

## Collective flows of protons and $\pi^-$ mesons in $^2\text{H} + \text{C}, \text{Ta}$ and $\text{He} + \text{C}, \text{Ta}$ collisions at 3.4 GeV/nucleon

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(Received 18 July 2011; published 29 December 2011)

Collective flows of protons and negative pions have been studied in  $^2\text{H} + \text{C}, \text{Ta}$  and  $\text{He} + \text{C}, \text{Ta}$  collisions at an energy of 3.4 GeV/nucleon. The data have been obtained by the 2-m Propane Bubble Chamber (PBC-500) at JINR. It is found that the directed flow of protons and  $\pi^-$  mesons characterized by  $d\langle P_x \rangle/d(y)$  increases with increase of the mass numbers of colliding nucleus pairs; the elliptic proton flow points out of the reaction plane and also strengthens as the system mass increases; the negative pion directed flow is in the same reaction plane as the proton flow for the lighter ( $^2\text{H} + \text{C}, \text{He} + \text{C}$ ) systems and in the opposite direction for the heavier ( $^2\text{H} + \text{Ta}, \text{He} + \text{Ta}$ ) systems. In  $^2\text{H} + \text{C}, \text{He} + \text{C}, \text{C} + \text{C}, \text{C} + \text{Ne}, ^2\text{H} + \text{Ta}, \text{He} + \text{Ta}, \text{C} + \text{Cu}$ , and  $\text{C} + \text{Ta}$  collisions, the linear dependence of directed and elliptic flow parameters from mass numbers of projectile and target nuclei,  $(A_P \cdot A_T)^{1/2}$ , is similar for protons while for  $\pi^-$  mesons the dependence of directed flow parameters is stronger. The ultrarelativistic quantum molecular dynamical model (URQMD) enlarged by the statistical multifragmentation model (SMM) satisfactorily describes the obtained experimental results for all pairs of nuclei. The data for such asymmetric nuclear collisions are obtained, to the best of our knowledge, for the first time.

DOI: [10.1103/PhysRevC.84.064915](https://doi.org/10.1103/PhysRevC.84.064915)

PACS number(s): 13.85.Hd, 13.85.Ni

### I. INTRODUCTION

Study of collective flows in nucleus-nucleus interactions at high and intermediate energies, such as the bounce-off [1] of compressed matter in the reaction plane (called the directed flow) and the elliptic flow in the transverse direction (the squeeze-out [2] of the participant matter out of the reaction plane at sufficiently low energies), is very important to learn more about the nuclear equation of state (see Refs. [3–7], and the excellent review in Ref. [8]). The equation of state (EOS) of nuclear matter is the relationship specifying how the pressure, or alternatively the energy per particle, depends on density and temperature. Many different methods were proposed for experimental studies of the flows in relativistic nuclear collisions, of which the most commonly used is the transverse momentum analysis technique proposed by Danielewicz and Odnyc [9].

The collective flows are mainly studied with respect to the reaction plane, which is defined by a beam direction and the impact parameter  $\mathbf{b}$  vector. In an experiment, the determination of the impact parameter  $\mathbf{b}$  is not possible, and therefore instead of  $\mathbf{b}$  a vector sum of transverse momenta of projectile and target nuclear fragments (first method), or participant protons (second method) are used. The fragmentation regions of projectile and target nuclei are not acceptable for experimental setups in some experiments, and therefore the reaction plane is defined by the second approach. The second approach is preferable also for the light nuclear systems, because the multiplicity of the participant protons is larger than the number of detected fragments.

Having determined the reaction plane, it is possible to find quantitative properties of the flows. At low and intermediate energies the average projection of a particle momentum on the reaction plane is used quite frequently, as well as the slope of its dependence on the particle rapidity. Coefficients of the Fourier decomposition of particle azimuthal distributions are very popular at high energies. For example, the elliptic flow has been explored by many collaborations including Alternating Gradient Synchrotron (AGS) [10,11], GSI [12], NA49 [13], and CERN/SPS [14,15] by means of the second harmonic coefficient of the Fourier analysis of the azimuthal distributions,  $v_2$ .

The collective flows of nucleons, pions, and nuclear fragments discovered in Refs. [16–19] are well established in collisions of heavy nuclei (for a review see Refs. [20,21], and Part V of [8]). The information about them in interactions of light and medium projectile nuclei with various target nuclei is very restricted. Recently there has been renewed interest in asymmetric systems since they offer more complete information than symmetric ones [22–24].

The flows of protons, pions, and  $\Lambda$  hyperons have been previously investigated [25–28] in light nucleus interactions with nuclei at energies of 3.4 and 3.7 GeV/nucleon by the authors of this paper. It is worth mentioning that the values of the elliptic flow excitation function,  $v_2$ , that we obtained for protons correspond to a very interesting energy region. According to the investigations in Au-Au collisions at Relativistic Heavy Ion Collider (RHIC) [29], an evolution from negative ( $v_2 < 0$ ) to positive ( $v_2 > 0$ ) elliptic flows has been observed in the energy interval of  $2.0 \leq E_{\text{beam}} \leq 8.0$  GeV/nucleon, and an apparent transition energy  $E_{\text{tr}} \approx 4$  GeV/nucleon has been noted. Therefore, the results obtained by us at the energies seem to be interesting from the viewpoint

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of an enrichment of the existing results in the above mentioned energy region. We believe that the results obtained in this paper will shed light on the nature of the flows.

We have studied the directed flows of protons and negative pions in C + C, Ne, Cu, Ta interactions at the energies 3.4 and 3.7 GeV/nucleon [27]. In this paper, we present collective flow results of protons and negative pions in  $^2\text{H} + \text{C}, \text{Ta}$  and  $\text{He} + \text{C}, \text{Ta}$  collisions at an energy of 3.4 GeV/nucleon, registered in the 2-m propane bubble chamber (PBC-500) of JINR. The reaction planes have been defined by participant protons because the protons with momentum  $p < 150 \text{ MeV}/c$  have not been detected within the PBC-500 (as far as their track lengths  $l < 2 \text{ mm}$ ) and protons with  $p < 200 \text{ MeV}/c$  are absorbed in the Ta target plate (the detector biases). The experimental results will be compared to the predictions of the ultrarelativistic quantum molecular dynamics model (URQMD) [30,31] enlarged by the statistical multifragmentation model [32].

The URQMD model is now widely applied for simulations of particle production and flow effects in various nucleus-nucleus interactions [8,33,34], although its original design was directed towards high energies. It considers the nuclear mean field, the Coulomb interactions, and stochastic binary collisions. It allows one to use “soft” or “hard” EOS, the momentum dependent interactions, and so on. An effective EOS of the model was determined in Ref. [35] (see also [36]). Below, we use version 1.3 of the model with default values of the model parameters corresponding, especially, to “hard” EOS. The program was run in the so-called potential mode.

Recently, the model was successfully applied for a description of the HADES Collaboration data on pion production in light nuclei collisions at 1–2A GeV energy range [37,38].

## II. EXPERIMENTAL DATA

For the experiment, the 2m propane bubble chamber (PBC-500) of JINR has been placed in a magnetic field of 1.5 T.

An investigation of the collective flow effects generally requires an analysis of collisions event by event, or exclusively. In this context, it has been important to put effort into the identification of  $\pi^+$  mesons, the admixture of which among positive charged particles was about 25%–27%. The identification has been carried out on the statistical basis using two-dimensional ( $P^\perp, P^\parallel$ ) particle distributions [26]. It had been assumed that  $\pi^-$  and  $\pi^+$  mesons contribute to a given cell within the ( $P^\perp, P^\parallel$ ) plane with equal probability. The difference in multiplicity of  $\pi^+$  and  $\pi^-$  in each event was required to be less than 3. After the performed identification, the admixture of  $\pi^+$  mesons among the protons is estimated to be less than 5%–7%. The remaining positive particles were assumed to be protons ( $^2\text{H}$  was excluded).

The procedures for separating out the  $^2\text{H} + \text{C}$  and  $\text{He} + \text{C}$  collisions in propane ( $\text{C}_3\text{H}_8$ ) and the processing of the data including particle identification and corrections have been described in detail in Refs. [39,40]. The analysis produced 4581 events of  $^2\text{H} + \text{C}$ , 1424 of  $^2\text{H} + \text{Ta}$ , 9737 of  $\text{He} + \text{C}$ , and 1532 of  $\text{He} + \text{Ta}$  collision. In the experiment, the projectile fragmentation products have been identified as those characterized by the momentum  $p > 3.5 \text{ GeV}/c$  and

angle  $\Theta < 3^\circ$ , and the target fragmentation products have been identified as protons with the momentum  $p < 0.25 \text{ GeV}/c$  in the target rest frame. The participant protons used for a determination of the reaction plane were determined as protons with  $p > 0.25 \text{ GeV}/c$ , different from the single charged projectile fragments.

## III. THE DIRECTED FLOW OF PROTONS

We have investigated the directed flow of protons in  $^2\text{H} + \text{C}$ ,  $^2\text{H} + \text{Ta}$ ,  $\text{He} + \text{C}$ , and  $\text{He} + \text{Ta}$  collisions at an energy of 3.4 GeV/nucleon using the transverse momentum analysis technique developed by Danielewicz and Odnyc [9]. The participant protons have been used for the determination of the reaction plane.

The analysis was carried out in the laboratory system. To eliminate the correlation of the particle with itself (autocorrelations) for each particle we estimated the reaction plane with the contribution of that particle removed from the definition of the reaction plane.

The reaction plane is spanned by the impact parameter vector  $\mathbf{b}$  and the beam axis. Within the transverse momentum method, the direction of  $\mathbf{b}$  is estimated event by event in terms of the vector constructed from particle transverse momenta:

$$\mathbf{Q}_j = \sum_{\substack{i=1 \\ i \neq j}}^n \omega_i \mathbf{P}_i^\perp, \quad (1)$$

where  $i$  is a particle index and  $\omega_i$  is the weight factor,  $\omega_i = y_i - y_c$ ,  $y_i$  is rapidity of  $i$ th particle, and  $y_c$  is the average rapidity of the participant protons in each nuclear systems [41]. A projection of the transverse momentum,  $\mathbf{P}_i^\perp$ , of a particle onto the estimated reaction plane is

$$P'_{xj} = \frac{\mathbf{P}_j^\perp \cdot \mathbf{Q}_j}{|\mathbf{Q}_j|}. \quad (2)$$

The dependence of the projection on the rapidity,  $y$ , was constructed for each interacting nuclear pair. For the subsequent analysis, the average transverse momentum in the reaction plane,  $\langle P'_{xj}(y) \rangle$ , is obtained by averaging over all events in the corresponding intervals of rapidity (Fig. 1). The transverse flow parameter  $F = d\langle P_x \rangle / d(y)$ , slope of the average momentum at the intersection point  $y = y_c$  [19], has been extracted (Table I). Since the component  $P_x$  of a particle momentum in the true reaction plane is systematically larger than the component  $P'_x$  in the estimated plane, we have to correct our data for the resolution of the reaction plane. To determine the correction factor, an angle  $\Phi$  between the true and estimated reaction planes has to be defined. The correction factor  $k = 1 / \langle \cos \Phi \rangle$ , where  $\langle \cos \Phi \rangle$  is given by the ratio [9,42]

$$\langle \cos \Phi \rangle = \frac{\langle \omega P'_x \rangle}{\langle \omega P_x \rangle} = \left\langle \frac{\omega \mathbf{P}_j^\perp \cdot \mathbf{Q}_j}{|\mathbf{Q}_j|} \right\rangle \sqrt{\frac{\langle Q^2 - \sum_{i=1}^n (\omega P_i^j)^2 \rangle}{\langle n^2 - n \rangle}}, \quad (3)$$

where  $n$  is proton multiplicity in an event. The  $\langle \cos \Phi \rangle$  values obtained for different systems are listed in Table I. The values of  $\langle P_x \rangle$  corrected for  $\langle \cos \Phi \rangle$  from Eq. (3) are shown in Figs. 1 and 2.

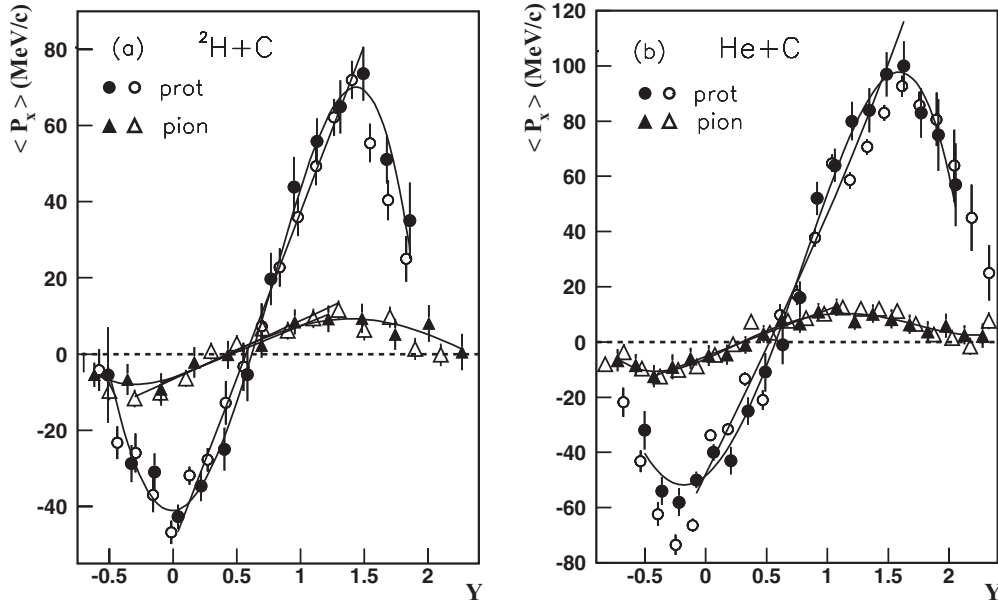


FIG. 1. The dependence of  $\langle P_x(Y) \rangle$  on the laboratory rapidity,  $Y$ , for protons and  $\pi^-$  mesons in (a)  $^2\text{H} + \text{C}$  and (b)  $\text{He} + \text{C}$  collisions. The closed symbols are the experimental data, and the open ones are the the model calculations. Straight lines represent the slope of data at midrapidity, obtained by fitting the data with first-order polynomial within the intervals of the rapidity  $0.25 < y < 1.70$  and  $0.10 < y < 1.85$ , respectively. The curved lines guide the eye over data.

For the analysis, a minimum of three participant protons,  $N_{\text{part}} \geq 3$ , are required for the reliable determination of the reaction plane. Figures 1 and 2 present the directed flow effects of protons and negative pions in  $^2\text{H} + \text{C}$ ,  $\text{He} + \text{C}$ ,  $^2\text{H} + \text{Ta}$ , and  $\text{He} + \text{Ta}$  collisions. As seen, the flow parameter  $F$  increases with the increase of the mass numbers of projectile  $A_P$  and target  $A_T$  nuclei (Table I). The dependences of  $F$  on  $(A_P \cdot A_T)^{1/2}$  for protons in  $^2\text{H} + \text{C}$ ,  $\text{He} + \text{C}$ ,  $\text{C} + \text{C}$ ,  $\text{C} + \text{Ne}$ ,  $^2\text{H} + \text{Ta}$ ,  $\text{He} + \text{Ta}$ ,  $\text{C} + \text{Cu}$ , and  $\text{C} + \text{Ta}$  collisions are shown in Fig. 3 together with our earlier results [28]. They are in the line with data for heavier colliding systems at lower energies (see Fig. 4 of [20]).

The obtained experimental results from  $^2\text{H} + \text{C}, \text{Ta}$  and  $\text{He} + \text{C}, \text{Ta}$  collisions were compared with the predictions of the ultrarelativistic quantum molecular dynamics model (URQMD) and our earlier experimental results from  $\text{C} + \text{C}$ ,  $\text{C} + \text{Ne}$ ,  $\text{C} + \text{Cu}$ , and  $\text{C} + \text{Ta}$  systems which were compared with the predictions of the quark-gluon string model (QGSM). A detailed description of the URQMD can be found in Ref. [30,31]. The URQMD is a microscopic transport model based on the covariant propagation of all

hadrons on classical trajectories in a combination with stochastic binary scatterings, a color string formation, and resonance decays. The URQMD model is designed as a multipurpose tool for studying a wide variety of heavy ion related effects ranging from multifragmentation and collective flow to particle productions and correlations in the energy range from Schwerionen-Synchrotron, GSI, Germany (SIS) to RHIC. At high densities ( $\rho > 2-3\rho_0$ ) and/or temperatures, one expects a phase transition or even a smooth crossover to quark gluon plasma (QGP). An existence of a coexistence region of plasma (strongly interacting quarks and gluons not confined to well separated hadrons) and hadron gas might lead to observable effects in the flow excitation function and other observables. The correspondently extended URQMD model can be used to study such phase transitions and effects. In the present version of the model (1.3) we consider the potential interactions between nucleons and excitations of the residual nuclei. The last one is needed for a determination of the reaction plane by the participant protons in the model, because some evaporated protons can be registered as participant ones.

TABLE I. The numbers of experimental and URQMD simulated events and multiplicities of protons (prior to the multiplicity cut), values of  $\langle \cos \Phi \rangle$  ( $\Phi$  is the angle between the true and the estimated reaction plane), and the flow parameter  $F$ .

$A_P + A_T$	$^2\text{H} + \text{C}$	$\text{He} + \text{C}$	$^2\text{H} + \text{Ta}$	$\text{He} + \text{Ta}$
$N_{\text{expt}}/N_{\text{prot}}$	4581/9630	9737/33008	1424/6632	1532/8354
$N_{\text{URQMD}}/N_{\text{prot}}$	27502/66510	31716/108871	8710/37200	8919/55518
$\langle \cos \Phi \rangle$	0.850	0.860	0.680	0.690
$F_{\text{expt}} (\text{MeV}/c)$	$87.3 \pm 6.3$	$94.9 \pm 5.2$	$131.3 \pm 6.6$	$138.7 \pm 5.4$
$F_{\text{URQMD}} (\text{MeV}/c)$	$83.5 \pm 2.6$	$89.9 \pm 1.3$	$130.3 \pm 3.9$	$137.2 \pm 2.1$

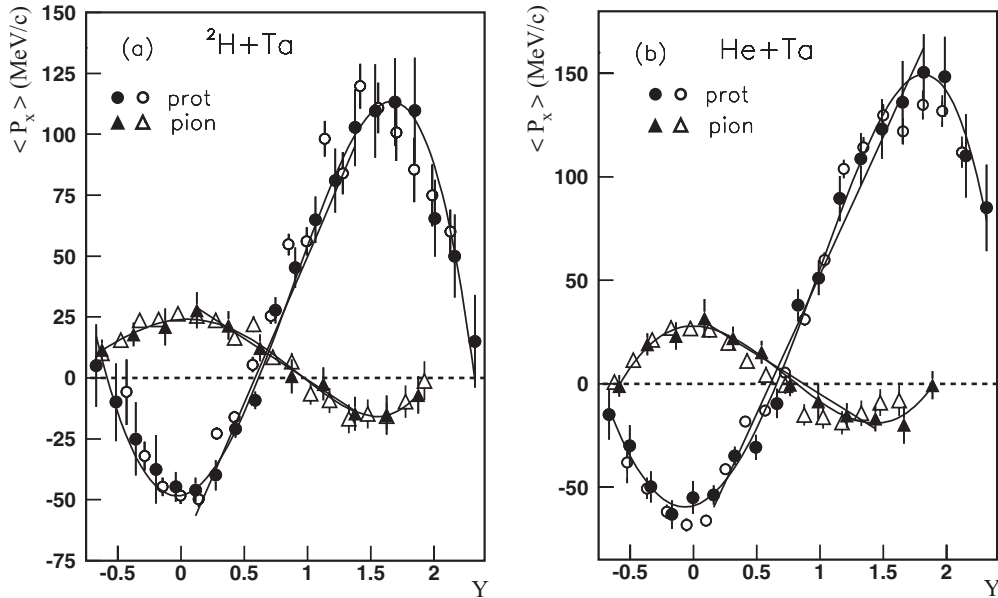


FIG. 2. The dependence of  $\langle P_x(Y) \rangle$  on the laboratory rapidity,  $Y$ , for protons and  $\pi^-$ -mesons in (a)  ${}^2\text{H} + \text{Ta}$  and (b)  $\text{He} + \text{Ta}$  collisions. Closed symbols correspond to experimental data and open ones to model. Straight lines represent the slope of data at midrapidity, obtained by fitting the data with first-order polynomial within the narrow intervals of the rapidity,  $0.07 < y < 1.35$  and  $-0.10 < y < 1.65$ , respectively. The curved lines guide the eye over data.

Fifty thousand events have been generated for  ${}^2\text{H} + \text{C}$  and  $\text{He} + \text{C}$  collisions, and 10 000 events for  ${}^2\text{H} + \text{Ta}$  and  $\text{He} + \text{Ta}$  collisions at an energy of 3.4 GeV/nucleon by using the URQMD model. The experimental selection criteria have been applied to the generated events. Then for further analysis, 27 502, 31 716, 8710, and 8919 events, respectively, have been selected. For URQMD events the projection of the transverse momentum onto the true reaction plane was determined. The values of the flow parameter  $F$  have been extracted for protons

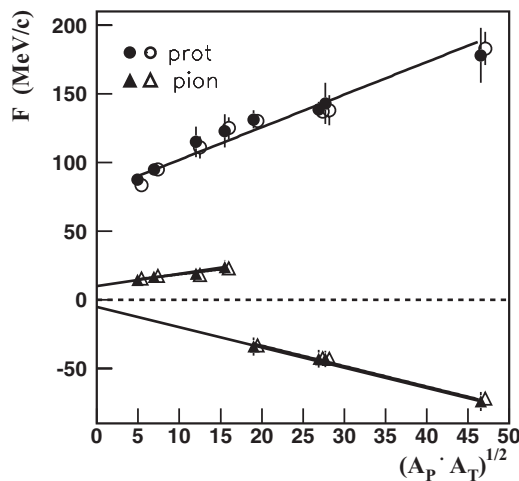


FIG. 3. The dependences of  $F$ , directed flow parameter on  $(A_P \cdot A_T)^{1/2}$  for protons (upper) and  $\pi^-$  mesons (lower) in  ${}^2\text{H} + \text{C}$ ,  $\text{He} + \text{C}$ ,  $\text{C} + \text{C}$ ,  $\text{C} + \text{Ne}$ ,  ${}^2\text{H} + \text{Ta}$ ,  $\text{He} + \text{Ta}$ ,  $\text{C} + \text{Cu}$ , and  $\text{C} + \text{Ta}$  collisions. Closed symbols correspond to experimental data and open ones to the QGSM and URQMD calculations.

from the dependences of  $\langle P_x(y) \rangle$  on the rapidity for each nuclear pair (Table I). As seen, there is quite good agreement in the experimental and theoretical distributions (Figs. 1–3).

#### IV. PROTON ELLIPTIC FLOW

We have investigated the proton elliptic flow in  ${}^2\text{H} + \text{C}$ ,  ${}^2\text{H} + \text{Ta}$ ,  $\text{He} + \text{C}$ , and  $\text{He} + \text{Ta}$  collisions at an energy of 3.4 GeV/nucleon. The azimuthal  $\phi$  distributions of the protons have been obtained and presented in Figs. 4 and 5 where  $\phi$  is the angle of the transverse momentum of each particle in the event with respect to the reaction plane ( $\cos\phi = P_x/P^\perp$ ). Due to low multiplicities of the participant protons in  ${}^2\text{H} + \text{C}$  and  ${}^2\text{H} + \text{Ta}$  interactions, we plotted in Fig. 4 the distributions on  $|\phi|$  in the interval 0–3.14 rad. The azimuthal angular distributions show maxima at  $\phi = 90^\circ$  and  $270^\circ$  with respect to the event plane. The maxima are associated with preferential particle emission perpendicular to the reaction plane (squeeze-out). To treat the data in a quantitative way, the azimuthal distributions have been fitted with the Fourier cosine-expansion (given the system invariance under reflections with respect to the reaction plane)  $dN/d\phi = a_0(1 + a'_1 \cos\phi + a'_2 \cos 2\phi)$ . The squeeze-out signature is the negative value of the coefficient  $a'_2$ , and this coefficient is the measure of the strength of the anisotropic emission. Compared to the coefficient  $a_2$  associated with a distribution relative to the true reaction plane, the coefficient  $a'_2$  is reduced,  $a'_2 = a_2 \langle \cos 2\Phi \rangle$  [28,43–45], where

$$\langle \cos 2\Phi \rangle = \frac{|((P'_X)^2 - (P'_Y)^2)|}{|((P_X)^2 - (P_Y)^2)|}. \quad (4)$$

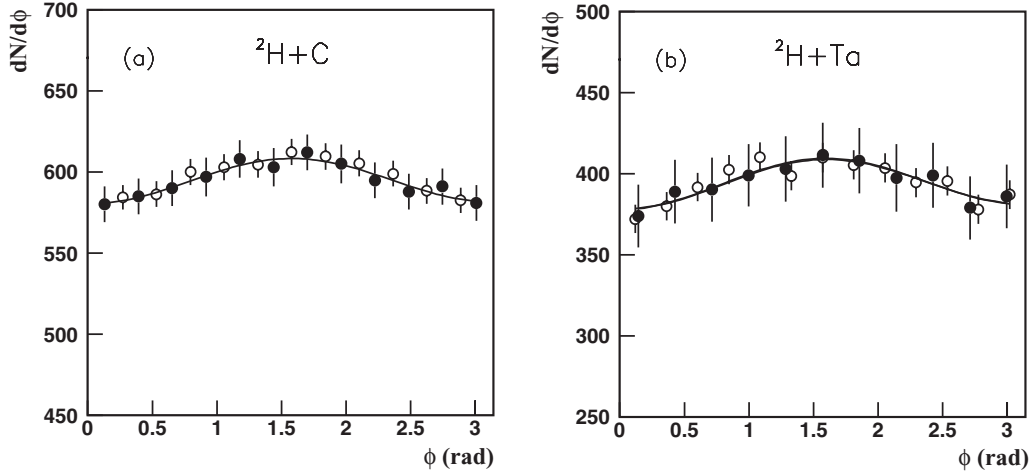


FIG. 4. The azimuthal distributions of the participant protons with respect to the estimated reaction plane in (a)  $^2\text{H} + \text{C}$  and (b)  $^2\text{H} + \text{Ta}$  collisions. Closed symbols correspond to experimental data and open ones to model. The lines represent fits to the equation  $dN/d\phi = a_0(1 + a'_0 \cos \phi + a'_2 \cos 2\phi)$ .

The numerator and denominator on the right-hand side are, respectively, obtained from

$$\langle (P'_X)^2 - (P'_Y)^2 \rangle = \left\langle 2 \left( \frac{\mathbf{P}_j^\perp \cdot \mathbf{Q}_j}{Q_j} \right)^2 - (P_j^\perp)^2 \right\rangle \quad (5)$$

and

$$|\langle (P_X)^2 - (P_Y)^2 \rangle| = \sqrt{\frac{\langle 2\bar{T} : \bar{T} - \sum_{i=1}^n (P_i^\perp)^4 \rangle}{\langle n^2 - n \rangle}}. \quad (6)$$

In the above, the transverse tensor  $\bar{T}$  is

$$T^{\alpha\beta} = \sum_{i=1}^n \left[ P_i^\alpha P_i^\beta - \frac{1}{2} (P_i^\perp)^2 \delta^{\alpha\beta} \right], \quad \alpha = x, y \quad (7)$$

and

$$\bar{T} : \bar{T} = \sum_{\alpha, \beta=x}^y T^{\alpha\beta} T^{\alpha\beta} - (T^{xx})^2 + (T^{yy})^2 + (T^{xy})^2. \quad (8)$$

With Eq. (5), we find that  $\langle \cos 2\Phi \rangle \approx 0.67$  for the protons of the given nuclear systems and the elliptical modulation parameters corrected according to  $\langle \cos 2\Phi \rangle$ , from the fits made under different cuts to analyzed particles, are provided in Table II. The elliptic anisotropy, quantified in terms of the  $a_2$  coefficient ( $a_2 = 2v_2$ ), was extracted from the azimuthal distributions of the protons with respect of the reaction plane at midrapidity. One can see that there is some indication that  $a_2$  increases with transverse momentum and projectile and target nuclei mass numbers for the above mentioned nuclear pairs (Table II). In Fig. 6, the results obtained in this paper are presented together with our earlier results for C + C, C + Ne, C + Cu, and C + Ta collisions [28]. They are comparable.

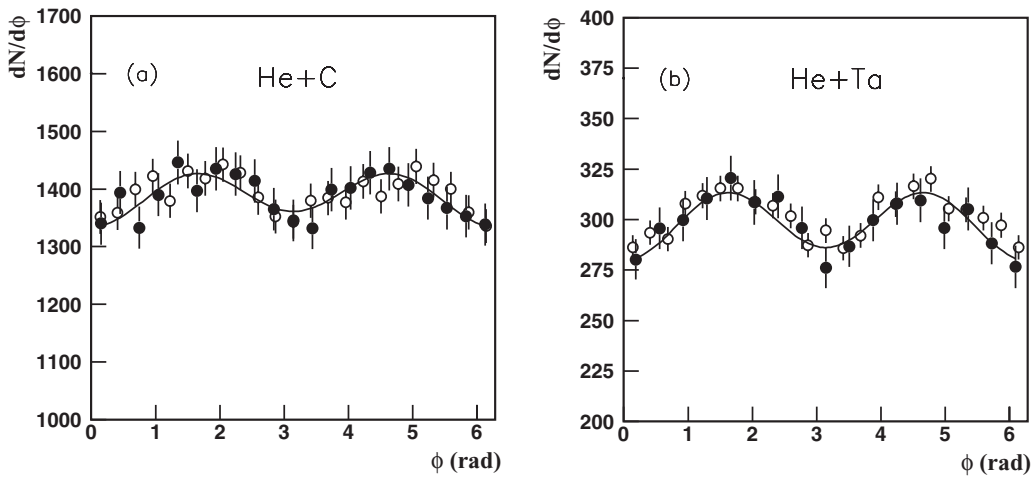


FIG. 5. The azimuthal distributions of the participant protons with respect to the estimated reaction plane in (a) He + C and (b) He + Ta collisions. Closed symbols correspond to experimental data and open ones to model. The lines represent fits to the equation  $dN/d\phi = a_0(1 + a'_0 \cos \phi + a'_2 \cos 2\phi)$ .

TABLE II. Characteristics of proton elliptic flow for experimental and URQMD-simulated collision events.

$A_P + A_T$	${}^2\text{H} + \text{C}$	$\text{He} + \text{C}$	${}^2\text{H} + \text{Ta}$	$\text{He} + \text{Ta}$
	$(-0.3 \leq y \leq 2.1)$	$(-0.3 \leq y \leq 2.3)$	$(-0.2 \leq y \leq 1.8)$	$(-0.2 \leq y \leq 1.8)$
$a_{2\text{expt}}$	$-0.034 \pm 0.006$	$-0.040 \pm 0.006$	$-0.059 \pm 0.009$	$-0.074 \pm 0.012$
$P_T > 200 \text{ MeV}/c$	$-0.041 \pm 0.008$	$-0.049 \pm 0.008$	$-0.076 \pm 0.010$	$-0.089 \pm 0.013$
$P_T > 300 \text{ MeV}/c$	$-0.048 \pm 0.009$	$-0.058 \pm 0.009$	$-0.093 \pm 0.011$	$-0.109 \pm 0.014$
$a_{2\text{mod}}$	$-0.037 \pm 0.006$	$-0.039 \pm 0.006$	$-0.057 \pm 0.007$	$-0.071 \pm 0.006$
$P_T > 200 \text{ MeV}/c$	$-0.043 \pm 0.007$	$-0.050 \pm 0.007$	$-0.075 \pm 0.008$	$-0.088 \pm 0.007$
$P_T > 300 \text{ MeV}/c$	$-0.051 \pm 0.008$	$-0.060 \pm 0.008$	$-0.091 \pm 0.009$	$-0.107 \pm 0.008$

The obtained experimental results have been compared to the predictions of the URQMD model. The experimental selection criteria have been applied to the generated events. The elliptic flow parameters with respect of the true reaction plane have been calculated (Table II) for URQMD events also. Quite good agreement between experimental and theoretical distributions has been obtained for proton elliptic flow in the above mentioned collisions (Figs. 5 and 6).

The elliptic flow has been investigated by various experimental groups for different systems. The elliptic flow measurements of charged hadrons in  $\text{Cu} + \text{Cu}$  and  $\text{Au} + \text{Au}$  collisions at  $\sqrt{S_{NN}} = 62.4$  and  $200 \text{ GeV}$  (the PHOBOS Collaboration) do not show any dependence on  $(A_P \cdot A_T)^{1/2}$  (see Ref. [46], Figs. 2(a) and 2(c)). The ALICE group has found about a 30% increase in the magnitude of  $v_2$  at going from  $\sqrt{S_{NN}} = 200 \text{ GeV}$  ( $\text{Au} + \text{Au}$ ) to  $2.76 \text{ TeV}$  ( $\text{Pb} + \text{Pb}$ ) (see Ref. [47], Fig. 4).

## V. DIRECTED FLOW OF $\pi^-$ MESONS

We have investigated the directed flow of the  $\pi^-$  mesons for our nuclear systems. Negative pions with momentum  $p >$

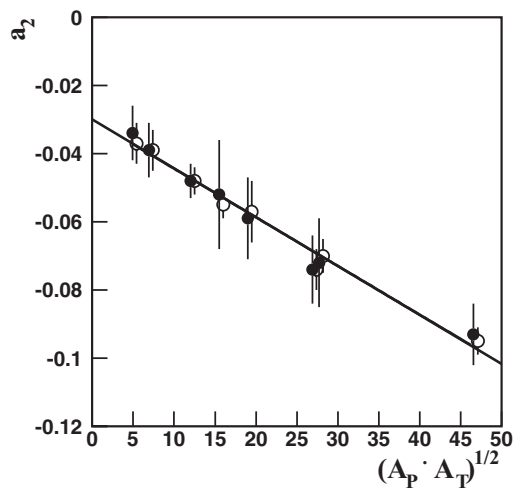


FIG. 6. The dependences of  $a_2$ -elliptic flow parameter on  $(A_P \cdot A_T)^{1/2}$  for protons in  ${}^2\text{H} + \text{C}$ ,  $\text{He} + \text{C}$ ,  $\text{C} + \text{C}$ ,  $\text{C} + \text{Ne}$ ,  ${}^2\text{H} + \text{Ta}$ ,  $\text{He} + \text{Ta}$ ,  $\text{C} + \text{Cu}$ , and  $\text{C} + \text{Ta}$  collisions. Closed symbols correspond to experimental data and open ones to the QGSM and URQMD calculations.

$50 \text{ MeV}/c$  have been detected within the chamber. There are no autocorrelations for pions, because the reaction plane is determined by protons. The average components of a pion transverse momentum in the reaction plane evaluated using Eq. (3) are presented in Figs. 1 and 2, and the values of flow parameter  $F$  at crossover from the fits to data are given in Table III.

The dependences of  $F$ , the directed flow parameter (absolute values), on  $(A_P \cdot A_T)^{1/2}$  for  $\pi^-$  mesons in  ${}^2\text{H} + \text{C}$ ,  $\text{He} + \text{C}$ ,  $\text{C} + \text{C}$ ,  $\text{C} + \text{Ne}$ ,  ${}^2\text{H} + \text{Ta}$ ,  $\text{He} + \text{Ta}$ ,  $\text{C} + \text{Cu}$ , and  $\text{C} + \text{Ta}$  collisions are plotted in Fig. 3. The results obtained in this paper are presented together with our earlier results [28] for pions and protons. As seen, the flow parameter  $F$  increases with the increase of the mass numbers of  $A_P$  and  $A_T$  nuclei (Table III.)

One can see from Table III that in  ${}^2\text{H} + \text{C}$  and  $\text{He} + \text{C}$  collisions the direction of pion flow is the same as the proton flow, while in  ${}^2\text{H} + \text{Ta}$  and  $\text{He} + \text{Ta}$  interactions the direction of the pion flow is opposite to the proton flow (antiflow). The FOPI Collaboration (see [48], Fig. 29) observed that positive charged pion flow shows antiflow only in the peripheral  $\text{Au} + \text{Au}$  interactions at  $1.5 \text{ A GeV}$ . The direction of negative charged pion flow coincides with the proton flow in the same interactions at all centralities. At higher energy ( $40 \text{ A GeV}$  and  $158 \text{ A GeV}$ ), the NA49 Collaboration found only antiflow of pions at all centralities [49]. According to our data, there is a possibility to study a transition from the flow to the antiflow in asymmetric nuclear collisions.

The anticorrelation of nucleons and pions was explained in Ref. [50] as due to an effect of multiple  $\pi N$  scattering. However, in Refs. [51–53] it was shown that the anticorrelation is a manifestation of the nuclear shadowing of the target and projectile spectators through both pion rescattering and reabsorptions. Quantitatively, the shadowing can produce

TABLE III. The number of experimental and URQMD simulated  $\pi^-$  mesons and the values of the flow parameter  $F$ .

$A_P + A_T$	${}^2\text{H} + \text{C}$	$\text{He} + \text{C}$	${}^2\text{H} + \text{Ta}$	$\text{He} + \text{Ta}$
$N_{\pi\text{expt}}$	3452	8776	1137	2200
$N_{\pi\text{mod}}$	23210	37786	11781	9770
$F_{\text{expt}} (\text{MeV}/c)$	$14.4 \pm 3.6$	$16.9 \pm 2.3$	$-34.0 \pm 6.5$	$-42.9 \pm 6.3$
$F_{\text{mod}} (\text{MeV}/c)$	$15.4 \pm 1.5$	$17.3 \pm 1.5$	$-33.2 \pm 3.1$	$-42.5 \pm 3.3$

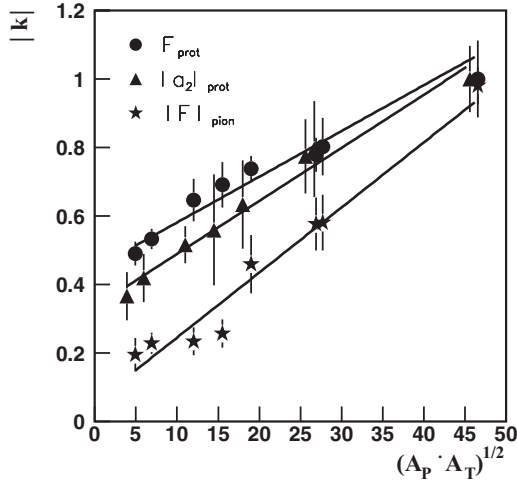


FIG. 7. The dependences of the normalized (divided on the maxima of those values) directed and elliptic flow parameters on  $(A_P \cdot A_T)^{1/2}$  for protons and  $\pi^-$  mesons in experimental  ${}^2\text{H} + \text{C}$ ,  $\text{He} + \text{C}$ ,  $\text{C} + \text{C}$ ,  $\text{C} + \text{Ne}$ ,  ${}^2\text{H} + \text{Ta}$ ,  $\text{He} + \text{Ta}$ ,  $\text{C} + \text{Cu}$ , and  $\text{C} + \text{Ta}$  collisions. The lines represent linear fits to the data.

in-plane transverse momentum components comparable to the momenta themselves and, thus, much larger than components due to collective motion for pions [54]. In our opinion, our results indicate that the flow behavior of  $\pi^-$  mesons in the light systems is due to the flow of  $\Delta$  resonances, whereas the antflow behavior in the heavier systems ( ${}^2\text{H} + \text{Ta}$  and  $\text{He} + \text{Ta}$ ) is the result of the nuclear shadowing effect. There the flow parameter  $F$ , the measure of the amount of collective transverse momentum transfer in the reaction plane, increases with the increase of projectile and target nuclei mass numbers (Table III). Our results agree with the results for different projectile and target nuclei configurations obtained at different energies and accelerators (GSI-SIS, AGS).

As shown in Refs. [55,56], the ratios of multiplicities of  $\pi^+$  and  $\pi^-$  meson coming from the decay of the  $\Delta^{++}$  and  $\Delta^0$  resonances to the multiplicities of the directly produced are about 62% and 48%, respectively, in the considered energy range and colliding nuclei, at least for  $\text{He} + \text{C}$  and  $\text{C} + \text{C}$  interactions at  $4.2A$  GeV/c. It means that  $\Delta$  decay is the dominant mechanism of pion production on the light targets. The shadowing can have influence on the directly produced mesons and  $\Delta$ 's, especially, in the collisions with heavy targets. The shadowing must be small in the interactions with light targets, which explains our observed flow of pions on the carbon target.

The obtained experimental results have been compared with the predictions of the URQMD model. The dependence of the projection of  $\pi^-$  mesons' transverse momentum onto the reaction plane (determined by the participant protons) on the rapidity  $y$  was calculated. There is quite good agreement in the experimental and theoretical distributions (Figs. 1–3).

In Fig. 7, the dependences of  $F$ -directed flow parameter (for protons and  $\pi^-$  mesons), and of  $a_2$ -elliptic flow parameter (for protons), on the  $(A_P \cdot A_T)^{1/2}$  in the above mentioned collisions are presented. In order to show both dependences in

the same figure, we have normalized the parameters on their maximum values,  $k = F/F_{\text{max}}$ ,  $k = a_2/a_{2\text{max}}$ . The values of  $F_{\text{max}}$  and  $a_{2\text{max}}$  for  $\text{C} + \text{Ta}$  interactions were taken from [26]. Because  $k = 1$  for  $\text{C} + \text{Ta}$  collisions, the results were shifted slightly on the figure to guide the eye. One can see from Fig. 7 that in  ${}^2\text{H} + \text{C}$ ,  $\text{He} + \text{C}$ ,  $\text{C} + \text{C}$ ,  $\text{C} + \text{Ne}$ ,  ${}^2\text{H} + \text{Ta}$ ,  $\text{He} + \text{Ta}$ ,  $\text{C} + \text{Cu}$ , and  $\text{C} + \text{Ta}$  collisions the linear dependences of the directed and elliptic flow parameters from system mass are similar for protons while for  $\pi^-$  mesons the dependence of the directed flow parameters is a little bit stronger.

## VI. CONCLUSIONS

The directed transverse collective flows of protons and  $\pi^-$  mesons and elliptic flow of protons emitted from  ${}^2\text{H} + \text{C}$ ,  $\text{He} + \text{C}$ ,  ${}^2\text{H} + \text{Ta}$ , and  $\text{He} + \text{Ta}$  reactions at energy 3.4 GeV/nucleon have been studied. We can summarize our results as follows:

- (i) The  ${}^2\text{H} + \text{C}$  system is the lightest studied one, and the  ${}^2\text{H} + \text{Ta}$  and  $\text{He} + \text{Ta}$  systems are fully (extremely) asymmetrical systems in which collective flow effects (directed and elliptic) have been ever detected (for protons and  $\pi^-$  mesons). As shown, the negative pions exhibit the directed flow consistent with that for protons in the  ${}^2\text{H} + \text{C}$  and  $\text{He} + \text{C}$  collisions. On the other hand, for the  ${}^2\text{H} + \text{Ta}$  and  $\text{He} + \text{Ta}$  interactions, the pion flows turn into antflow with the pion average in-plane momenta becoming opposite to those for protons. The directed flow parameter  $F$  increases with the increase of the mass of projectile and target nuclei for both protons and negative pions. For protons, the increase is from  $87.3 \pm 6.3$  ( ${}^2\text{H} + \text{C}$ ) to  $138.7 \pm 5.4$  ( $\text{He} + \text{Ta}$ ) (MeV/c) (whereas for negative pions the increase is from  $14.4 \pm 3.6$  MeV/c for  ${}^2\text{H} + \text{C}$  interactions to  $48.7 \pm 7.2$  MeV/c for  $\text{He} + \text{Ta}$  ones);
- (ii) The proton elliptic flow parameter  $a_2$  increases with the increase of the mass numbers of projectile  $A_P$  and target  $A_T$  nuclei from  $0.034 \pm 0.006$  for  ${}^2\text{H} + \text{C}$  interactions to  $0.074 \pm 0.012$  for  $\text{He} + \text{Ta}$  ones.
- (iii) The flow measurements have been described by the ultrarelativistic quantum molecular dynamics model (URQMD). There is quite good agreement between the experimental and the theoretical distributions.
- (iv) In  ${}^2\text{H} + \text{C}$ ,  $\text{He} + \text{C}$ ,  $\text{C} + \text{C}$ ,  $\text{C} + \text{Ne}$ ,  ${}^2\text{H} + \text{Ta}$ ,  $\text{He} + \text{Ta}$ ,  $\text{C} + \text{Cu}$ , and  $\text{C} + \text{Ta}$  collisions, the linear dependence of directed and elliptic flow parameters from mass numbers of projectile and target nuclei,  $(A_P \cdot A_T)^{1/2}$ , is similar for protons while for  $\pi^-$  mesons the dependence of directed flow parameters is stronger.

## ACKNOWLEDGMENTS

One of us (L.Ch.) would like to thank the board of directors of the Laboratory of Information Technologies of JINR for the warm hospitality. This work was partially supported by the Georgian Shota Rustaveli National Science Foundation under Grant No. GNSF/ST08/4-418.

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