LONGITUDINAL ASPECTS

OF SLOW EXTRACTION

Combined notes on :

Stochastic and other means of rf acceleration to 'feed' the resonance, 'Empty bucket' stabilisation of the spill and General longitudinal strategy

presented by

R. Capp

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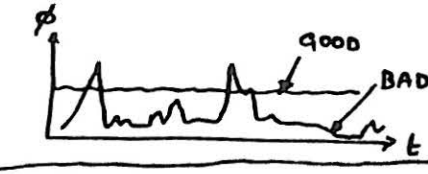
PS, CERN

INTRODUCTION

what is the problem?

The non uniform structure of the extracted particle flux

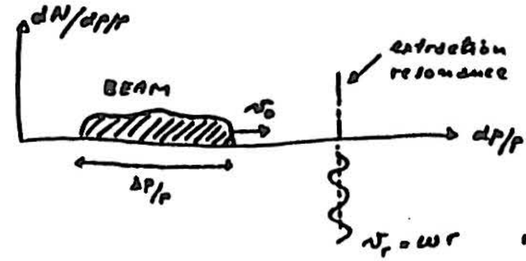


ϕ = # of p. extr. / sec

1) low frequency : 50 ÷ 1000 Hz due to power supply ripple
2) medium frequency: 10 ÷ 1000 kHz " revolution freq. structure
3) high frequency : 1 ÷ 100 MHz " RF freq. structure

Generally the "low frequency" is considered the most annoying one.

in a "standard" extraction



$v_r = \omega r$ r = ripple amplitude in $\Delta p/p$ units

$v_0 = \frac{\Delta P/P}{T_s}$ T_s = spill length

if $v_0 << v_r \Rightarrow$ HIGH ripple $\Rightarrow$ BAD

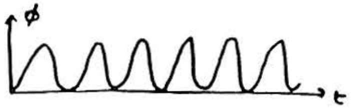
$v_0 >> v_r \Rightarrow$ LOW ripple but T_s SMALL example : $\omega = 2\pi \cdot 50\,Hz$, $r = 10^{-4}$, $\frac{\Delta p}{p} = 3 \cdot 10^{-3}$

then for $v_0 = \omega r$: $T_S = 100\ ms$

DUTY FACTOR F

$$F = \frac{\langle \phi \rangle^2}{\langle \phi^2 \rangle} = \frac{1}{1 + \frac{1}{2}\left(\frac{\omega r}{v_0}\right)^2} \qquad 0 < F < 1$$

if $\omega r = v_0 \Rightarrow F = 2/3$



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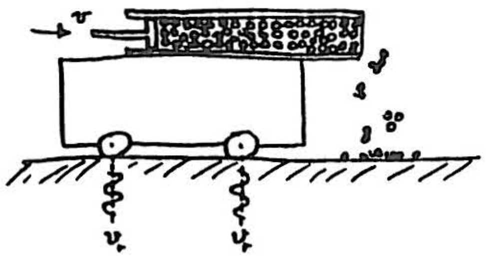
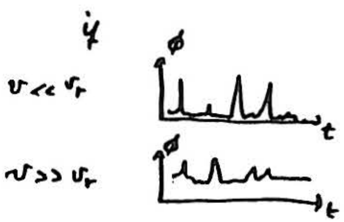
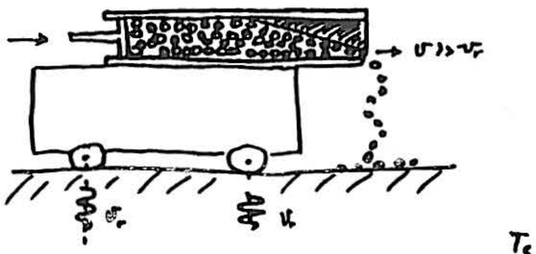
a mechanical analogy :







T_S is large

DIFFUSION ... A REMINDER

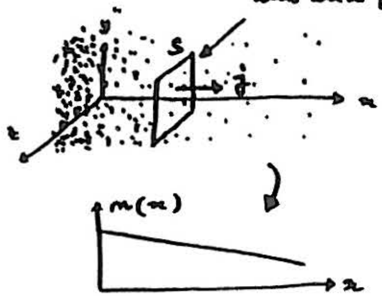
1^{st} Analogy : MOLECULAR DIFFUSION

Remarks :

1) one has a molecular diffusion when the spatial distribution of p. is not uniform.
2) diffusion is always towards low concentration

n = # of p./m^3 = concentration = density

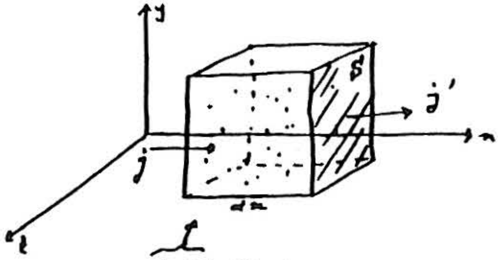
j = current density
= # of p. traversing S in one second
(S ↓ unit area [m^2])



NB : if n = constant $\Rightarrow j = 0$

$$j = -D \frac{\partial n}{\partial x} \qquad (*)$$

where D = diffusion coefficient



$dV = S\,dx$

\# of p in dV = dN = n dV

input flux $\phi_{IN} = j S$

output " $\phi_{OUT} = j' S$

accumulation rate $= \phi_{IN} - \phi_{OUT} = (j - j') S = -\dfrac{\partial j}{\partial x} S\,dx$ (**)

↓

but also

↓

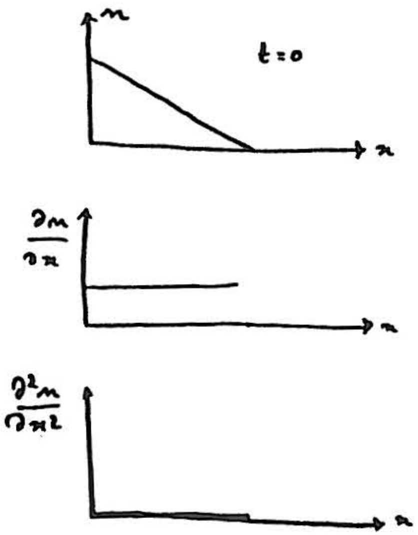
accumulation rate $= \dfrac{\partial n}{\partial t} dV = \dfrac{\partial n}{\partial t} S\,dx$ (***)

Combining (**) & (***) $\dfrac{\partial n}{\partial t} = -\dfrac{\partial j}{\partial x}$

and (*)

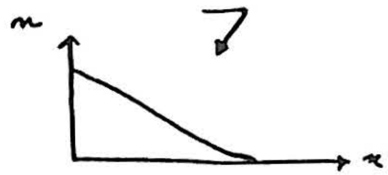
$$\boxed{\frac{\partial n}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial n}{\partial x}}$$ DIFFUSION EQUATION

EXAMPLE #1



⇓

t = ∞

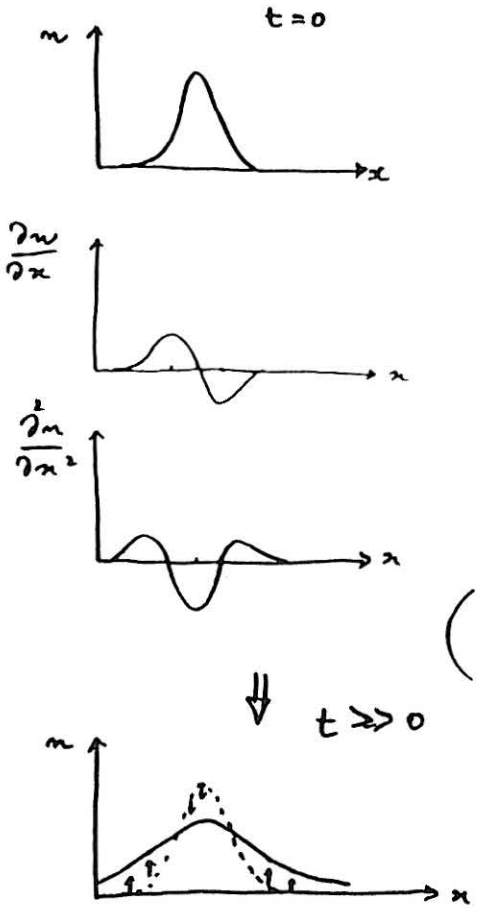


i.e. no change

↓

STEADY STATE

EXAMPLE # 2



$$\left(\frac{\partial n}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial n}{\partial x} \right)$$

... BACK TO STEADY STATE ...

$n = n(x, \cancel{t})$

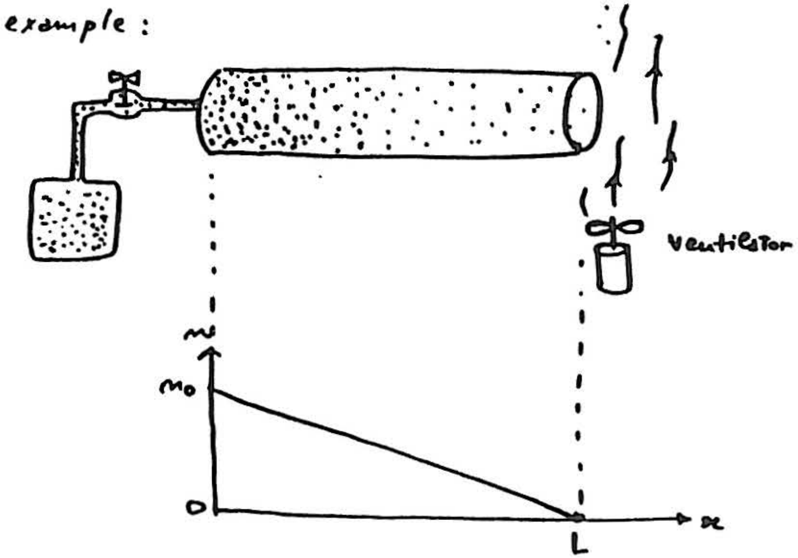
i.e. the concentration is stationary

$$\frac{\partial n}{\partial t} = 0 \quad \Rightarrow \quad \frac{\partial}{\partial x} D \frac{\partial n}{\partial x} = 0$$

i.e.

$$- D \frac{\partial n}{\partial x} = \text{constant} = j = \text{constant flux} = \text{no accumulation}$$

example :

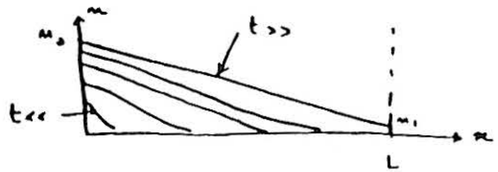


$$j = - D \frac{\partial n}{\partial x} = D \frac{n_0}{L}$$

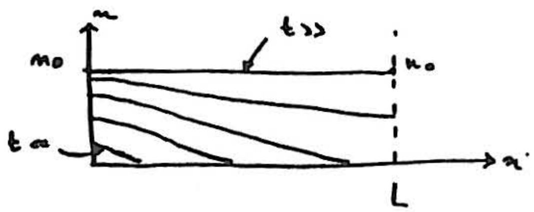
Remarks:

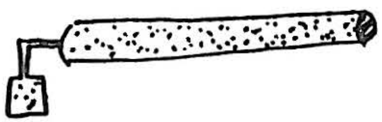
How the steady state is reached?

case #1: OPEN PIPE



case #2: CLOSED PIPE





2nd Analogy: THERMAL DIFFUSION

that is a transfer of energy (or heat)

Remarks:

1) one has thermal diffusion when there is a temperature (T) difference
2) the direction is from HOT (T>>) to COLD (T<<)

as in molecular diffusion exchanging $n \to T$ we get

the energy density current $j_E = -D \frac{\partial T}{\partial x}$

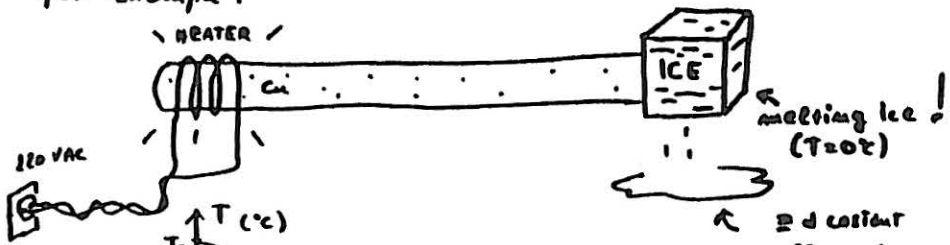
and the DIFFUSION EQ.

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial T}{\partial x}$$

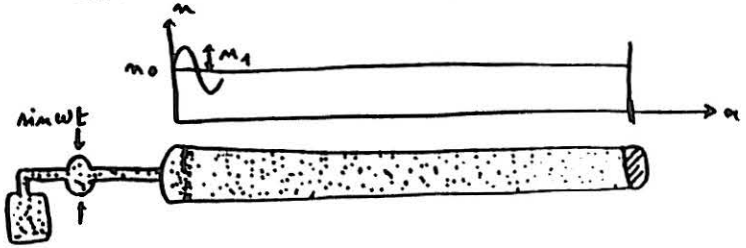
the STEADY STATE, that is $T_{stationary} = T(x, \not{t})$

means $D \frac{\partial T}{\partial x} = -j_E = \text{constant}$

for example:



PERTURBATION PROPAGATION



Given a perturbation like:

$$n = n_1 \sin \omega t$$

will this perturbation propagate like a travelling wave like

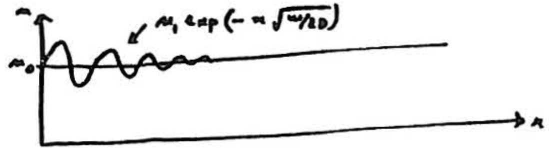
$$n(x \neq 0) = n_0 + n_1 \sin(\omega t - kx) \quad ?$$

$$\left.\begin{aligned} \frac{\partial n}{\partial t} &= n_1 \omega \cos(\omega t - kx) \\ \frac{\partial^2 n}{\partial x^2} &= -k^2 n_1 \sin(\omega t - kx) \end{aligned}\right\} \text{ they do not satisfy: } \frac{\partial n}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial n}{\partial x}$$

the answer is: no. BUT

$$\boxed{n = n_0 + n_1 \exp\left[-x\sqrt{\frac{\omega}{2D}}\right] \cdot \sin\left[\omega t - x\sqrt{\frac{\omega}{2D}}\right]}$$

will!



NB: ① $k = \frac{2\pi}{\lambda} = \sqrt{\frac{\omega}{2D}}$ & $\boxed{v = \lambda f = \frac{\omega}{k} = \sqrt{2D\omega}}$

② if D ≫ ⇒ the damping is small

③ the group velocity: $v_g = \frac{d\omega}{dk} = \sqrt{8D\omega} > v$

BACK TO OUR ACCELERATOR

n [# p/m³] → ψ [# p/$\frac{\Delta p}{p}$] = proton density in $\frac{\Delta p}{p}$ space

j [# p/m² s] → ϕ [# p extr./s]

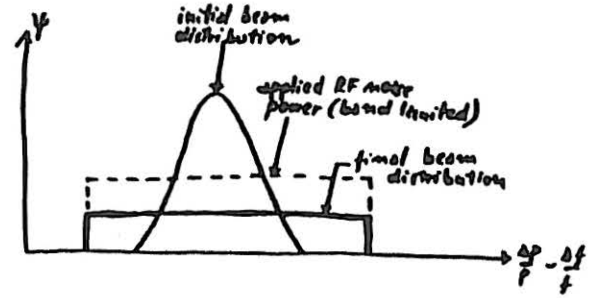
$$j = -D\frac{\partial n}{\partial x} \quad \rightarrow \quad \phi = -D\frac{\partial \psi}{\partial \frac{\Delta p}{p}}$$

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial n}{\partial x} \quad \rightarrow \quad \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial \frac{\Delta p}{p}} D \frac{\partial \psi}{\partial \frac{\Delta p}{p}}$$

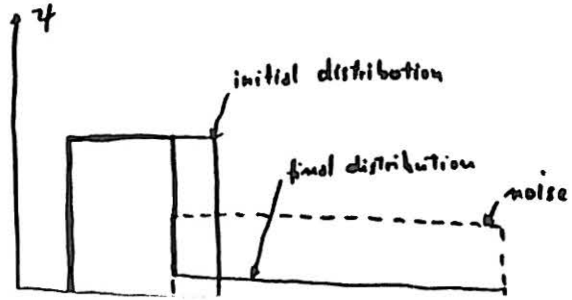
APPLICATIONS:

1) Beam shaping

See also Fig. 1



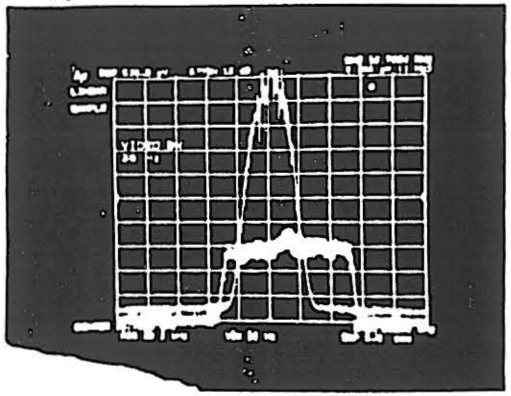
2)



Test stochastic extr.: Fig 1. Beam shaping in LEAR

Shaping: $f = 16.76$ MHz

$T = 49$ MeV

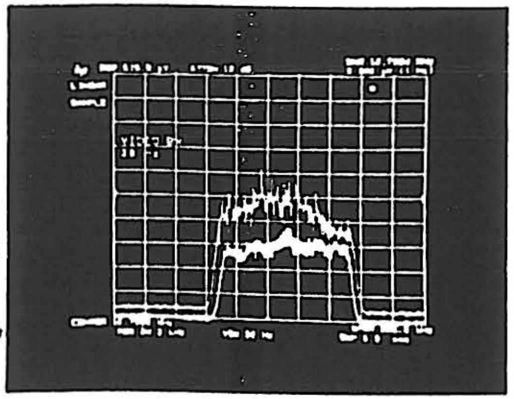


1) Beam distribution

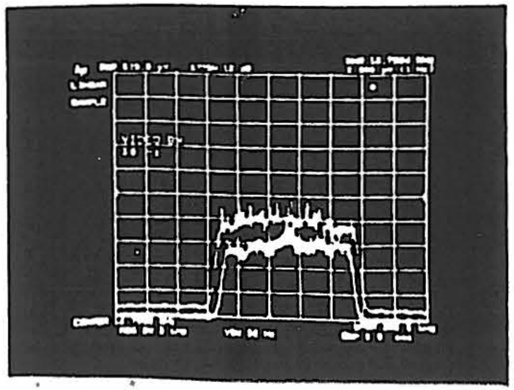
2) Noise spectrum

$P_A = 0.6$ W

Initial conditions

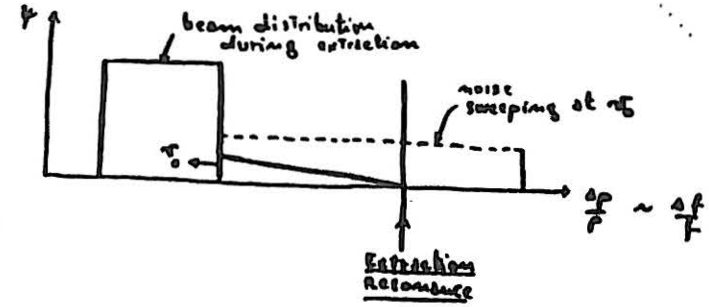


10 sec later



20 sec. later

3) STOCHASTIC EXTRACTION



See also Fig 2.

REMARKS:

- For a p. in Brownian motion, the rms distance from the origin after a time t (where $r=0$ at $t=0$) is:

$$\langle r^2 \rangle = 2\, n\, D\, t \qquad n = \# \text{ of space dimensions}$$

- The rms energy gain given by a noise with bandwidth W (overlapping only one harmonic of the revolution frequency) and rms voltage V_m is

$$\frac{dE^2}{dt} = \frac{f_0}{W} \frac{(eV_m)^2}{T_0} = \frac{1}{W} \left(\frac{eV_m}{T_0} \right)^2$$

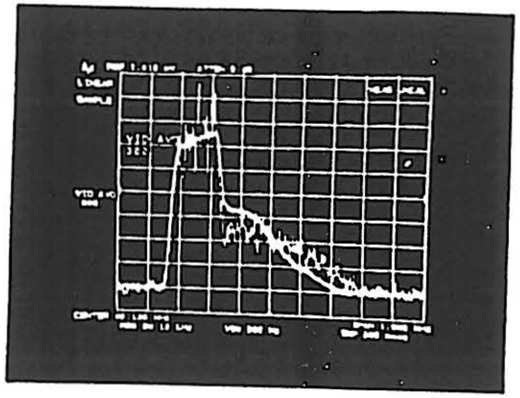
from $D = \frac{1}{2} \frac{d(\Delta p/p)^2}{dt}$

and knowing that $\Delta p = \frac{1}{\beta c} \Delta E$, $p = eB\rho$, $f_0 = \frac{\beta c}{2\pi R}$

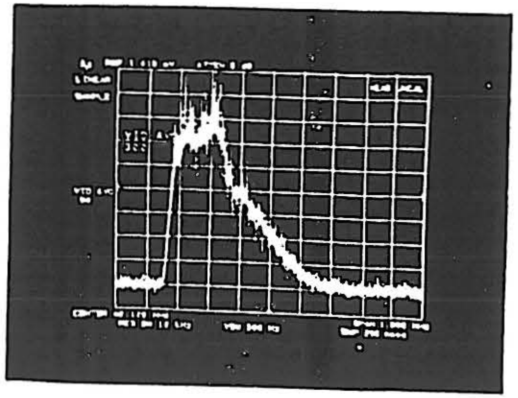
we obtain

$$D = \frac{1}{2W} \left(\frac{V_m}{2\pi R \rho B} \right)^2$$

- $$F = \frac{1}{1 + \frac{1}{2} \frac{W\tau^2}{D}} \Rightarrow \text{if } \boxed{D > W\tau^2} \Rightarrow \text{ok (Fig 1)}$$



Sweep 10 s
$P_A = 0.6$ W



Sweep 100 s
$P_A = 0.16$ W

Fig. 2. Stochastic extraction in LEAR
Beam distribution during extraction

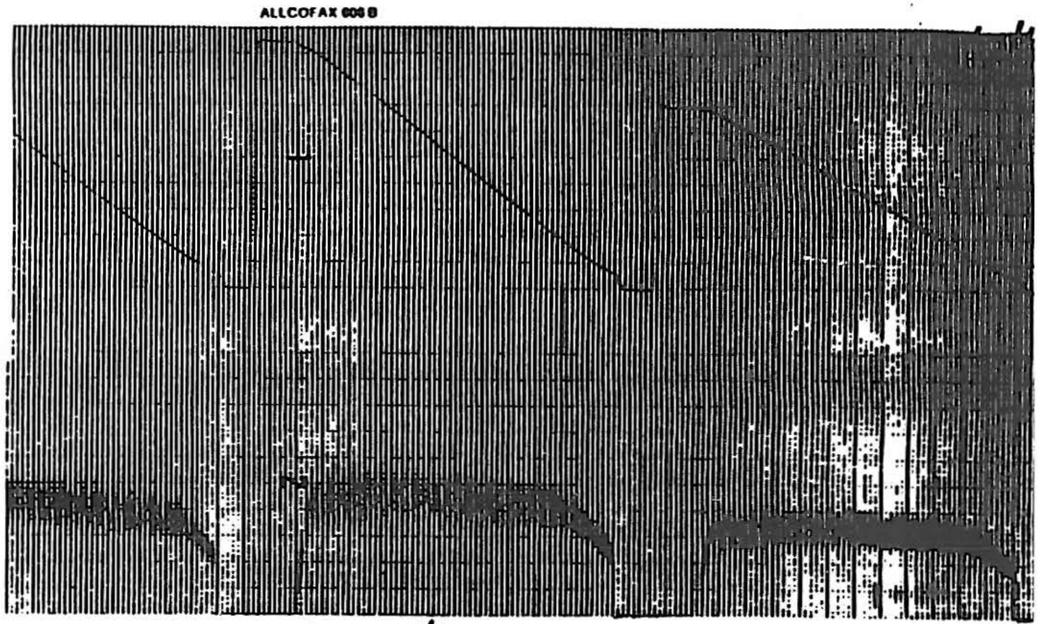
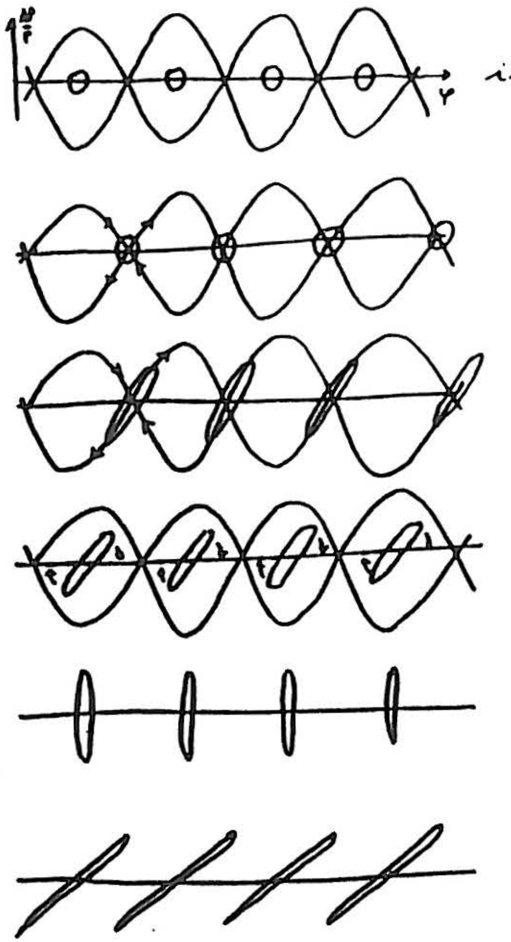


Fig. 3 First 1h stochastic extraction in LEAR
Top trace : circulating beam current
bottom " : extracted flux

PRESENT PS SLOW EXTRACTION

a) Phase jump debunching (24 GeV/c)

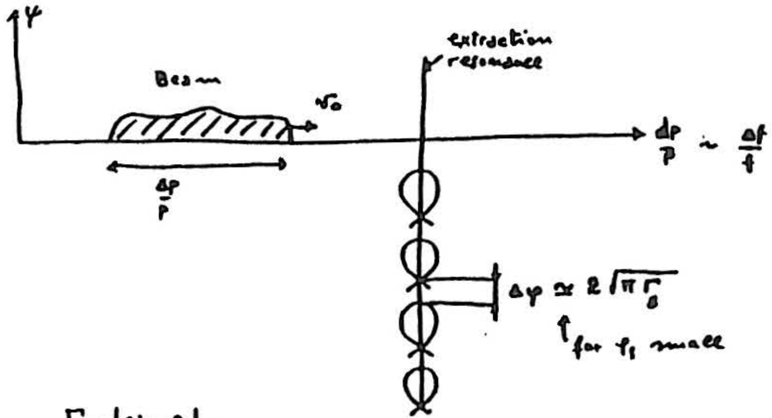


$$T_d = \frac{2\pi}{h f_0 |\eta| \, \Delta p/p}$$

b) The beam is 'pushed' to the resonance by decreasing the B field

c) $\omega\tau \simeq 2\pi \cdot 50 \cdot 10^{-4} = .03$, $\nu_0 = \frac{3 \cdot 10^{-3}}{100\ ms} \simeq .01$ $\Rightarrow F \simeq 0.2$

EMPTY BUCKET CHANNELLING



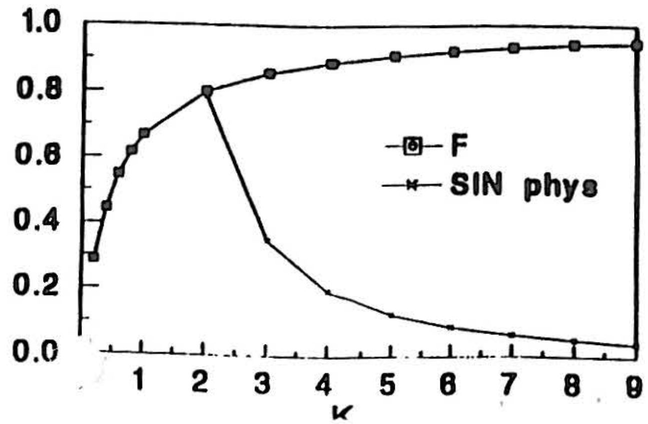
$$\Gamma_s = |\sin\varphi_s| = $$

$$= 2\pi R \rho \frac{\dot{B}}{V_{RF}} \left(\frac{\gamma^2}{\gamma^2 - \gamma_t^2}\right) \rightarrow \text{if } \Delta B \neq 0 \ \& \ f = \text{const.}$$

$$\boxed{\nu = \frac{\Delta p/p}{T_s} \simeq \nu_0 \kappa \qquad \text{where} \quad \kappa \simeq \frac{2\pi}{\Delta\varphi} \simeq \sqrt{\frac{\pi}{\Gamma_s}}}$$

for example: if $\Gamma \simeq \sin 1.4° \simeq .025 \Rightarrow \kappa \simeq 10$

$$F \simeq \frac{1}{1 + \frac{1}{2}\left(\frac{\omega\tau}{\kappa\nu_0}\right)^2}$$



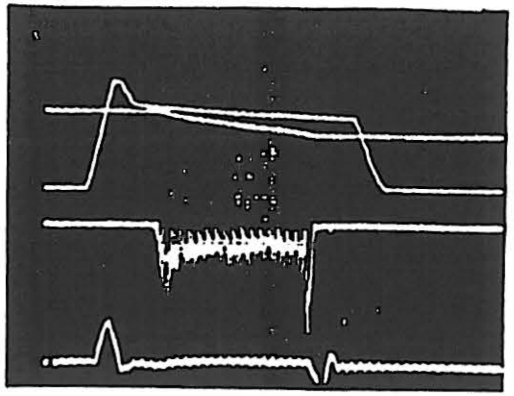


Fig. 4: Empty-bucket channelling

I_p

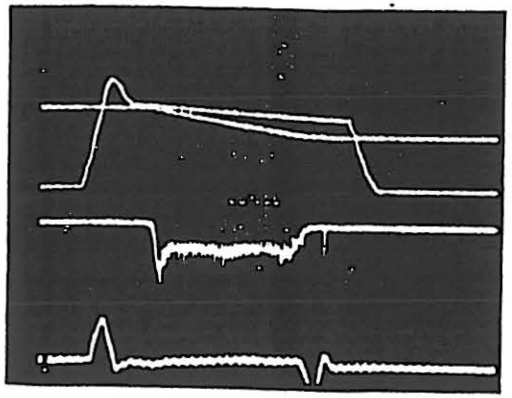
BQ

SE 62 spill

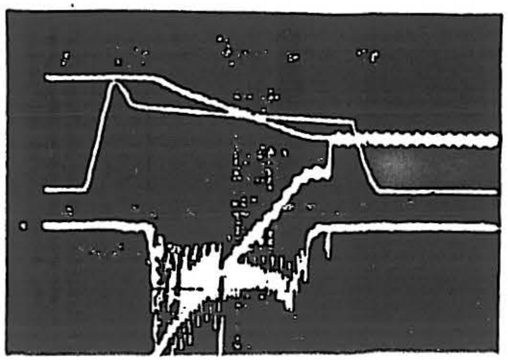
(7) No RF

Servo loop current

Hor. - 100 ms/div.



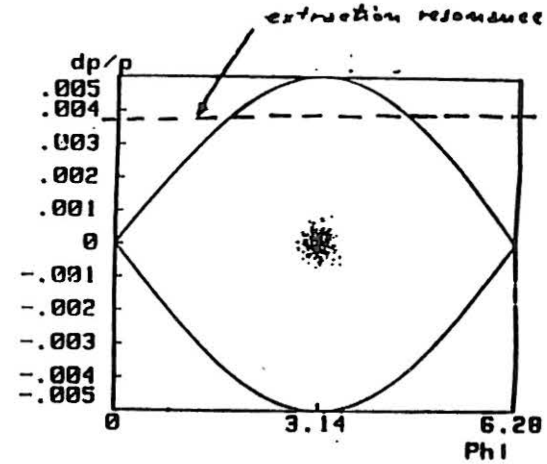
(8) RF on



(9) RF on only during the 2nd half of extraction

(4th trace: losses in s.s. 61)

NOISY BUCKETS



1) Moving, with a perturbation on the radial loop, the beam close to the extr. resonance
2) Shaking, with noise in the phase loop, the bunch to increase E_ℓ

⇓

Should provide a fair control of the spill

NB:
- tails during E_ℓ blow-up are welcome!
- The strong RF structure should not be a problem for medical applications
- no debunching
- no gap relays
- facilitate beam diagnostic (intensity, position, ...)

SUMMARY

- STOCHASTIC EXTRACTION
  - + good F for long spills (>> 1 sec.)
  - + hardware simplicity
  - + operational "
  - − moderate spill control
  - − problems for short spill (< 1 sec.)
- PHASE JUMP DEBUNCHING
  - + operationally simple & fast
  - − distribution not very rectangular
  - − hardware complicated
- RESONANCE FEEDING WITH B SLOPE
  - + hardware simplicity (?)
  - − poor F
  - − poor spill control
- EMPTY BUCKET CHANNELLING
  - + good for short spills (< 1 sec.)
  - − hardware complicated
  - − poor spill control
- NOISY BUCKETS
  - + hardware simplicity
  - + good spill control
  - + no debunching
  - + facilitate instrumentation
  - − strong RF structure (may be acceptable ?)
- UNSTACKING
  - + facilitate instrumentation
  - − RF structure (may be acceptable ?)
  - − hardware complicated
  - − operationally "
  - − poor F

References

1) S. van der Meer, Stochastic extraction, a low ripple version of resonant extraction, CERN/PS/AA 78-6

2) W. Hardt, Ch. Steinbach, R.Cappi, Ultraslow extraction with good duty factor, Proc. of the XI th. Int. Conf. on High En. Accel., CERN, July 7-11, 1980, p.335-340 or CERN/PS/OP/DL 80-16

3) D.Boussard, M.Gyr, K.H.Kissler, T.Linnecar, Slow extraction at 400GeV/c with stochastic RF noise, SPS Improvement report No. 179, 24th July,1980

3a) R. Giannini, W.Hardt, R.Cappi, Ultraslow extraction, Proc. of LEAR workshop, Erice, May 9-16, 1982 or CERN/PS/LEA 82-3

4) see for ex. The Feynman Lectures on Physics

5) Ch. Steinbach, R. Cappi, Improvement of the low frequency duty factor of slow extraction by RF phase displacement, CERN/PS/OP 80-10

6) S.Hansen, A. Hofmann, E. Peschardt, F. Sacherer, W. Schnell, Longitudinal bunch dilution due to RF noise, Proc. 1977 PAC, Chicago, March 16-18 1977 or CERN-ISR-RF-TH / 77-25