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RARE KAON PROCESSES AS A PROBE OF THE TOP QUARK AND GRAVITINO MASSES IN LOCAL SUPERSYMMETRIC (SUSY) THEORIES

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ABSTRACT

A detailed analysis of the rare K processes in local SUSY theories is given. The top quark mass cannot be much higher than O(100 GeV) without making the gravitino mass lower than O(20 GeV), its present lower bound in local SUSY theories from PETRA and PEP.

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1. Grand Unified Theories (GUTs) suffer from the gauge hierarchy problem. Supersymmetry seems to provide a natural way out from this impasse[1]. Alas, global supersymmetry does not fit the bill[2]. Realistic models[3] demand the primordial SUSY breaking scale to be[3]

$$M_S \sim \sqrt{M_W M_P} \simeq O(10^{10}\ \mathrm{GeV}) \tag{1}$$

where M_W denotes the electroweak scale and M_P is the celebrated Planck mass (10^{19} GeV). In such a case, gravitational interactions[4] cannot be neglected if we want to solve the gauge hierarchy problem, since, for example, there are contributions[5] to scalar (Higgs, squarks, sleptons) masses proportional to the gravitino mass

$$m_{3/2} \sim \frac{M_S^2}{M_P} \sim M_W \tag{2}$$

and thus comparable to the maximum allowable scalar masses[5].

Among the many advantages of local SUSY theories, the, in general, automatic soft breaking[6] of global SUSY, as a direct consequence of spontaneous local SUSY breaking, is of great importance. The spectacular thing about soft global SUSY breaking emerging from supergravity is that its very specific form[5],[6] seems to satisfy all the very stringent criteria coming from low energy phenomenology. It is well known that the absence of flavour-changing neutral currents (FCNC) seems to be the Achilles' heel of many of the substitutes or extensions of the standard $SU(3) \otimes SU(2) \otimes U(1)$ model, such as composite or technicolour models. In supersymmetric theories, FCNC imposes severe limits[7],[8],[9] on the mass difference between squarks of the same charge but of different generation. That sounds rather bad for softly broken SUSY theories in general, since all squark masses are more or less arbitrary[10]. It should not come as a big surprise that in local SUSY theories, all squarks have more or less the same mass[5]: gravity is flavour-blind! There are certainly non-gravitational (i.e., non-universal) corrections to squark masses, but the situation is still generally under control.

Before jumping to conclusions, there is a point to be clarified. As we discussed before, in local SUSY theories, all scalar particles, including the Higgs doublets H_2 [responsible for the $SU(2) \otimes U(1)$ breaking] are getting[5] a positive contribution to their $(\mathrm{mass})^2$ at M_P: $M_{H2}^2 \sim m_{3/2}^2 \sim M_W^2$. There are two known ways of making $M_{H_2}^2$ negative at low energies ($\sim M_W$), thus triggering the spontaneous breaking of $SU(2) \otimes U(1)$:

i) by introducing a light singlet[6];

ii) by introducing a large Yukawa coupling[11], presumably related to the presently unknown top quark mass (if we want to avoid the introduction of a fourth generation), so that its negative contribution eventually wins over the positive contribution of gauge interactions to $M^2_{H_2}$. Thus we end up with a negative M^2_{H2} at the Fermi scale $(\sim M_W)$ [11].

The first case i) is disastrous, as it brings up[12] the gauge hierarchy problem again, while in case ii), we are necessarily led to a lower bound for the top quark mass[11]

$$m_t > O(60\ \mathrm{GeV}) \tag{3}$$

The educated reader will notice immediately that in this case we clash head-on with the Buras upper bound[13] on the top quark mass of 30 ~ 40 GeV from rare kaon decays.

In this paper, we show that in local SUSY theories the Buras limit is pushed naturally to higher masses ($\sim$ 100 GeV), without even using loop-holes in the original Buras argument, involving long-distance (LD) contributions to the K_L-K_S mass difference, or without utilizing the vacuum insertion approximations à la Gaillard and Lee[14] in calculating the $\langle \bar{K}_0 | (\bar{s}_L \gamma^\mu d_L)^2 | K_0 \rangle$ $(\equiv M)$ matrix element. Interestingly enough, following the standard arguments, i.e., no LD substantial contributions and bag model[15], or, even better, PCAC type calculations[16] of M,

$$M_{\text{vacuum ins.}} \approx 2.38\ M_{\text{bag}} \approx 3\ M_{\text{PCAC}} \tag{4}$$

we find that rare K decay phenomenology becomes restrictive. It does not allow top quark masses larger than O(100 GeV) if we do not want to have gravitino masses, and thus approximately squark or slepton masses, below 20 GeV, which is the present lower bound from PETRA and PEP[17].

2. The system under consideration at scales smaller than the grand unifying scale M_X is the standard $SU(3) \otimes SU(2) \otimes U(1)$ model, involving three generations of fermions and two Higgs doublets H and $\tilde{H}$. The superpotential is

$$W = \sum_{i,j} \left\{ f^*_{ji}\, Q_j H U^c_i + \tilde{f}^*_{ji}\, Q_j \tilde{H} D^c_i \right\} \tag{5}$$

with Q_j the quark doublet superfield and U_i^c, D_i^c the singlets under the weak group SU(2). We have ignored the leptons for this discussion. In writing Eq. (5), we bear in mind that at scales $\sim M_W$, the breaking of $SU(2) \otimes U(1)$ occurs radiatively[11]. i and j denote generation indices. The soft breaking terms induced by supergravity involving the scalar fields have the form[5),6)]

$$\sum_{\text{scalars}} m_i^2 |\phi_i|^2 + m_{3/2} \sum_{i,j} \left(A_{ji} f_{ji}^* Q_j H U_i^c + \tilde{A}_{ji} \tilde{f}_{ji}^* Q_j \tilde{H} D_i^c + \text{h.c.} \right) \tag{6}$$

where now all fields in Eq. (6) denote scalars. $m_{3/2}$ is the gravitino mass. Just below the Planck scale M_P, all m_i's are equal to $m_{3/2}$ and the A_{ij}, $\tilde{A}_{ij}$'s are equal to A, which is a characteristic of the local supersymmetry breaking. The charge $\pm\frac{2}{3}$ quarks p_i, p_i^c are related to the mass eigenstates u_{i_L}, u_{i_R} through

$$u_{i_L} = (U_L)_{ij}\, p_j \quad , \quad u_{i_R} = (U_R)_{ij}\, \bar{p}_j^c \tag{7a}$$

where $U_{L,R}$ diagonalize the fermion mass matrix. We also rotate the squark states sp_i, sp_i^c, (partners of p_i, p_i^c respectively) and define new states by

$$su_i = (U_L)_{ij}\, sp_j \quad , \quad tu_i = (U_R^*)_{ij}\, (sp^c)_j \quad . \tag{7b}$$

Provided that A_{ij} are generation-blind, we have for the up-squark mass matrix using Eq. (6)

$$\sum_i \left(m_{L_i}^2 |su_i|^2 + m_{R_i}^2 |tu_i|^2 \right) + \sum_i \left[A\, m_{3/2}\, m_{u_i} (su_i)(tu_i) + \text{h.c.} \right] . \tag{8}$$

m_{Li}^2, m_{Ri}^2 contain $m_{3/2}^2$, m_{ui}^2 as well as contributions stemming from non-vanishing D terms. The crucial point to notice is that Eq. (8) does not contain flavour-changing terms, owing to the fact that A_{ij}'s were assumed to be generation-independent. While this is true at scales approaching M_P, it is in general not guaranteed down at scales M_W. However, following the renormalization group equations for A_{ij}'s from M_X down to M_W, one can see that the ratios

$A_{ij}(M_W)/A_{ij}(M_X)$ are, to a very good approximation, generation-independent. If, for instance, one starts with $A = 3$ and $M_{gaugino}/m_{3/2} = 1$ at M_X, and takes the top Yukawa coupling $f_{33} = 0.1$ at M_X, then one has for the A_{ij}, $\tilde{A}_{ij}$ of the up- and down-quarks at M_W

$$A_{ij} = 6 \, \mathrm{diag}\,(1.041,\ 1.041,\ 1.033)^{*}$$

$$\tilde{A}_{ij} = 6 \begin{pmatrix} 1.044 & 1.044 & 1.041 \\ 1.044 & 1.044 & 1.041 \\ 1.044 & 1.044 & 1.041 \end{pmatrix}$$

Though A_{ij}'s have suffered large renormalizations by a factor of two, ratios of different A_{ij}'s or $\tilde{A}_{ij}$'s are $1 + O(10^{-3})$. From now on, any reference to A will mean the value of A at scales $O(M_W)$.

Another source of such a non-alignment of the squark mass matrix M^2_{sq} and the fermion mass matrix $M_q^+ M_q$ may be mass corrections to M^2_{sq} due to Higgses, which may generate a radiative departure from this alignment[18]. However, this latter source is controllable and may not be very important. Therefore, here we supersymmetrize in a standard way, taking into account the new diagrams which contribute to the K_L-K_S mass difference and to $K_L \to \mu^-\mu^+$ involving squark and wino fields.

The mass matrix (8) leads, for each flavour index i, to four mass eigenstates doubly degenerate with masses

$$m^{+2}_{su_i} = m^{+2}_{tu_i} = \frac{1}{2}\left[m^2_{Li} + m^2_{Ri} + \left((m^2_{Li} - m^2_{Ri})^2 + 4A^2 m^2_{3/2} m^2_{u_i}\right)^{1/2}\right]$$

$$m^{-2}_{su_i} = m^{-2}_{tu_i} = \frac{1}{2}\left[m^2_{Li} + m^2_{Ri} - \left((m^2_{Li} - m^2_{Ri})^2 + 4A^2 m^2_{3/2} m^2_{u_i}\right)^{1/2}\right] \quad (9)$$

In our calculations, for definiteness, we put $m^2_{Li} \simeq m^2_{Ri}$; our results change little in the general case.

*) Without loss of generality, we have taken A_{ij}'s diagonal at M_X.

3. In Figs. 1 and 2, we give the supersymmetric graphs which contribute to the K_L-K_S mass difference and to $K_L \to \mu^-\mu^+$ *). For the sd → sd effective Lagrangian, we have from the graph of Fig. 1

$$\mathcal{L}^{susy}_{sd\to sd} = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi\sin^2\theta_W}(\bar{s}_L\gamma^\mu d_L)^2\sum_{i,j\geq 2}^{3}(U^+_{sj}U_{id}U^+_{si}U_{jd})\,\mathcal{B}_s(i,j) \tag{10}$$

We prefer to conform to the notation of Refs. 13) and 19), so that U_{ij} denotes elements of the Kobayashi-Maskawa matrix. $B_s(i,j)$ is a complicated function whose explicit form will be given below. The ordinary quark contribution is given by the same expression (10) when $B_s(i,j)$ is replaced by $B(x_i,x_j)$ of Refs. 13) and 19). For the $K_L \to \mu^-\mu^+$ process, the supersymmetric contribution of the diagrams in Fig. 2 gives the effective Lagrangian

$$\mathcal{L}^{susy}_{sd\to\bar{\mu}\mu} = \frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi\sin^2\theta_W}(\bar{s}_L\gamma^\mu d_L)(\bar{\mu}_L\gamma_\mu\mu_L)\sum_{i,j\geq 2}^{3}4U^+_{si}U_{id}\,G_s(i) \tag{11}$$

Again, the ordinary non-supersymmetric contribution is given by the same expression (11), with $G_s(i)$ being replaced by $G(x_i)$ of Refs. 13) and 19). For completeness, we give the explicit expression of B_s and G_s appearing in Eqs. (10) and (11):

$$\mathcal{B}_s(i,j) = \frac{1}{4n}\sum_{m,n=\pm}\left[a(s^m_i, s^n_j) + b(t^m_i, t^n_j) + 2(mn)\, c(s^m_i, t^n_j)\right]$$

$$G_s(i) = \frac{1}{4}\sum_{m=\pm}\left[(f(s^m_i) - f(s^m_1)) - \frac{1}{n}(g(s^m_i,\lambda) - g(s^m_1,\lambda)) - \delta_{i}f(t^m_i)\right] \tag{12}$$

$$-\frac{1}{4}\sum_{m,n=\pm}(mn)\left[(\phi(s^m_i, s^n_i) - \phi(s^m_1, s^n_1)) - \delta_i\,\phi(t^m_i, t^n_i)\right]$$

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*) The wino w_1 and w_2 masses are taken equal inside the Feynman integrations. This is a very good approximation. In our numerical calculations, later on, these will be taken to be $\approx M_W$.

a, b, c are given by

$$a(s_i,s_j') = \frac{1}{4}\left[g(s_i,s_j') - g(s_i,s_i') + g(s_i,s_i') - g(s_i,s_j')\right]$$
$$b(t_i,t_j') = \frac{1}{4}\,\delta_i\delta_j\; g(t_i,t_j') \tag{13}$$
$$c(s_i,t_j) = \frac{1}{4}\,(\delta_i\delta_j)^{1/2}\, g(s_i,t_j)$$

whereas the functions g, f, d are given by

$$g(x,y) = \frac{1}{(x-y)}\left[\left(\frac{x}{1-x}\right)^2 \ln x - (x \rightleftarrows y)\right] + \frac{1}{(1-x)(1-y)}$$
$$f(x) = \frac{x}{(1-x)^2}\ln x + \frac{1}{(1-x)}\,, \quad \phi(x,y) = \frac{1}{8(x-y)}\left[\frac{x^2}{(1-x)}\ln x - (x\rightleftarrows y)\right] + \frac{3}{16} \tag{14}$$

The arguments $s_i^{\pm}$, $t_i^{\pm}$ as well as η, δ_i and λ appearing in (12) are

$$s_i^{\pm} = \left(\frac{m^{\pm}_{su_i}}{m_{\tilde{W}}}\right)^2,\; t_i^{\pm} = \left(\frac{m^{\pm}_{tu_i}}{m_{\tilde{W}}}\right)^2,\; \eta = \left(\frac{m_{\tilde{W}}}{M_W}\right)^2,\; \delta_i = \left(\frac{m_{u_i}}{M_W}\right)^2,\; \lambda = \left(\frac{m_{s_\nu}}{m_{\tilde{W}}}\right)^2 \tag{15}$$

with $m_{\tilde{W}}$ the wino mass, m_{s_ν} the sneutrino mass, and M_W the W boson mass. The essential point to keep in mind, putting aside all this formalism, is that B_s, G_s now contain information for the gravitino mass $m_{3/2}$ and the constant A of local SUSY through the squark masses $m^{\pm}_{su_i,tu_i}$.

Before dealing with the general case, we briefly recall why one has a bound for the top quark mass in the ordinary theory by combining the K_L-K_S mass difference and the bound for the short-distance $K_L \to \mu^-\mu^+$ amplitude. Ignoring for the sake of argument the contribution of up- and charm-quarks, we have for the K_L-K_S mass difference

$$\frac{\Delta m_K}{m_K} = \langle \bar{K}_0 | (\bar{s}d)^2 | K_0\rangle \left(\frac{m_t^2}{M_W^2}\right) \Theta \tag{16}$$

where Θ denotes a function of the Kobayashi-Maskawa angles and m_t is assumed large enough. To parametrize $<\bar{K}_0|\ldots|K_0>$, it is often useful to introduce the parameter R [see, for instance, Ref. 13)] so that

$$\langle \bar{K}_0 | (\bar{s}d)^2 | K_0 \rangle = K \left(\frac{.42}{R}\right) \tag{17}$$

where K is a constant. The parameter R is defined so that for $R = 0.42$, one has the vacuum insertion approximation, while for $R = 1$, one has the bag model. The PCAC estimate already discussed corresponds to $R \simeq 1.3$, slightly larger than that of the bag models. For the short-distance $K_L \to \mu^-\mu^+$ amplitude, one has a bound

$$\sqrt{\Theta}\left(\frac{m_t^2}{M_W^2}\right) \leq (\text{constant})\,|s_1 c_3| \tag{18}$$

so that, combining (16) and (18) on account of (17), we get

$$\left(\frac{m_t^2}{M_W^2}\right) \leq (\text{constant})\,\frac{(s_1 c_3)^2}{R} \tag{19}$$

One sees therefore that m_t cannot increase arbitrarily. The bound given by Eq. (19) becomes stronger with higher values of R. The analysis which has just been presented for the Buras limit on m_t in an ordinary theory is very much simplified, since actually the complete expressions for $\Delta m_K/m_K$ and the $K_L \to \mu^-\mu^+$ amplitude should be used.

In the supersymmetric theory for the K_L-K_S mass difference, one has

$$\frac{\Delta m_K}{m_K} = \frac{f}{R}\left[\eta_c A_c \mathcal{B}_{cc} + \eta_t A_t \mathcal{B}_{tt} + \eta_{ct} A_{ct} \mathcal{B}_{ct}\right] \tag{20}$$

with "f" a numerical constant equal to $1.6\ 10^{-10}$, and where the functions B_{ij} are the sums of $B_S(i,j)$ and the $B(x_i,x_j)$'s of the non-supersymmetric case. $A_{c,\ t,\ ct}$ depend on the Kobayashi-Maskawa angles, and $\eta_{c,\ t,\ ct}$ are QCD correction factors. To help the reader, we have followed very closely the notation of Ref. 13). From the $K_L \to \mu^- \mu^+$ short-distance bound, we get

$$|A_{s.d.}(K_L \to \mu^- \mu^+)| = |\eta_c' A_c' G_c + \eta_t' A_t' G_t| \leq (.85\ |s_1 c_1|) 10^{-2} \tag{21}$$

where we have a prime on the η, A's to distinguish them from those of Eq. (20). $G_{c,t}$ contain the contributions both of ordinary graphs and the supergraphs. A bound on m_t can now arise, combining Eqs. (20) and (21).

The Kobayashi-Maskawa angles are not known, except for the Cabibbo angle $\theta_1 \equiv \theta_c$. However, they are constrained experimentally to be within some regions[20]. In numerical calculations, we scan the whole allowed region of angles, imposing the constraint that the CP violating parameter ε_m is of the order $\approx 2.10^{-3}$ [21),22) *)].

To present a rather model-independent analysis, we take m_{Li}^2 and m_{Ri}^2 in Eq. (9) so that $m_{Li}^2 \simeq m_{Ri}^2 \simeq r^2 m_{3/2}^2 + m_{ui}^2$ where "r" is a renormalization factor**) and we have assumed that the D term contribution is negligible. For squarks, "r" is of the order of three when the gaugino mass M_0 at M_X is taken to be equal to $m_{3/2}$. When $M_0 << m_{3/2}$, the "r" is of the order of unity. For the sleptons, $r \simeq O(1)$ because of the absence of strong interactions. In general, "r" is model-dependent. Since we do not want to commit ourselves to a particular model, we rescale the gravitino mass $m_{3/2}$ and the A defining the rescaled quantities

$$m_g = r\, m_{3/2} \quad , \quad A_r = A/r \tag{22}$$

We remind the reader that A is the renormalized value of the A parameter at scales $O(M_W)$.

That done, the mass spectrum as given by Eq. (9) is

$$m_{su_i}^{\pm\,2} = m_{tu_i}^{\pm\,2} \simeq m_g^2 + m_{u_i}^2 \pm A_r m_g m_{u_i} \tag{23}$$

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*) Actually, $|\varepsilon_m| \approx |\varepsilon| \approx 2.10^{-3}$ if one employs the fact that $|\varepsilon'/\varepsilon| \sim |2\xi/2\xi + \varepsilon_m| \lesssim O(10^{-2})$ [see Ref. 21)]. In our numerical calculations, we constrain $|\varepsilon_m/2.10^{-3}|$ to lie within 0.2 and 5. That is, we allow for a "tolerance" factor of about $\simeq 5$.

**) Actually, the different squark fields have different renormalization factors, but for our calculations to a good approximation, we have put them equal.

The relation of the physical gravitino mass $m_{3/2}$ to m_g and of the A to the effective A_r is given through Eq. (22). Notice that as $r \gtrsim 1$, $m_{3/2} \lesssim m_g$ and $A \gtrsim A_r$. Given the choice of "R" we are then left with three parameters m_t, m_g and A_r at M_W, which enter into the equations, and hence for a fixed A_r the bound on $K_L \to \mu^-\mu^+$ can give us the allowed m_g, m_t values when scanning the whole range of the angles, as discussed previously.

In Fig. 3, we plot the ratios $B_{tt}/B_{tt}^{ordinary}$ as a function of m_g for two typical top quark masses m_t = 20 GeV and 70 GeV. The A's were taken as $A_r = 0$ and $|A_r| = 2$. We observe that the ratios are very close to unity, approaching one asymptotically when m_g grows. The corresponding ratios for B_{ct} and B_{cc} stay even closer to one, due to the smallness of the charm-quark masses. This shows that the supersymmetry effect on the K_L-K_S mass difference is quite negligible. On the contrary, the effect of supersymmetry on $K_L \to \mu^-\mu^+$ can be quite substantial. In fact, graphs with winos and squarks carry a negative sign relative to the ordinary graphs while they are of the same order of magnitude, resulting in a cancellation which makes the $A(K_L \to \mu^-\mu^+)$ amplitude smaller than that of the ordinary theory $A^{ordin.}(K_L \to \mu^-\mu^+)$. This weakens the bound of Eq. (21), thus allowing for higher values of m_t. These conclusions are independent of the specific values of A_r chosen. In order to have a feeling of what may happen, we take $\theta_2 = 5^0$, $\theta_3 = 13^0$ and $\delta = 0.5^0$ for $|A_r| = 2$, m_g = 60 GeV and m_t = = 20 GeV in the free quark model. In this case, $|\varepsilon| \simeq 2.10^{-3}$. For these values, the bound of the right-hand side of Eq. (21) is $\simeq (0.20)10^{-2}$, while $|A^{ordin.}(K_L \to \mu^-\mu^+)|$ is $(0.25)10^{-2}$, exceeding this upper bound. However, at the same time, $A(K_L \to \mu^-\mu^+)$ stays well below the bound since $|A/A^{ordinary}| \simeq 0.5$.

In Fig. 4, we have plotted the boundaries of the allowed regions in the m_t, m_g plane for the cases $A_r = 0$ and $|A_r| = 2$ for the free quark model (FQM), and also when QCD corrections are taken into account. For other values of A_r within $0<|A_r|<2$, the boundary lies between those of $A_r = 0$ and $|A_r| = 2$, and hence there is no substantial change. The values taken for the parameter R are close to unity (bag models) and close to 1.3 (PCAC calculation); for the "vacuum insertion" values, $R \simeq 0.42$, we found no bound on m_t, m_g at least in the range $m_t \leq$ 300 GeV. For the bag models, we observe that in order to have $m_t \simeq$ O(100 GeV), we need $m_g \simeq$ O(20 GeV) and consequently light gravitino masses $m_{3/2} <$ O(20 GeV). Also, if one pushes up m_g to have rather large values $\simeq$ O(150 GeV), the top quark mass is forced to be $\lesssim$ O(50 GeV) in the QCD case and $\lesssim$ O(70 GeV) in the FQM. This may indicate that models with large top quark mass ($\gtrsim$ 100 GeV) may be in trouble and violate the Buras limit. However, one cannot derive a definite conclusion unless all the details of the specific models are taken into account.

For values of $R \simeq 1.3$ (PCAC), the situation is even more restrictive (see Fig. 4), since one has $m_t \lesssim O(80\ \text{GeV})$ in the FQM and $m_t \lesssim O(70\ \text{GeV})$ in the QCD case. If the PCAC approach is the most reliable one for calculating the value of R, then one draws the conclusion that a reasonable upper bound for the top quark mass is 80 GeV. Notice that, for $R \simeq 1.3$, in the QCD case, the Buras upper bound on m_t in conjunction with the lower bound of ~ 60 GeV, coming from the radiative breaking scenario[11] of the electroweak $SU(2) \times U(1)$, forces the m_t to be between ~ 60 and ~ 70 GeV, while at the same time m_g and hence $m_{3/2}$ are less than $O(50\ \text{GeV})$.

For other values of $|A_r|$ larger than two, the squark masses are not positive definite, as is seen from Eq. (23), and hence one has to impose the condition $(m^{\pm}_{su_i})^2, (m^{\pm}_{tu_i})^2 \geq 0$. In reality, these masses should be greater than $\simeq 20$ GeV, as dictated by experimental information from PETRA and PEP. This constrains the allowed region for m_t, m_g a great deal, especially for $|A_r| \geq 3$, in addition to the constraints coming from the Buras limit. In Fig. 5, we have depicted the disallowed (shadowed) regions when $(m^{\pm}_{st})^2 \geq m_o^2$, m_o being the experimental lower bound on m_{squark} which presently has a value of about 20 GeV. We observe that for $|A_r| < 2$, the boundary of the allowed region in the m_t, m_g plane is the segment of an ellipse passing through the points (m_o,o) and (o,m_o) and whose semi-major axis lies on the diagonal and has a length of $\sqrt{2}\, m_o/(2-|A_r|)^{1/2}$. For $|A_r| = 2$, the boundary becomes the two straight lines $m_g = \pm\, m_o + (\tan \pi/4) m_t$. Finally, for $|A_r| > 2$, the corresponding boundary consists of the segments of two hyperbolae passing through (m_o,o) and (o,m_o), with the lines $m_g = (\tan\theta) m_t$ and $m_g = (\cot\theta) m_t$ respectively as asymptotes; the angle θ is given by $\pi/4 - \text{arc}\tan(|A_r|-2/|A_r|+2)^{1/2}$. The allowed area in this case is very limited, especially for large values of $|A_r|$. This, in combination with the Buras limit, imposes tighter constraints on the top quark mass.

Finally, we come to examine a realistic model based on $N = 1$ supergravity. We take the model of Ellis, Hagelin, Nanopoulos and Tamvakis[11] and consider their case of having $m_t = 70$ GeV and $m_t = 88$ GeV respectively. The effective A_r in the first case is approximately $A_r \simeq 0.6$, in the second case $A_r \simeq 0.85$, and in both cases m_g is related to the physical gravitino mass $m_{3/2}$ through $m_g \simeq 2.5\, m_{3/2}$. This is because the top squark renormalization factor is about 2.5. In Fig. 3, we have plotted the ratio $B_{tt}/B_{tt}^{ordinary}$ for both cases, and we see that the ratio is close to unity, as expected from the previous general analysis.

In Fig. 6, we give the maximum value of the parameter R allowed as a function of m_g. We observe that in the first case, one can have $0.9 \lesssim R \lesssim 1$ (bag models) for $m_g \lesssim 65$ GeV, that is, for a relatively light gravitino mass $m_{3/2} \lesssim 25$ GeV. The second case is not consistent with bag or PCAC type calculations, as expected, since the top quark mass in this case is quite heavy (88 GeV).

4. In conclusion, we have shown that in $N = 1$ local Supersymmetric theories, the Buras upper bound on the top quark mass can be pushed upwards ($\sim$ 100 GeV) without too many problems. There is enough space for the top quark mass to be between its lower bound of (50-60 GeV) necessary for radiative $SU(2) \times U(1)$ breaking and its super-Buras bound of (80-100 GeV). Interestingly enough, large values ($\sim$ 100 GeV) for the top quark force the gravitino to have a low mass of about O(20 GeV).

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FIGURE CAPTIONS

Fig. 1 : Graphs contributing to the $sd \to sd$ effective Lagrangian involving squarks and winos.

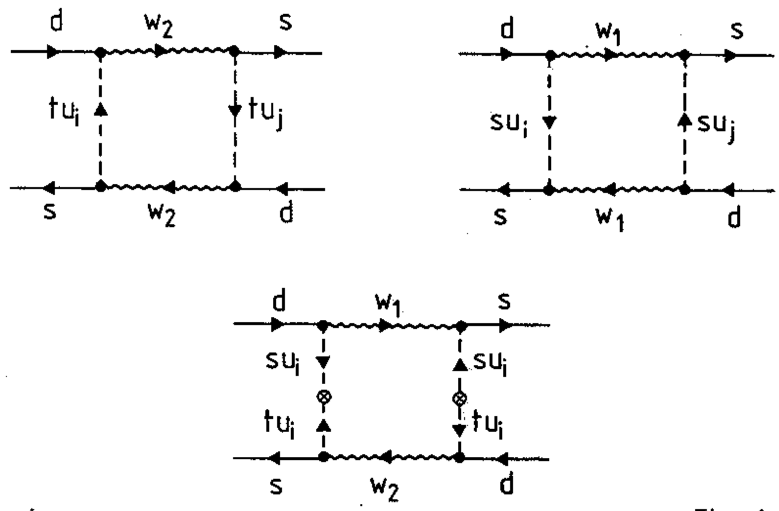
Fig. 2 : Graphs contributing to the $\bar{s}d \to \mu^-\mu^+$ effective Lagrangian involving squarks and winos.

Fig. 3 : Values for the ratios $B_{tt}/B_{tt}^{ordinary}$ as functions of the rescaled gravitino mass m_g for top quark masses $m_t = 20$ GeV and $m_t = 70$ GeV. The solid lines correspond to $A_r = 0$ and the dashed lines to $|A_r| = 2$. The dashed-dotted lines are the Ellis, Hagelin, Nanopoulos and Tamvakis model (EHNT) for $m_t = 70$ GeV and 88 GeV respectively.

Fig. 4 : Boundaries of the allowed region in the m_t, m_g plane for $R \simeq 1$ (bag models) and $R \approx 1.3$ (PCAC), when the Buras limit is imposed. The solid line is for $A_r = 0$ and the dashed line for $|A_r| = 2$. FQM and QCD stand for the free quark and QCD models respectively. The allowed region in each case lies on the left of the boundary.

Fig. 5 : The disallowed regions (shadowed) in the m_t, m_g plane when the constraint $m_{st} \geq m_o$ is imposed for $|A_r| < 2$, $|A_r| = 2$, and $|A_r| > 2$. (See also main text).

Fig. 6 : The maximum value of R (R^{max}) as a function of the mass m_g for the model of Ellis, Hagelin, Nanopoulos and Tamvakis for the cases $m_t = 70$ GeV (solid curve) and $m_t = 88$ GeV (dashed curve).



d
w_2
s
tu_i
tu_j
s
w_2
d
d
w_1
s
su_i
su_j
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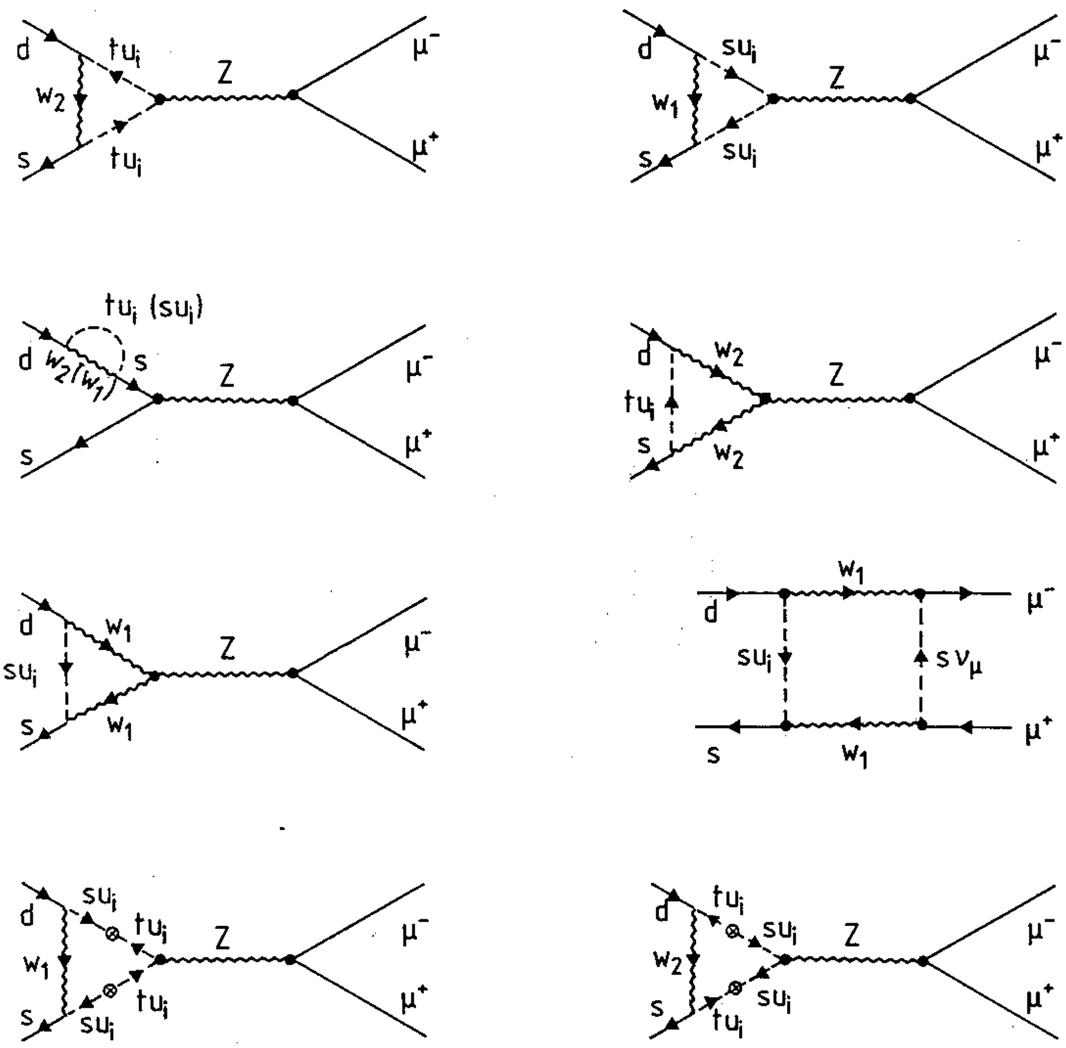
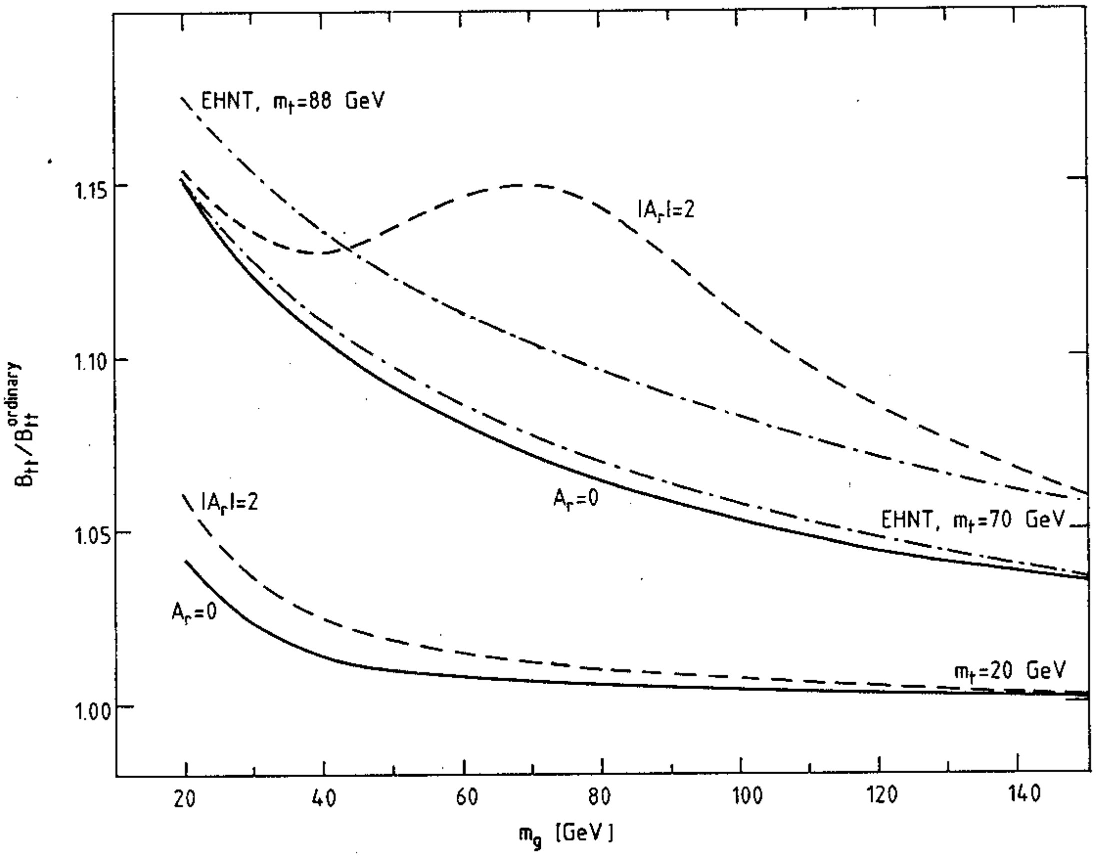
w_1
d
d
w_1
s
su_i
su_i
tu_i
tu_i
s
w_2
d

Fig. 1



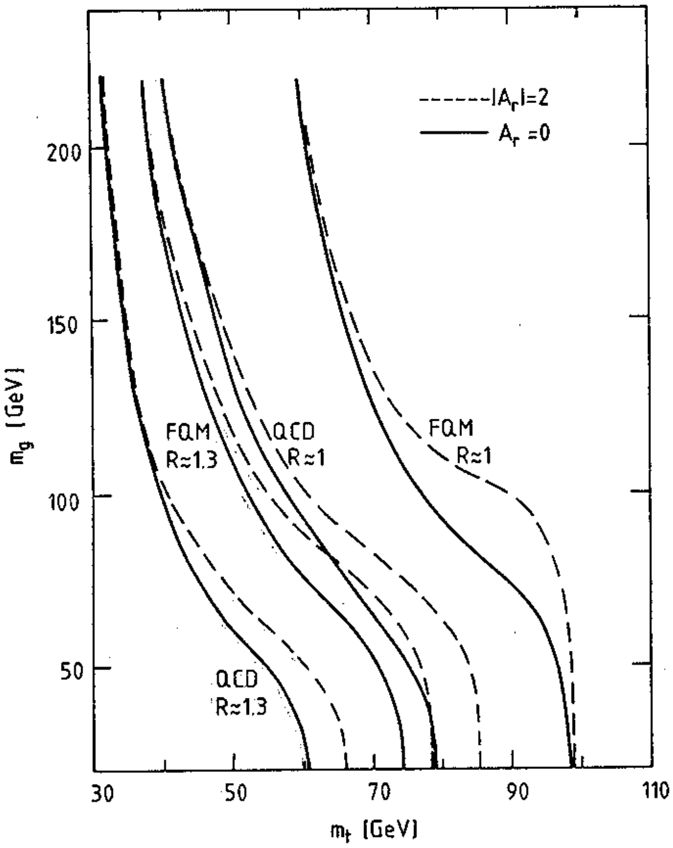
d
tu_i
w_2
s
tu_i
Z
μ^-
μ^+
d
su_i
w_1
s
su_i
Z
μ^-
μ^+
tu_i (su_i)
d
$w_2(w_1)$
s
s
Z
μ^-
μ^+
d
w_2
tu_i
s
w_2
Z
μ^-
μ^+
d
w_1
su_i
s
w_1
Z
μ^-
μ^+
w_1
d
su_i
$s\nu_\mu$
s
w_1
μ^-
μ^+
d
su_i
tu_i
w_1
s
su_i
tu_i
Z
μ^-
μ^+
d
tu_i
su_i
w_2
s
tu_i
su_i
Z
μ^-
μ^+

Fig. 2



EHNT, m_t=88 GeV
$|A_r|$=2
1.15
1.10
1.05
1.00
$B_{tt}/B_{tt}^{ordinary}$
A_r=0
EHNT, m_t=70 GeV
$|A_r|$=2
A_r=0
m_t=20 GeV
20
40
60
80
100
120
140
m_g [GeV]

Fig. 3



$|A_r|$=2
A_r =0
200
150
100
50
m_g [GeV]
FQM
R≈1.3
QCD
R≈1
FQM
R≈1
QCD
R≈1.3
30
50
70
90
110
m_t [GeV]

Fig. 4

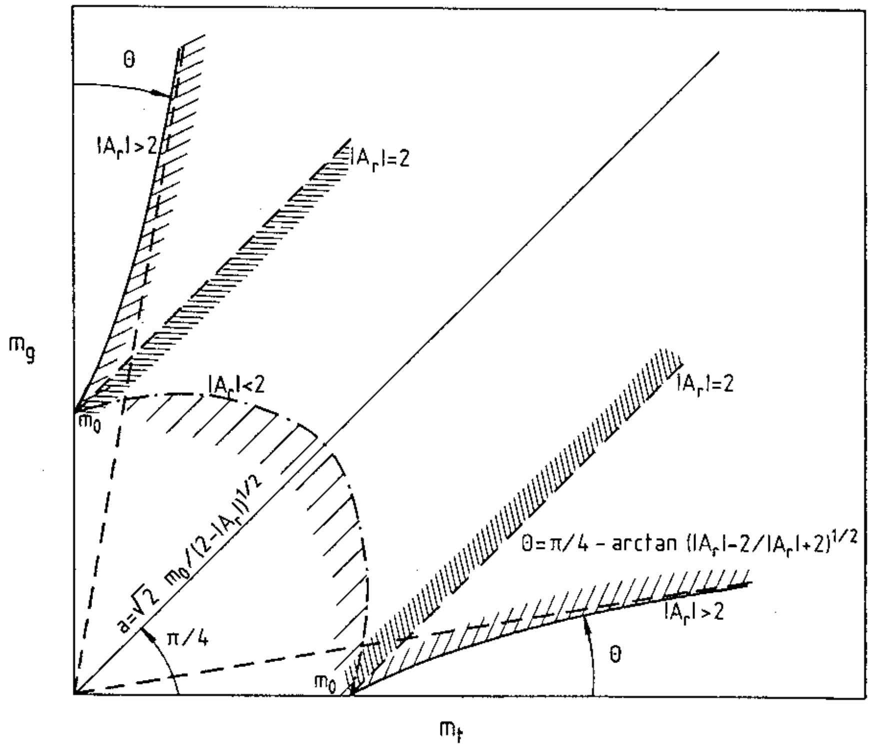

θ
$|A_r|>2$
$|A_r|=2$
m_g
$|A_r|<2$
m_0
$a=\sqrt{2}\ m_0/(2-|A_r|)^{1/2}$
$\pi/4$
m_0
$|A_r|=2$
$\theta=\pi/4-\arctan(|A_r|-2/|A_r|+2)^{1/2}$
$|A_r|>2$
θ
m_t

Fig. 5

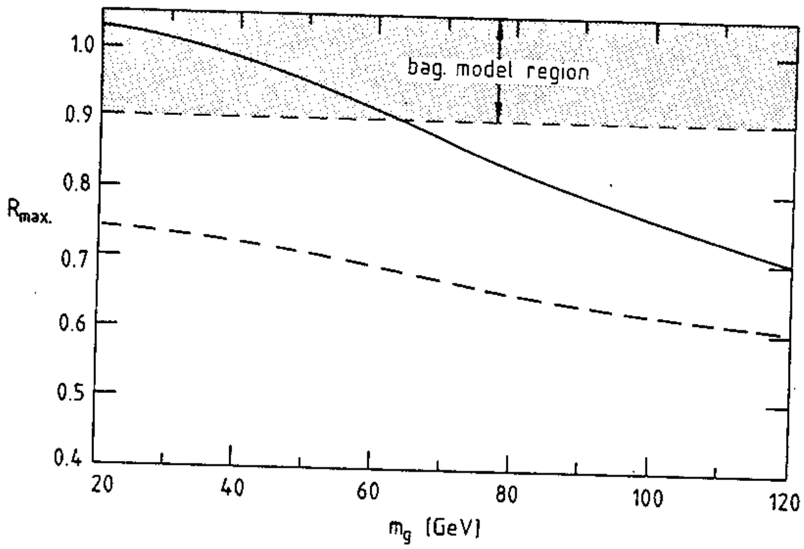

bag. model region
$R_{max.}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
20
40
60
80
100
120
m_g (GeV)

Fig. 6