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E R R A T U M

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HIGHER DIMENSIONAL RENORMALIZATION GROUP INVARIANT VACUUM CONDENSATES IN QUANTUM CHROMODYNAMICS

by S. Narison and R. Tarrach

On p. 1, the last two lines should be replaced by:

of the factorization hypothesis[1] with the renormalization group. We work with one flavour and at the one loop level; a generalization of our results to several flavours is straightforward.

On p. 2, Eq. (5c) should read

$$\langle \psi A^\mu_a O_4 \bar{\psi} \rangle = Z_{4,3} \langle \psi A^\mu_a O_{3B} \bar{\psi} \rangle^{(0)} + Z_{4,4} Z_F \langle \psi A^\mu_a O_{4B} \bar{\psi} \rangle^{(2)} + Z_{4,5} \langle \psi A^\mu_a O_{5B} \bar{\psi} \rangle^{(0)},$$

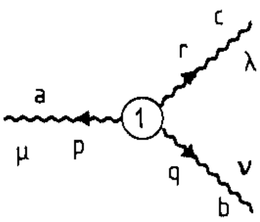
On p. 4, the end of the 6th line should read:

... the quark mass m equal to zero,

On p. 6, the 1st line after Eq. (13f) should start with:

where $Z_A = Z_\alpha^{-1}$ ...

In Fig. 3, the first Feynman rule should read:



$$-\frac{3}{2}\, ig\, f_{abc} \Big[p^\lambda q^\mu r^\nu - p^\nu q^\lambda r^\mu + qr\left(p^\nu g^{\mu\lambda} - p^\lambda g^{\mu\nu}\right) + pq\left(r^\mu g^{\mu\lambda} - r^\nu g^{\mu\lambda}\right) + pr\left(q^\lambda g^{\mu\nu} - q^\mu g^{\nu\lambda}\right)\Big]$$

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HIGHER DIMENSIONAL RENORMALIZATION GROUP INVARIANT VACUUM CONDENSATES IN QUANTUM CHROMODYNAMICS

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A B S T R A C T

The renormalization of the five- and six-dimensional scalar gauge invariant composite operators is performed. The corresponding renormalization group invariant vacuum condensates are studied. Our main results are that the six-dimensional gluon condensate does not mix with the quartic quark condensates nor the other way round. We also show that the factorization hypothesis of the four-quark operator does not lead to a renormalization group invariant quark vacuum condensates. So, one has to be careful when keeping the six-dimensional operators in the QCD sum rule analysis.

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The vacuum condensates are the main input in the quantum chromodynamics (QCD) sum rule approach[1)] for the understanding of low energy hadron physics. The vacuum condensates of lowest dimensions dominate the sum rules at intermediate values of q^2 ($\sim$ 1 to 2 GeV^2). They are, up to dimension six:

$$\langle \bar{\psi}\psi \rangle , \tag{1a}$$

$$\langle G_a^{\mu\nu} G_{\mu\nu}^a \rangle , \tag{1b}$$

$$\langle \bar{\psi} \sigma_{\mu\nu} \frac{\lambda_a}{2} G_a^{\mu\nu} \psi \rangle , \tag{1c}$$

$$\langle f_{abc} G_a^{\mu\nu} G_{\nu\rho,b} G_{\mu,c}^{\rho} \rangle , \tag{1d}$$

$$\langle \bar{\psi} \Gamma_1 \psi \bar{\psi} \Gamma_2 \psi \rangle , \tag{1f}$$

where ψ is the quark field, $G_a^{\mu\nu} \equiv \partial^\mu B_a^\nu - \partial^\nu B_a^\mu + g f_{abc} B_b^\mu B_c^\nu$ is the gluonic field strength. B_a^ν is the gluon field, $\sigma^{\mu\nu} \equiv \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$, Γ_i is any combination of Dirac matrices, λ^a is the $SU(3)_c$ Gell-Mann matrices and f_{abc} the structure constants of the $SU(3)_c$ algebra. These vacuum condensates, once renormalized, depend on the renormalization point μ and are, therefore, not a convenient way of characterizing the vacuum. Instead, one can build with them, renormalization group invariant quantities[2)], once a choice of a renormalization scheme prescription has been done[3)], which are μ-independent and lead thus to a more meaningful description of the physical vauum. For the two lowest dimensional vacuum condensates, these expressions are well known[4,5)]:

$$\phi_1 = m \langle \bar{\psi}\psi \rangle , \tag{2a}$$

$$\phi_2 = m \gamma \langle \bar{\psi}\psi \rangle + \frac{\beta}{4} \langle G_a^{\mu\nu} G_{\mu\nu}^a \rangle , \tag{2b}$$

where we consider only one flavour; γ and β are respectively the quark mass anomalous dimension and the Callan-Symanzik β function. The quantities ϕ_1 and ϕ_2 given in Eq. (2) are known to be μ-independent to all orders in perturbation theory. For the five- and six-dimensional vacuum condensates, the equivalent expressions to Eq. (2) are not known, even to lowest order. The aim of this note is to present a complete study of the five-dimensional vacuum condensate and a partial study of the six-dimensional ones, analysing critically the compatibility of the factorization hypothesis[1)] with one flavour and at the one loop level; a generalization of our results to several flavours is straightforward.

1. RENORMALIZATION OF THE FIVE-DIMENSIONAL VACUUM CONDENSATES

There are five independent gauge invariant scalar operators which, in principle, mix under renormalization. They are:

$$O_1 = i m^2 \bar{\psi}\psi \;, \tag{3a}$$

$$O_2 = -\frac{i}{4} m\, G_a^{\mu\nu} G_{\mu\nu}^a \;, \tag{3b}$$

$$O_3 = -m \bar{\psi}(\not{D} + im)\psi \;, \tag{3c}$$

$$O_4 = ig \bar{\psi} \sigma^{\mu\nu} \frac{\lambda_a}{2} G_{\mu\nu}^a \psi \;, \tag{3d}$$

$$O_5 = i \bar{\psi}(\not{D} + im)(\not{D} + im)\psi \;, \tag{3e}$$

where $D_\mu \equiv \partial_\mu - ig\,(\lambda_a/2)\, B_\mu^a$ is the covariant derivative. O_3 and O_5 vanish if one uses the classical equations of motion. We are interested in the renormalization of O_4 using the background field method[6] which has been very convenient for the renormalization of the four-dimensional operators in Eq. (2) to two-loops[5]. The relation between the renormalized operator O_4 to the bare composite operators O_{jB}^0, written in terms of bare fields and parameters is introduced via the renormalization constants $Z_{4,j}$ as:

$$O_4 = Z_{4,j}\, O_{jB}^0 \;. \tag{4}$$

The study of the following three zero-momentum insertion Green's functions allows the determination of the $Z_{4,j}$:

$$\langle \psi O_4 \bar{\psi} \rangle = Z_{4,1} \langle \psi O_{1B} \bar{\psi} \rangle^{(0)} + Z_{4,3} \langle \psi O_{3B} \bar{\psi} \rangle^{(0)} + \langle \psi O_{4B} \bar{\psi} \rangle^{(2)} + Z_{4,5} \langle \psi O_{5B} \bar{\psi} \rangle^{(0)} \;, \tag{5a}$$

$$\langle A_a^\mu O_4 A_b^\nu \rangle = Z_{4,2} \langle A_a^\mu O_{2B} A_b^\nu \rangle^{(0)} + \langle A_a^\mu O_{4B} A_b^\nu \rangle^{(2)} \;, \tag{5b}$$

$$\langle \psi A_a^\mu O_4 \bar{\psi} \rangle = Z_{4,3} \langle \psi A_a^\mu O_{3B} \bar{\psi} \rangle^{(0)} + Z_{4,4} Z_F \langle \psi A_a^\mu O_{4B} \bar{\psi} \rangle^{(2)} + Z_{4,5} \langle \psi A_a^\mu O_{5B} \bar{\psi} \rangle \;, \tag{5c}$$

where A_a^μ is the gluon background field, Z_F is the quark wave function renormalization constants, O_{jB} means the bare composite operator written in terms of renormalized fields and parameters, the indices (0) and (2) mean lowest order and second order contributions. The Feynman rules needed for the computation of the above expressions are given in Fig. 1. The diagrams which contribute at the one-loop level to Eq. (5) are given in Fig. 2. We find in the t'Hooft minimal renormalization scheme[3] for the dimension of the space-time $n = 4 + 2\varepsilon$ and for $SU(N)_c$:

$$Z_{4,1} = -3\left(\frac{N^2-1}{2N}\right)\left(\frac{\alpha_s}{\pi}\right)\frac{1}{\epsilon} , \tag{6a}$$

$$Z_{4,2} = 2\left(\frac{\alpha_s}{\pi}\right)\frac{1}{\epsilon} , \tag{6b}$$

$$Z_{4,3} = -Z_{4,5} = -\frac{1}{2} Z_{4,1} , \tag{6c}$$

$$Z_{4,4} = 1 - \frac{1}{8}\left(N - \frac{5}{N}\right)\left(\frac{\alpha_s}{\pi}\right)\frac{1}{\epsilon} . \tag{6d}$$

The results in Eq. (6) together with the ones shown in Eq. (2) allow us to write a new vacuum condensate which renormalizes multiplicatively:

$$\langle O_4 + x O_1 + y O_2 \rangle = Z_{4,4} \langle O^0_{4B} + x O^0_{1B} + y O^0_{2B} \rangle \tag{7}$$

and from which a renormalization group invariant condensate can be obtained as:

$$\phi_3 = \alpha_s^z \langle O_4 + x O_1 + y O_2 \rangle . \tag{8}$$

The values of x, y, and z to one-loop are:

$$x = -6\frac{(N^2-1)(8N^2+4N-3)}{(N^2+1)(8N^2-2N-3)} , \tag{9a}$$

$$y = -\frac{24N}{8N^2-2N-3} , \tag{9b}$$

$$z = -\left(\frac{3}{2N}\right)\cdot\frac{N^2-5}{11N-2} , \tag{9c}$$

where the lowest-order expressions of β and γ have been used, i.e.

$$\beta = \left(\frac{\alpha_s}{\pi}\right)\left(-\frac{11}{6}N + \frac{1}{3}\right) + \mathcal{O}(\alpha_s^2) \,, \tag{10a}$$

$$\gamma = \left(\frac{\alpha_s}{\pi}\right)\left(\frac{3}{2}\right)\left(\frac{N^2-1}{2N}\right) + \mathcal{O}(\alpha_s^2) \,. \tag{10b}$$

2. RENORMALIZATION OF THE SIX-DIMENSIONAL VACUUM CONDENSATES

The study of the renormalization of the six-dimensional vacuum condensates is technically much more involved. Even, putting the quark mass one equal to zero, and for only one flavour, there are ten independent scalar gauge invariant operators. We will choose them as:

$$O_1 = \frac{g}{4} f^{abc} G_{\mu\nu,a} G_b^{\nu\rho} G_{\rho,c}^{\mu} \,, \tag{11a}$$

$$O_2 = g^2 \bar{\psi}\psi\bar{\psi}\psi \,, \tag{11b}$$

$$O_3 = g^2(4\bar{\psi}\psi\bar{\psi}\psi + 11\bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi) \,, \tag{11c}$$

$$O_4 = g^2\left(6\bar{\psi}\psi\bar{\psi}\psi + \frac{11}{2}\bar{\psi}\sigma^{\mu\nu}\psi\bar{\psi}\sigma_{\mu\nu}\psi\right) , \tag{11d}$$

$$O_5 = g^2(4\bar{\psi}\psi\bar{\psi}\psi - 11\bar{\psi}\gamma^\mu\gamma^5\psi\bar{\psi}\gamma_\mu\gamma^5\psi) \,, \tag{11e}$$

$$O_6 = g^2(\bar{\psi}\psi\bar{\psi}\psi + 11\bar{\psi}\gamma^5\psi\bar{\psi}\gamma^5\psi) \,, \tag{11f}$$

$$\begin{aligned} O_7 = & -\frac{1}{2}\left(\partial^\mu G_{\mu\nu,a} + g f_{abc} B_b^\mu G_{\mu\nu,c}\right)\left(\partial^\rho G_{\rho,a}^{\nu} + g f_{amn} B_m^\rho G_{\rho,n}^{\nu}\right) \\ & - \frac{1}{2} g\left(\partial^\mu G_{\mu\nu,a} + g f_{abc} B_b^\mu G_{\mu\nu,c}\right)\bar{\psi}\gamma^\nu \frac{\lambda^a}{2}\psi \,, \end{aligned} \tag{11g}$$

$$O_8 = i\bar{\psi}\not{D}^3\psi \,, \tag{11h}$$

$$O_9 = ig\bar{\psi}\sigma^{\mu\nu}\frac{\lambda^a}{2}G_{\mu\nu,a}\not{D}\psi \,, \tag{11i}$$

$$\begin{aligned} O_{10} = {} & g\left(\partial^\mu G_{\mu\nu,a} + g f_{abc} B_b^\mu G_{\mu\nu,c}\right)\bar{\psi}\frac{\lambda_a}{2}\gamma^\nu\psi - \frac{g^2}{2}\Big(\bar{\psi}\psi\bar{\psi}\psi - \\ & \frac{1}{2}\left(1-\frac{2}{N}\right)\bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi - \frac{1}{2}\bar{\psi}\gamma^\mu\gamma^5\psi\bar{\psi}\gamma^\mu\gamma^5\psi \\ & - \bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi\Big) \,. \end{aligned} \tag{11j}$$

Operators which do not contribute at zero momentum have not been included. Bianchi and Fierz identities have been repeatedly used. The colour indices in the quartic quark terms are summed according to $\bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi = \bar{\psi}_\alpha\Gamma\psi^\alpha\bar{\psi}_\beta\Gamma\psi^\beta$. The operators O_3 to O_6 are written in such a way that they lead to zero value of vacuum condensates if the vacuum saturation hypothesis (factorization) is used[1]:

$$\langle\bar{\psi}\Gamma_1\psi\bar{\psi}\Gamma_2\psi\rangle = \frac{1}{N^2}\left\{\mathrm{Tr}\,\Gamma_1\,\mathrm{Tr}\,\Gamma_2 - \mathrm{Tr}(\Gamma_1\Gamma_2)\right\}\langle\bar{\psi}\psi\rangle^2 , \tag{12}$$

where the normalization factor N is defined as $\langle\bar{\psi}_\alpha\psi_\beta\rangle = (\delta_{\alpha\beta}/N)\langle\bar{\psi}\psi\rangle$ and the subscripts α,β include spin, colour and flavour. The operators O_7 to O_{10} vanish if the classical equations of motion are used. In the following, we would like to address ourselves to two questions. First, how the operators of the first group (O_1 and O_2) in Eq. (11) get renormalized and second, is the factorization hypothesis compatible with the renormalization group invariance or not? Indeed, if the operators of the second group (O_3 to O_6) do not mix under renormalization with the ones of the first group, then four renormalization group invariant vacuum condensates involving only operators of the second group can be written down; the factorization hypothesis just implies that these constants are zero. If this is not the case, the factorization hypothesis means putting a non-trivial function of μ equal to zero, which certainly cannot be a good approximation for all values of μ.

The study of the renormalization of O_1 and O_2 requires the calculation of the following Green's functions (with $O_i = \sum_j Z_{i,j} O^0_{jB}$):

$$\langle A_a^\mu A_b^\nu A_c^\lambda O_1\rangle = Z_{1,1} Z_A \langle A_a^\mu A_b^\nu A_c^\lambda O_{1B}\rangle^{(2)} + Z_{1,7}\langle A_a^\mu A_b^\nu A_c^\lambda O_{7B}\rangle^{(0)} , \tag{13a}$$

$$\langle A_a^\mu A_b^\nu A_c^\lambda O_2\rangle = Z_{2,1}\langle A_a^\mu A_b^\nu A_c^\lambda O_{1B}\rangle^{(0)} + Z_{2,7}\langle A_a^\mu A_b^\nu A_c^\lambda O_{7B}\rangle^{(0)} , \tag{13b}$$

$$\langle\psi\psi O_1\bar{\psi}\bar{\psi}\rangle = \sum_{j=2}^{6} Z_{1,j}\langle\psi\psi O_{jB}\bar{\psi}\bar{\psi}\rangle^{(0)} + Z_{1,10}\langle\psi\psi O_{10B}\bar{\psi}\bar{\psi}\rangle^{(0)} , \tag{13c}$$

$$<\psi\psi O_2 \bar\psi\bar\psi> = Z_{2,2}\, Z_F^2 Z_A^{-1} <\psi\psi O_{2B} \bar\psi\bar\psi>^{(2)} + \sum_{j=3}^{6} Z_{2,j} <\psi\psi O_{jB} \bar\psi\bar\psi>^{(0)} + Z_{2,10} <\psi\psi O_{10B} \bar\psi\bar\psi>^{(0)}, \tag{13d}$$

$$<\psi A_a^\mu O_1 \bar\psi> = <\psi A_a^\mu O_{1B} \bar\psi>^{(2)} + \sum_{j=7}^{10} Z_{1,j} <\psi A_a^\mu O_{jB} \bar\psi>^{(0)}, \tag{13e}$$

$$<\psi A_a^\mu O_2 \bar\psi> = <\psi A_a^\mu O_{2B} \bar\psi>^{(2)} + \sum_{j=7}^{10} Z_{2,j} <\psi A_a^\mu O_{jB} \bar\psi>^{(2)}, \tag{13f}$$

where $Z_A = Z^{-1}$ is the background field renormalization constant. The needed Feynman rules are given in Fig. 3. The diagrams which have to be computed are shown in Fig. 4. The calculation is done in the Feynman gauge; this is no limitation since only the renormalization constants which mix with the operators of the third group (O_7 to O_{10}) can be gauge dependent but these give vanishing condensates. The result is:

$$Z_{1,1} = 1 - \left(\frac{\alpha_s}{\pi}\right)\left(\frac{1}{\epsilon}\right)\left(\frac{1}{12}\right)(2+7N), \tag{14a}$$

$$Z_{1,2} = Z_{1,3} = Z_{1,4} = Z_{1,5} = Z_{1,6} = Z_{1,8} = Z_{1,9} = 0, \tag{14b}$$

$$Z_{1,7} = -\frac{3}{8}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{1}{\epsilon}\right) \cdot N, \tag{14c}$$

$$Z_{2,2} = 1 - \left(\frac{\alpha_s}{\pi}\right)\left(\frac{1}{\epsilon}\right)\left(286N + 38 - \frac{185}{N}\right)\left(\frac{1}{132}\right), \tag{14d}$$

$$Z_{2,1} = Z_{2,7} = Z_{2,8} = Z_{2,9} = 0, \tag{14e}$$

$$Z_{2,3} = -\left(\frac{\alpha_s}{\pi}\right)\left(\frac{1}{\epsilon}\right)\left(\frac{1}{528}\right)\left(\frac{2}{N} - 1\right), \tag{14f}$$

$$Z_{2,4} = \left(\frac{\alpha_s}{\pi}\right)\left(\frac{1}{\epsilon}\right)\left(\frac{1}{88}\right)\left(1 - \frac{2}{N}\right), \tag{14g}$$

$$Z_{2,5} = -\left(\frac{\alpha_s}{\pi}\right)\left(\frac{1}{\epsilon}\right)\frac{1}{528} , \tag{14h}$$

$$Z_{2,6} = -\left(\frac{\alpha_s}{\pi}\right)\left(\frac{1}{\epsilon}\right)\frac{1}{33} , \tag{14i}$$

$$Z_{2,10} = -\left(\frac{\alpha_s}{\pi}\right)\left(\frac{1}{\epsilon}\right)\frac{1}{12} . \tag{14j}$$

Notice that O_1 [Eq. (11a)] does not mix with any other operators of the first [Eq. (11b)] and second [Eqs. (11c) to (11f)] groups. Thus, at the one-loop level

$$\langle O_1 \rangle = Z_{1,1} \langle O^0_{1B} \rangle , \tag{15}$$

and it is easy to build a renormalization group invariant condensate proportional to it:

$$\phi_4 = (\alpha_s)^{-\frac{2+7N}{11N-2}} \langle O_1 \rangle . \tag{16}$$

O_2 [Eq. (11b)] however mixes with the operators of the second group [Eqs. (11c) to (11f)], and one can easily check that the operators of the second group also mix with O_2 under renormalization. This means that the factorization hypothesis is incompatible with the renormalization group and that putting all the second group condensates equal to zero leads to a μ-dependence of $\langle O_2 \rangle$ inconsistent with the renormalization group. If among all the quartic quark condensates $\langle O_2 \rangle$ is only retained, then there is absolutely no meaning in any renormalization group improvement of the result obtained.

Thus, we conclude from the analysis of six-dimensional vacuum condensates, that the gluon condensate $\langle O_1 \rangle$ in Eq. (11a) does not mix with the quartic quark condensate nor the other way round. We also show that the four-fermion vacuum condensate $\langle O_2 \rangle$ [Eq. (11b)] mixes under renormalization with other four-fermion condensates [Eqs. (11c) to (11f)] which vanish under the factorization hypothesis introduced by SVZ[1]. So, the only four fermion vacuum condensate, $\langle O_2 \rangle$, retained in the QCD sum rules analysis, cannot be made renormalization group invariant within the factorization hypothesis; its value is μ-dependent and the factorization hypothesis prevents any renormalization group improvement.

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REFERENCES

1) M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385.

2) For a review on the renormalization group, see, for example,
A. Peterman, Phys. Reports 53 (1979) 159.

3) For a review on comparison of renormalization schemes, see, for example,
S. Narison, Phys. Reports 84 (1982) 263.

4) J.C. Collins, A. Duncan and S.D. Joglekar, Phys. Rev. D 16 (1977) 438.

5) R. Tarrach, Nucl. Phys. B196 (1982) 45.

6) L.F. Abbott, Nucl. Phys. B185 (1981) 189 and references therein.
H. Kluberg-Stern and J.B. Zuber, Phys. Rev. D 12 (1975) 3159.

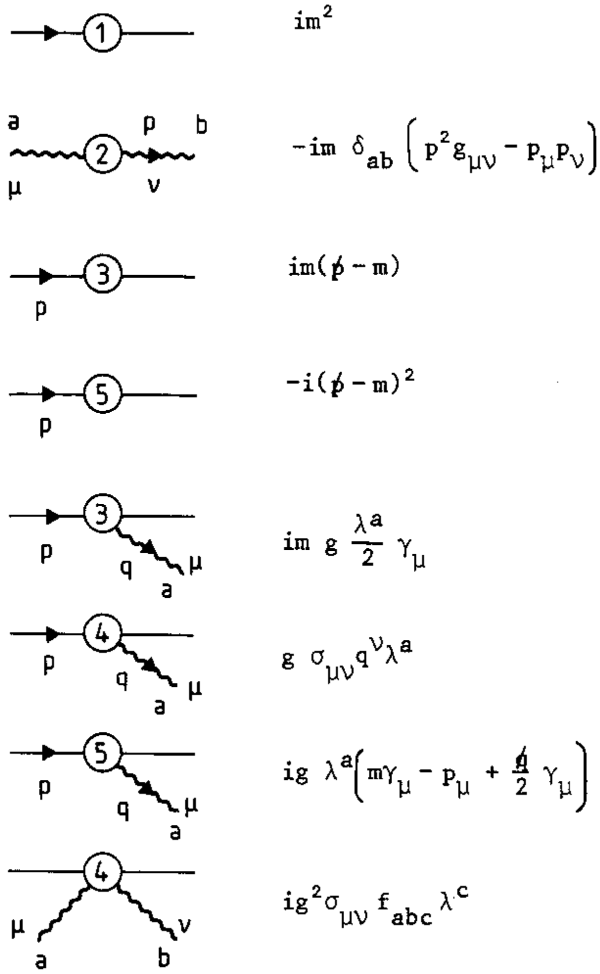
Figure captions

Fig. 1 : Feynman rules for the operators of dimension five in the background field gauge. i denotes the operator O_i and m is the quark mass.

Fig. 2 : Contributing diagrams for the renormalization of the dimensional five operators to lowest order of QCD.

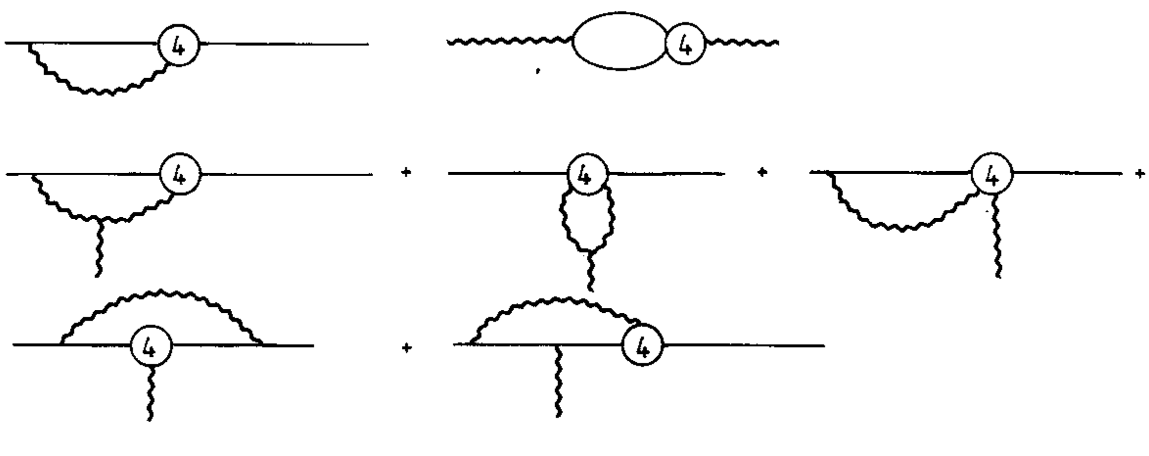
Fig. 3 : Feynman rules for the operators of dimension six in the background field gauge and for massless quarks.

Fig. 4 : Contributing diagrams for the renormalization of the dimensional six operators to lowest order of QCD in the case of massless quarks.



1
im^2
a p b
2
μ ν
$-im\ \delta_{ab}\left(p^2 g_{\mu\nu} - p_\mu p_\nu\right)$
3
p
$im(\not{p} - m)$
5
p
$-i(\not{p} - m)^2$
3
p q μ a
$im\ g\ \frac{\lambda^a}{2}\ \gamma_\mu$
4
p q μ a
$g\ \sigma_{\mu\nu} q^\nu \lambda^a$
5
p q μ a
$ig\ \lambda^a\left(m\gamma_\mu - p_\mu + \frac{\not{q}}{2}\ \gamma_\mu\right)$
4
μ ν a b
$ig^2 \sigma_{\mu\nu}\ f_{abc}\ \lambda^c$

Fig. 1



4
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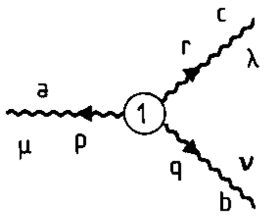
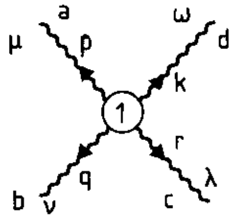
Fig. 2



$$-\frac{3}{2}\, ig \left[p^{\lambda} q^{\mu} r^{\nu} - p^{\nu} q^{\lambda} r^{\mu} + qr\left(p^{\nu} g^{\mu\lambda} - p^{\lambda} g^{\mu\nu}\right) \right.$$

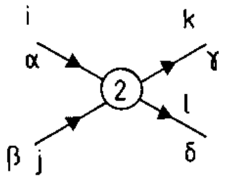
$$\left. + \, pq\left(r^{\mu} g^{\nu\lambda} - r^{\nu} g^{\mu\lambda}\right) + pr\left(q^{\lambda} g^{\mu\nu} - q^{\mu} g^{\nu\lambda}\right)\right]$$



$$-\frac{3}{2}\, g^{2} \left\{ f_{xab} f_{xcd} \left[rk\left(g^{\mu\lambda} g^{\nu\omega} - g^{\mu\omega} g^{\nu\lambda}\right) + g^{\mu\omega} r^{\nu} k^{\lambda} \right. \right.$$

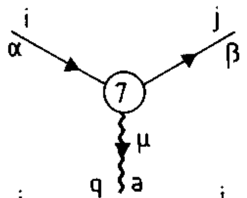
$$\left. + \, g^{\nu\lambda} r^{\omega} k^{\mu} + g^{\lambda\omega}\left(k^{\nu} r^{\mu} - k^{\mu} r^{\nu}\right) - g^{\mu\lambda} r^{\omega} k^{\nu} - g^{\nu\omega} r^{\mu} k^{\lambda} \right]$$

$$\left. + \text{ five other terms required by Bose statistics} \right\}$$

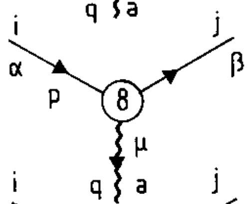


$$g^{2}\left(\delta_{ki}\delta_{\ell j}\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{kj}\delta_{\ell i}\delta_{\gamma\beta}\delta_{\alpha\delta}\right)$$

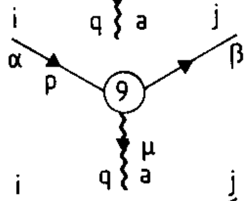
and the analogues for O_3 to O_6 and O_{10}



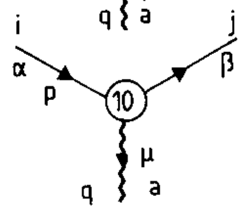
$$\frac{g}{2}\, \frac{\lambda^{a}_{\beta\alpha}}{2} \left(q^{2}\gamma^{\mu} - q^{\mu}\not{q}\right)_{ji}$$



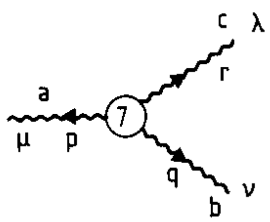
$$-g\, \frac{\lambda^{a}_{\beta\alpha}}{2} \left[(p-q)^{2}\gamma^{\mu} - \not{q}\gamma^{\mu}\not{p} + 2p^{\mu}\not{p}\right]_{ji}$$



$$-2g\, \frac{\lambda^{a}_{\beta\alpha}}{2} \left(\not{q}\gamma^{\mu}\not{p} - q^{\mu}\not{p}\right)_{ji}$$



$$-g\, \frac{\lambda^{a}_{\beta\alpha}}{2} \left(q^{2}\gamma^{\mu} - q^{\mu}\not{q}\right)_{ji}$$



$$ig\, f_{abc} \left\{ p^{2}\left(q^{\nu} g^{\mu\lambda} + 2r^{\nu} g^{\mu\lambda} - r^{\mu} g^{\nu\lambda} - r^{\lambda} g^{\mu\nu} - 2q^{\lambda} g^{\mu\nu} + q^{\mu} g^{\mu\lambda}\right) \right.$$

$$+ \, p^{\mu}\left(p^{\nu} r^{\lambda} + 2p^{\nu} q^{\lambda} - p^{\lambda} q^{\nu} - 2p^{\lambda} r^{\nu} - pq g^{\nu\lambda} + pr g^{\nu\lambda}\right)$$

$$\left. + \text{ two other terms required by Bose statistics} \right\}$$

Fig. 3

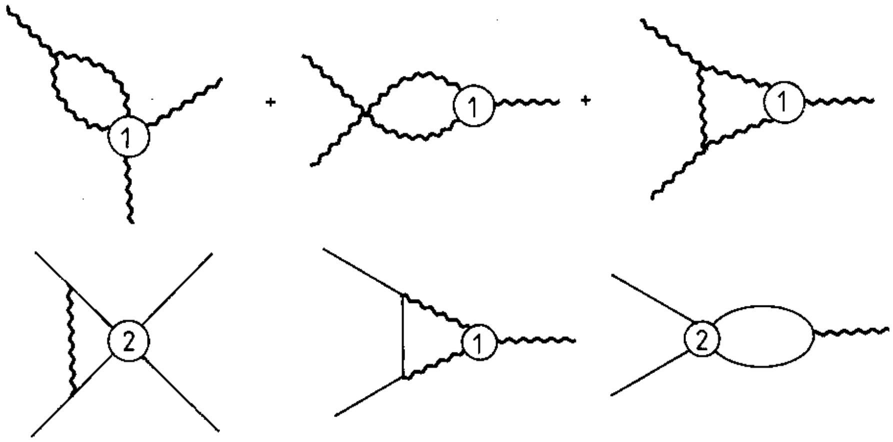


Fig. 4