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REDUCTION OF THE APPARENT IMPEDANCE OF WIDE BAND ACCELERATING CAVITIES BY RF FEEDBACK

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1. Introduction

In the CERN SPS proton synchrotron, the four accelerating cavities are of the travelling wave structure type[1]. The major technological advantages of this choice are on the power side; the power amplifiers, which always see a matched load, are located in a surface building where they can be easily maintained and expanded if necessary[2]. Moreover, the bandwidth of the travelling wave structures (±0.7 MHz at -3 dB) whose centre frequency corresponds to the transition crossing frequency (200.22 MHz), is large enough to avoid any tuning during the acceleration cycle.

The filling time of the cavity (700 ns for a five section cavity) is only a small fraction of one machine revolution (T = 23 μs). Therefore, contrary to the situation with high Q cavities, beam loading appears as a transient effect which results in different stable phase angles for different bunches[3]. Whenever the bunches are not sitting at their proper phase angle, for instance at injection or at transition, bunch oscillations will be excited.

At the 10 GeV/c injection energy, (f_{RF} = 199.5 MHz), the cavities present to the beam an impedance very similar to that of a detuned RLC cavity, with the result that strong dipole and quadrupole instabilities occur[3].

The accelerated beam current was limited to 2.5×10^{13} protons by these two effects mixed together, which resulted in capture losses increasing with the injected beam intensity. Amongst the various solutions envisaged to cure this problem[4], the RF feedback, which would alleviate both transient beam loading and instability effects, looked most promising.

2. Representation of a travelling wave cavity

A particle crossing a travelling wave cavity sees a total RF voltage V_t, which is the sum of the component produced by the RF power generator, and the beam-induced component[1]. Using the same notations as for RLC type cavities we can write:

$$V_t = Z_1 I_g + Z_2 I_b \qquad (1)$$

where I_g is a quantity proportional to the RF drive, I_b is the RF component of beam current, and Z_1, Z_2 are impedances pertaining to the cavity. In the case of a standing wave cavity, one has $Z_1 = Z_2$, which is the usual resonator impedance. For a travelling wave structure, which looks like a matched transmission line from the power generator, Z_1 is purely real, but falls off away from synchronism, due to the transit time factor. Z_2, on the contrary, is complex and rather similar to the impedance of an RLC cavity (note that at resonance $Z_2 = Z_1/4$):

$$Z_1 = R \; \frac{\sin \tau/2}{\tau/2} \qquad (2)$$

$$Z_2 = \frac{R_0}{4}\left[\left(\frac{\sin \tau/2}{\tau/2}\right)^2 - 2j\,\frac{\tau - \sin\tau}{\tau^2}\right] \qquad (3)$$

where τ is the transit time angle $\ell\, \delta\omega / v_g$ (ℓ = cavity length, $\delta\omega$ frequency deviation from the centre frequency of the cavity, v_g group velocity). For a five-section cavity in the SPS, $\tau = \pi$ at the injection frequency (199.5 MHz) and R_0 = 5.6 MΩ.

As Z_1 vanishes for $\tau = \pm 2\, k\pi$, but Z_2 does not, the beam loading effect cannot be corrected everywhere by acting through the RF power amplifier, and we can only hope to reduce its effect in the range $-2\pi < \tau < +2\pi$. Transient beam loading effect is given by the value of Z_2 at the frequencies $f_{RF} \pm nf_{rev}$, whereas the threshold and growth rate for instabilities depend on the value of Z_2 at the frequencies $f_{RF} \pm nf_{rev} \pm mf_s$, ($f_{rev} = 1/T$, f_s = synchrotron frequency, m,n integers).

In a standing wave cavity, the total voltage V_t can be monitored by a single probe in the resonator itself. In the case of a travelling wave structure, it is possible to synthesize a signal proportional to V_t by using one probe per cell, and combining all the probe signals, taking into account the time of flight of the beam from cell to cell[4]. All four SPS cavities are equipped with such networks, which can eventually be grouped together to generate a signal proportional to the total RF voltage seen by the beam.

3. The RF feedback

Figure 1 shows the principle of the RF feedback. The total voltage V_t seen by the beam is reinjected into the feedback cavity via its power amplifier. Without any delay in the system, the open loop feedback gain would be GZ_1 and the cavity impedance seen by the beam $Z_2' = Z_2/(1 + GZ_1)$ could be much smaller than Z_2.

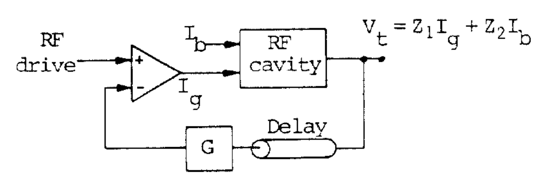

Fig. 1 Principle of the RF feedback

Unfortunately, the long delay in the system (2.3 μs) will severely limit the bandwidth and the RF feedback could hardly correct more than the n = 0 beam loading component. In order to circumvent the problem, we observe that we need a large gain G, only in the vicinity of the frequencies $f_{RF} \pm nf_{rev}$. Outside these bands the phase rotation due to the long delay is unimportant if the gain is made low enough. If, in addition, the total delay of the system is made exactly equal to one machine turn, the open loop phase of the feedback system is always zero for each frequency $f_{RF} \pm nf_{rev}$. The two ingredients needed to make the RF feedback work for n not too small are therefore: a transfer function with comb filter shape and a total delay of one machine turn.

The transfer function of the comb filter $H(j\omega)$ can be written:

$$H(j\omega) = \frac{G_0}{1 - K\,\exp(-j\,\Delta\omega\, T)} \qquad (4)$$

where $\Delta\omega = \omega - \omega_{RF}$, G_0 and K (0 < K < 1) are constants. $H(j\omega)$ can be realised using various technological solutions (i.e. digital filter, N path filter). The total open loop transfer function GZ_1, including the one turn delay T is now:

$$GZ_1 = \frac{G_0 Z_1}{1 - K\exp(-j\Delta\omega T)}\,\exp(-j\Delta\omega T) = \frac{G_0 Z_1}{\exp(j\Delta\omega T) - K} \qquad (5)$$

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The apparent impedance of the cavity Z_2' becomes:

$$Z_2' = Z_2 \cdot \frac{\exp j\Delta\omega T - K}{\exp j\Delta\omega T - K + G_0Z_1} = Z_2 \alpha\, e^{j\theta} \quad (6)$$

In the simplest case where G_0 is real and constant, the function Z_2'/Z_2 is represented in the complex plane by a circle centred on the real axis. Z_2'/Z_2 is real for the frequencies $f_{RF} \pm kf_{rev}$ and $f_{RF} \pm (k + \frac{1}{2})f_{rev}$.

$$\frac{Z_2'}{Z_2} = \frac{1 - K}{1 - K + G_0Z_1} \quad \text{for} \quad \Delta\omega = 2\pi k\, f_{rev} \quad (7)$$

$$\frac{Z_2'}{Z_2} = \frac{1 + K}{1 + K - G_0Z_1} \quad \text{for} \quad \Delta\omega = 2\pi(k + \tfrac{1}{2})f_{rev} \quad (8)$$

The system remains stable if $G_0Z_1 < 1 + K$. The minimum apparent impedance Z_2' which can be obtained is therefore:

$$Z_2'_{min} = Z_2 \frac{\exp j\Delta\omega T - K}{\exp j\Delta\omega T + 1} \quad (9)$$

K must be close to unity to achieve a low impedance at $\Delta\omega = 2\pi\, nf_{rev}$ in order to reduce the transient beam loading effect. However, the filling time of the filter increases when K approaches unity, and consequently the transient response of the RF feedback is slowed down. On the other hand, to minimize the absolute value of the impedance at the synchrotron sidebands, one should have $K = \cos(2\pi\, mf_s/f_{rev})$. As will be seen later, K is rather determined by practical considerations leading to simplification of the hardware.

Remark The maximum open loop gain G_0Z_1 is limited to the value 1 + K because, at the frequencies $f_{RF} + (k + \frac{1}{2})f_{rev}$, the gain has changed sign due to the one turn overall delay of the system. In order to avoid this delay, one could imagine a filter synthesized by an array of individual resonators, each channel having its own phase shifter adjusted to provide the correct phase at the nf_{rev} frequencies. It can be shown[4], that although no delay is physically implemented in the hardware, the overall transfer function also exhibits an opposite sign at the frequencies $f_{RF} + (k + \frac{1}{2})f_{rev}$. This leads to the same gain limitation, which therefore appears to be a general one for a multiband feedback having a non-zero delay.

4. Effect on the instability

The bunch to bunch instability threshold for mode m,n is determined[5] by the synchrotron frequency shift which is, for a relatively narrow band impedance, proportional to the quantity:

$$R = Z_2(f_{RF} + nf_{rev} + mf_s) - Z_2^*(f_{RF} - nf_{rev} - mf_s) \quad (10)$$

With the RF feedback Z_2 becomes $Z_2' = \alpha_+ e^{j\theta_+} Z_2$ for $f = f_{RF} + nf_{rev} + mf_s$ and $Z_2' = \alpha_- e^{j\theta_-} Z_2$ for $f = f_{RF} - nf_{rev} - mf_s$.

In the simple case G_0Z_1 = constant one has $\alpha_+ = \alpha_-$, $\theta_- = -\theta_+$ and consequently $R' = \alpha\, e^{j\theta}R$.

The synchrotron shift curve is therefore simply reduced in size by the factor α ($\alpha < 1$) and rotated around the origin of the complex plane.

This is only true for modes with n small where the frequencies of interest $f \simeq f_{RF} \pm nf_{rev}$ are close enough to satisfy the condition G_0Z_1 = constant. In reality, Z_1 is frequency dependent because of the transit time factor, and G_0 (which includes the RF power amplifier gain) is also band limited. Figure 2 shows the curves R'(n) for the dipole (m = ±1) and quadrupole mode (m = ±2) at injection in the SPS. The transfer function of the power amplifier is modelled by a simple tuned circuit with a ±2 MHz bandwidth, and

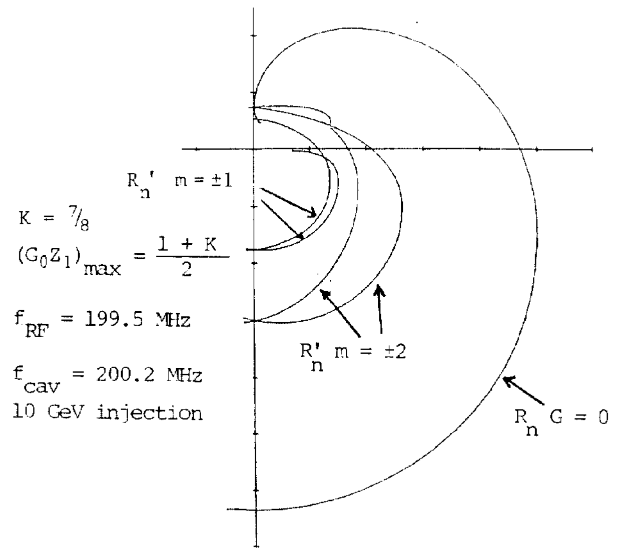

Fig. 2 Synchrotron frequency shift with and without RF feedback

the gain of the loop is half of its maximum value in order to stay sufficiently far from the instability limit. The effect of the RF feedback is to reduce the size of the curve, and therefore to increase the beam intensity threshold, by a factor ∿ 3 for the dipole mode and ∿ 2 for the quadrupole mode.

5. Implementation of the system (Fig. 3)

The V_t signal corresponding to the four accelerating cavities is translated down in frequency by coherent mixing with f_{RF} using two quadrature mixers. Two high pass networks following the input mixers remove the n = 0, m = 0,1,2.. components of the signal, which are taken care of by the usual amplitude and phase loops of the cavities. The comb filtering and delay functions are provided by the two identical digital filters described later.

After filtering, an RF signal is reconstructed by up mixing with the RF frequency; it is then added to the normal RF drive signal to feed the power amplifier of the feedback cavity.

Careful design of the input and output mixer circuitry provides a very good rejection (> 35 dB) of the unwanted sidebands. Rejection of the carrier is relatively unimportant, as it is automatically corrected by the cavity amplitude and phase loops.

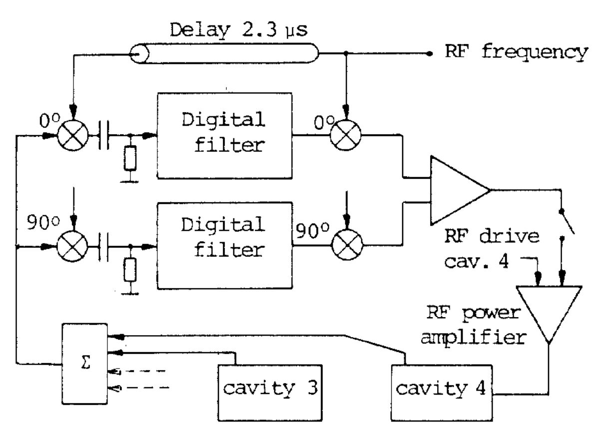

Fig. 3 Layout of the RF feedback system

The relative phasing of the input and output mixers, obtained in our case by a 2.3 μs delay, is such that a constant open loop RF phase can be maintained even when f_{RF} varies during acceleration. Stability of the open loop gain (amplitude and phase) is not very critical for a feedback system, contrary to a compensation scheme[4], and as a result no frequent adjustments of gain and RF phase are needed.

6. The digital filter

The heart of the system is the digital filter, schematically displayed in Fig. 4. It is derived from a single pole low pass recursive filter by using a memory with N locations instead of one. At every clock pulse, the contents of the memory are shifted by one location. If the sampling frequency is made equal to Nf_{rev}, it can be shown[6] that the transfer function of the filter is given by equation (4).

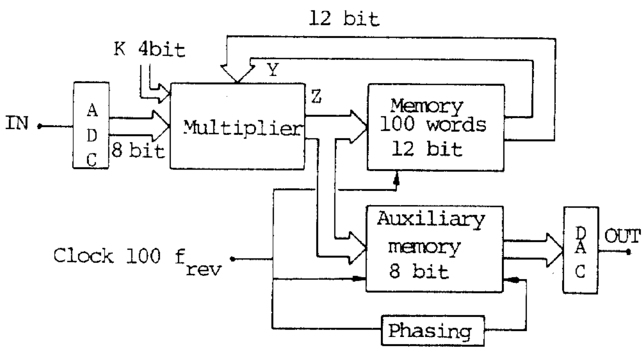

Fig. 4 The digital filter and delay

The overall bandwidth of the electronics, ultimately limited to $N\,f_{rev}/2$ (Nyquist frequency) must be larger than the cavity bandwidth. Fig. 5 shows the overall transfer function of the digital filter including the input and output anti-aliasing filters (designed to have a linear phase response). N is equal to 100, which gives a cycle time ($1/Nf_{rev}$) of 230 ns.

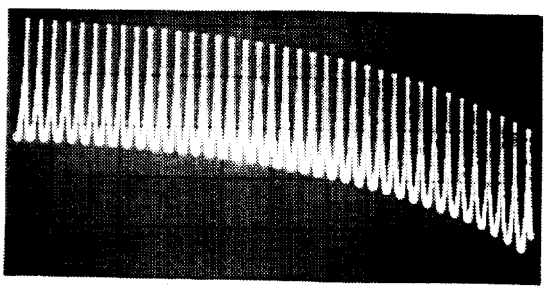

Fig. 5 Digital filter amplitude response 10 dB/div

Implementation of the one-turn delay is straightforward in digital technology: a second memory is inserted between the comb filter itself and the output digital to analog converter. Additional circuitry is necessary to control the precise timing of the memories from the revolution frequency clock.

The speed of the various elements, limited by the cycle time, is critical. For A-D conversion one should employ flash converters and fast parallel multipliers in the filter loop. To simplify the multiplier array, K is defined as a four-bit word only (K = 7/8). The memories used are of the "First IN. First OUT" type.

With the number of bits selected (Fig. 4) the effect of quantization errors, usually amplified in a recursive filter, is negligible (< -30 dB). This has been checked beforehand by running a computer simulation program.

7. Results with beam

The RF feedback system has been used on the injection flat bottom, and around transition where the transient beam loading effect is most important. The input signals of the two digital filters represent the beam loading components on the RF cavities. The effect of the RF feedback is clearly demonstrated in Fig. 6 (first batch injected). The response time of the feedback (which depends upon the value of K) is only a few turns and small enough compared to one synchrotron period.

The dipole mode instabilities can be monitored using the set-up described in reference (3). They disappear completely with RF feedback ON. The overall result of the system is that the capture losses remain as low as a few percent even in the case of triple batch injection. Moreover, the suppression of the beam loading effects greatly eases the setting-up procedure of the entire RF low-level system, which behaves in a

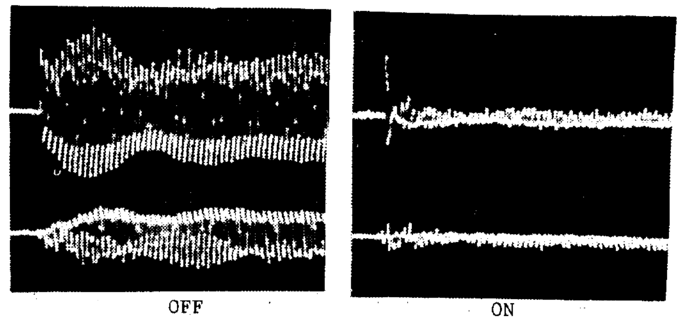

Fig. 6 Input signals of digital filters (injection)

much more understandable way. To go much beyond 3×10^{13} particles accelerated, it might be necessary to share the reinjected RF power among several cavities.

8. Conclusion

Travelling wave cavities, with their inherent advantages for the RF power designer, can be made attractive also for the machine physicist. Their undesirable effects (transient beam loading and instabilities) can now be avoided using modern electronic technology.

The use of real time digital filtering techniques in feedback applications for accelerators has been demonstrated in this particular example. It looks a very promising technology and other applications are already envisaged. For instance, a multiband reject filter having an overall bandwidth of ∿ 40 MHz is being constructed for damping transverse instabilities in the SPS.

Acknowledgements

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