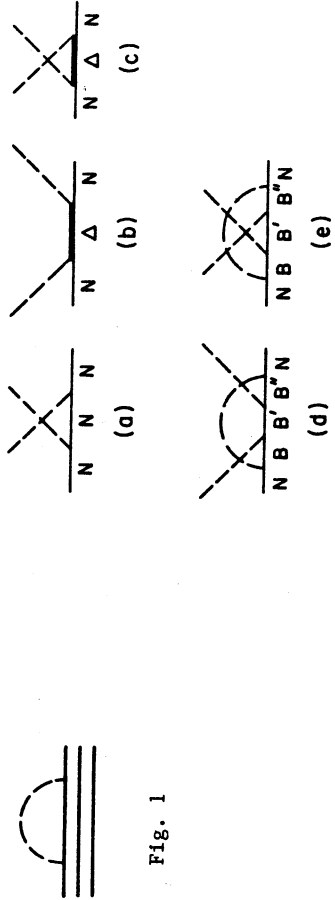


Fig. 2

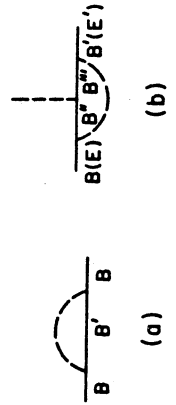


Fig. 3

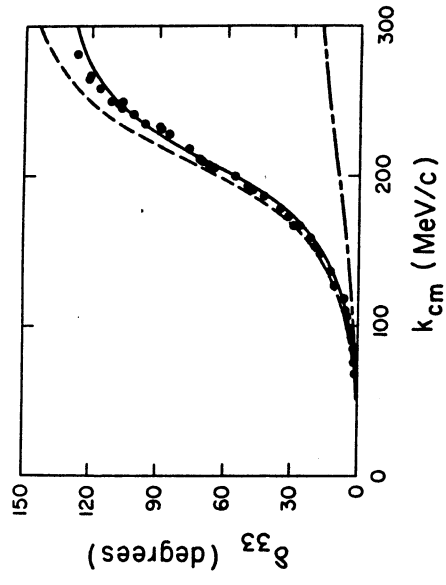


Fig. 4

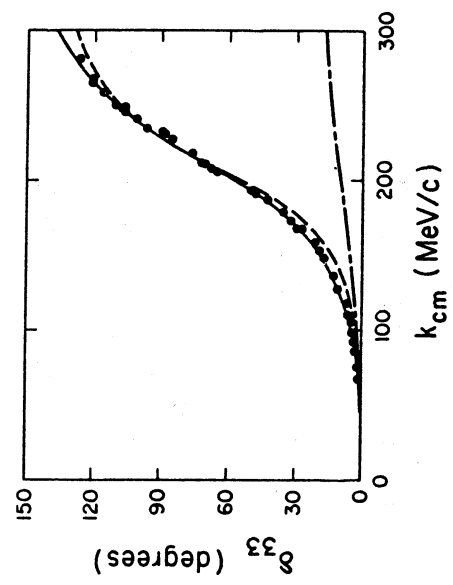


Fig. 5