

Enhanced cosmological ${}^6\text{Li}$ abundance as a potential signature of residual dark matter annihilations

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Residual late-time dark matter particle annihilations during and after big-bang nucleosynthesis may alter the predicted cosmological abundances of the light elements. Within the constrained minimal supersymmetric extension of the standard model with a neutralino as the lightest supersymmetric particle, we find negligible effects on the abundances of deuterium, ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$ predicted by homogeneous big-bang nucleosynthesis, but potentially a large enhancement in the predicted abundance of ${}^6\text{Li}$. This enhancement may be as much as 2 orders of magnitude in the focus-point WMAP strip and in the coannihilation and funnel regions for large $\tan\beta$ for small $m_{1/2}$, and the effect is still significant at large $m_{1/2}$. However, the potential ${}^6\text{Li}$ enhancement is negligible in the part of the coannihilation strip for $\tan\beta = 10$ that survives the latest LHC constraints. A similar enhancement of the ${}^6\text{Li}$ abundance may also be found in a model with common, nonuniversal Higgs masses (the NUHM1).

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I. INTRODUCTION

The success of homogeneous big-bang nucleosynthesis (BBN) is one of the linchpins of cosmology. Using the baryon-to-photon ratio, η , inferred from measurements of the cosmic microwave background radiation [1], homogeneous BBN predicts successfully the astrophysical abundances of deuterium, ${}^3\text{He}$ and ${}^4\text{He}$ [2–7]. On the other hand, there are issues with the abundances of ${}^7\text{Li}$ [7] and potentially with ${}^6\text{Li}$ [8]. In particular, the predicted abundance of ${}^7\text{Li}$ is considerably larger than the range suggested by observations [9–18],¹ and there are suggestions that the astrophysical value of the ${}^6\text{Li}$ abundance may be much higher than predicted by homogeneous BBN [14]. However, one should note that the line asymmetries which have been interpreted as ${}^6\text{Li}$ could be the result of convective processes affecting ${}^7\text{Li}$ [20].

We and others have investigated previously whether the late decays of massive particles, such as the gravitino in the constrained minimal supersymmetric extension of the standard model (the CMSSM) with a neutralino as the lightest supersymmetric particle (LSP), could improve significantly the ${}^6\text{Li}$ and ${}^7\text{Li}$ abundances predicted by homogeneous BBN [21–45]. We did not find a solution to the ${}^6\text{Li}$ problem, but we did find a region of supersymmetric parameter space where gravitino decays might alleviate or even solve the ${}^7\text{Li}$ problem [45,46]. On the other hand

the ${}^7\text{Li}$ problem might have a more banal solution, such as the existence of a suitable carbon, boron, or beryllium resonance [47].

In addition to decays, the late-time annihilations of cold dark matter may also affect the abundances of the light elements [25,35,36,48,49]. In particular, these annihilations may have a significant effect on the abundance of ${}^6\text{Li}$ [36]. In [36] it was argued that ${}^6\text{Li}$ production may occur if the s -wave annihilation cross section is sufficiently large, and it was assumed that the relic density of the annihilating dark matter particles is controlled largely by the s -wave part of the cross section. However, in supersymmetric models where the LSP is a neutralino, such as the CMSSM, the relic density is in fact largely determined by the p -wave part of the cross section, which by the time of BBN is essentially ineffective. Therefore, a reanalysis of the suggestion of [36] in the context of the CMSSM and related models is timely, and is the subject of this paper.

In this paper we study the possible effects on the cosmological light-element abundances of residual late-time annihilations of neutralino LSPs during or after BBN [35,36,48].² We find negligible effects on the abundances of deuterium, ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$ predicted by homogeneous BBN, but potentially a large enhancement in the predicted

¹A globular cluster star with a ${}^7\text{Li}$ abundance comparable to the BBN prediction has recently been observed [19]: this value may be due to production by a previous generation of stars.

²Recent papers have also considered the BBN consequences of weakly interacting massive particle (WIMP) models having residual annihilations increased by Sommerfeld or Briet-Wigner enhancements [50]. In the CMSSM model, neither effect occurs, due to the lack of a light boson and of extreme degeneracy in the funnel, respectively.

abundance of ${}^6\text{Li}$, as suggested in [36]. The physics of this effect is the following. It is well understood that the famous $A = 5$ gap in the spectrum of stable nuclei impedes the production of heavier nuclei in BBN. The dominant mechanism for making ${}^6\text{Li}$ in annihilating-particle scenarios is initiated by p and n spallation of ${}^4\text{He}$. This yields many $A = 3$ nuclei with only a tiny reduction in ${}^4\text{He}$ abundance. The tritium and ${}^3\text{He}$ nuclei are produced with large, nonthermal energies, and subsequently slow down due to ionization losses, but have some probability of inducing $t(\alpha, n){}^6\text{Li}$ or ${}^3\text{He}(\alpha, p){}^6\text{Li}$ reactions first. In this way, an amount of ${}^6\text{Li}$ may be produced that is large relative to the standard homogeneous BBN abundance, without making large amounts of extra deuterium and $A = 3$ or reducing the ${}^4\text{He}$ abundance, and leaving the ${}^7\text{Li}$ abundance unaffected.

In the CMSSM [51], it is assumed that all supersymmetry-breaking gaugino masses have a common value $m_{1/2}$ at some grand unification scale before renormalization, and likewise all the soft supersymmetry-breaking scalar masses are assumed to have a common m_0 . The other parameters of this model are the (supposedly) universal trilinear parameter A_0 (taken here to be $A_0 = 0$) and the ratio of Higgs vacuum expectation values,

$\tan\beta$. In addition, one must specify the sign of the Higgs mixing mass, μ , which is generally taken to be positive in the CMSSM so as to improve compatibility with measurements of $g_\mu - 2$ and $b \rightarrow s\gamma$ decays. As is well-known, in the CMSSM there are strips in the $(m_{1/2}, m_0)$ planes for fixed A_0 and $\tan\beta$ along which the relic density of the neutralino LSP, χ , lies within the range favored by WMAP and other astrophysical observations [52]. At relatively low values of m_0 there is generically a coannihilation strip close to the boundary where the LSP would become charged that extends, for large $\tan\beta$, into a funnel at large $m_{1/2}$ where the LSPs annihilate rapidly via direct-channel heavy Higgs resonances. At relatively high values of m_0 , there is a focus-point strip close to the boundary of consistent electroweak symmetry breaking. In light of present experimental constraints from the LHC and elsewhere, plausible values of $\tan\beta$ range between ~ 10 and ~ 55 [53]. Representative $(m_{1/2}, m_0)$ planes for these values of $\tan\beta$ are shown in Fig. 1, and are discussed below in more detail. In this paper, we explore the effects on cosmological light-element abundances of residual late-time $\chi\chi$ annihilations for CMSSM parameters along the coannihilation/funnel and focus-point strips for these reference values of $\tan\beta$, and also remark on additional possibilities in

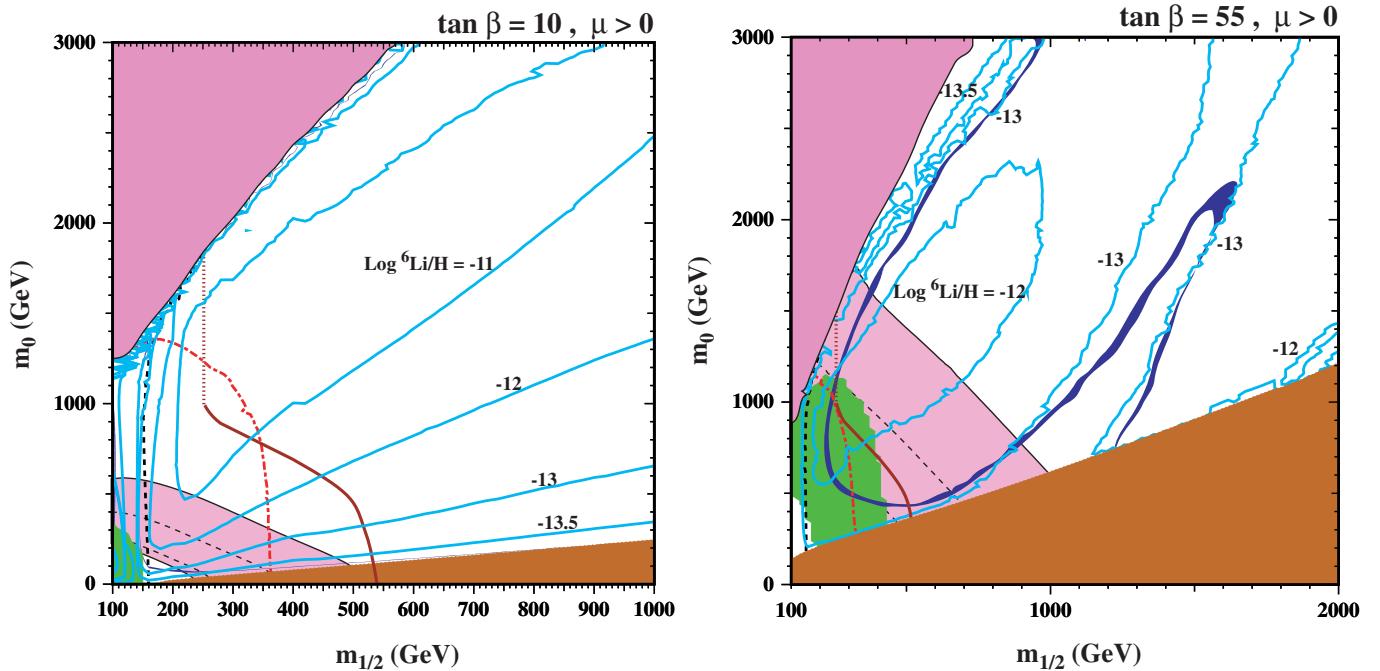


FIG. 1 (color online). Left: The CMSSM $(m_{1/2}, m_0)$ plane for $A_0 = 0$ and $\tan\beta = 10$. Right: The corresponding plane for $\tan\beta = 55$, both with $\mu > 0$, displaying contours of the ${}^6\text{Li}$ abundance including the effects of late-time $\chi\chi$ annihilations. Contours of the ${}^6\text{Li}$ abundance are colored light blue, and the WMAP-compatible [1] strips of parameter space are shaded dark blue. The brown shaded region at large $m_{1/2}$ and small m_0 is excluded because the LSP would be charged, and in the pink shaded region at small $m_{1/2}$ and large m_0 there would be no consistent electroweak vacuum. Also shown are the exclusion by LEP searches for the Higgs boson (red dashed-dotted line) and charginos (black dashed line), and by LHC searches for sparticles (purple solid and dotted line) [59]. The green shaded region at relatively low $m_{1/2}$ is excluded by $b \rightarrow s\gamma$ [69], and the paler pink region is favored by $g_\mu - 2$ at the 1 - (2) - σ level [71], as indicated by the dashed (solid) black lines.

one- and two-parameter generalizations of the CMSSM with nonuniversal soft supersymmetry-breaking contributions to Higgs masses (the NUHM1 and NUHM2) [54–56].

We find that such late annihilations have no significant effect on the cosmological abundances of deuterium, ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$ in any of the CMSSM scenarios studied. However, they may enhance the ${}^6\text{Li}$ abundance by up to 2 orders of magnitude: in some instances, we find ${}^6\text{Li}/\text{H} \sim 10^{-12}$, compared to the value $\sim 10^{-14}$ found in standard homogeneous BBN [57,58]. This possibility arises at low $m_{1/2}$ along the focus-point strips for $\tan\beta = 10$ and 55, and also along the coannihilation strip for $\tan\beta = 55$. The values of ${}^6\text{Li}/\text{H}$ along these strips decrease to $\sim 10^{-13}$ at large $m_{1/2}$, which is typical also of values in the funnel region for $\tan\beta = 55$. On the other hand, we find no substantial enhancement of ${}^6\text{Li}/\text{H}$ along the coannihilation strip for $\tan\beta = 10$ for values of $m_{1/2}$ consistent with the LHC constraints [53,59], though values as large as $\sim 10^{-13}$ might have been reached at lower $m_{1/2}$. The possibility of a large ${}^6\text{Li}$ enhancement is extended in the NUHM1 to a large range of m_0 with $\tan\beta = 10$ and low $m_{1/2}$.

II. THE COSMOLOGICAL LITHIUM PROBLEMS

Cosmologically, the dominant lithium isotope is ${}^7\text{Li}$, whose abundance is commonly inferred from observations of low-metallicity halo dwarf stars. These indicate a plateau of lithium versus metallicity [9], with

$$\left(\frac{{}^7\text{Li}}{\text{H}}\right)_{\text{halo}^*} = (1.23_{-0.16}^{+0.34}) \times 10^{-10}, \quad (1)$$

whereas observations of globular clusters [12] yield somewhat higher values:

$$\left(\frac{{}^7\text{Li}}{\text{H}}\right)_{\text{GC}} = (2.35 \pm 0.05) \times 10^{-10}. \quad (2)$$

For comparison, the standard BBN result for ${}^7\text{Li}/\text{H}$ is $(5.12_{-0.62}^{+0.71}) \times 10^{-10}$ [7], and the difference between this and (1) and (2), constitutes the cosmological ${}^7\text{Li}$ problem. As already mentioned, this might be resolved by new physics beyond the standard model such as late decays of massive gravitinos [46], or by some undocumented standard model effect such as a suitable carbon, boron, or beryllium resonance [47]. The cosmological ${}^7\text{Li}$ problem is not our focus in this paper.

${}^6\text{Li}$ has been observed in some halo stars [60] with $[\text{Fe}/\text{H}] \sim -2$, and with an isotopic ratio that is

$$\left(\frac{{}^6\text{Li}}{{}^7\text{Li}}\right)_{\text{halo}^*} \sim 0.05. \quad (3)$$

These observations are consistent with the results of galactic cosmic-ray nucleosynthesis [58,61,62]—though see below for results at lower metallicity. This confirms that most of the lithium is in the form of ${}^7\text{Li}$, leaving unscathed the cosmological ${}^7\text{Li}$ problem.

However, a recent paper has reported the presence of a similar isotopic abundance in halo stars over a broad range of metallicities that extends to significantly lower values ($[\text{Fe}/\text{H}] \sim -1$ to -3) [14,60]. The inferred plateau ${}^6\text{Li}/\text{H}$ ratio $\sim (6$ to $25) \times 10^{-12}$ is about 1000 times higher than the ${}^6\text{Li}/\text{H}$ ratio predicted by standard homogeneous BBN [57,58], namely, ${}^6\text{Li}/\text{H} \sim 10^{-14}$. The isotopic ratio (3) cannot be explained by conventional galactic cosmic-ray nucleosynthesis, at the lowest metallicities: this is the cosmological ${}^6\text{Li}$ problem. The reliability of the ${}^6\text{Li}$ plateau at very low metallicity has been questioned [20], so the ${}^6\text{Li}$ problem should be taken with a grain of salt. But in any case, these exciting if controversial results demonstrate that ${}^6\text{Li}$ abundances at levels ${}^6\text{Li}/\text{H} \lesssim \text{few} \times 10^{-12}$ are at or near the reach of present observational techniques.

Thus the current observational situation is evolving, but without question is interesting: at the very least, the present results serve as upper limits to primordial ${}^6\text{Li}$, and impose bounds on nonstandard BBN. At most, current data may already point to a primordial ${}^6\text{Li}$ problem which would *demand* new BBN physics, and probe its details. Our focus in this paper is to determine the ${}^6\text{Li}$ production and its observational implications in the context of some of the most popular supersymmetric dark matter scenarios.

It has been proposed that some decaying-particle scenario might produce ${}^6\text{Li}$ at the plateau level with some destruction of ${}^7\text{Li}$ [41,63–67], offering the possibility of solving both lithium problems simultaneously. However, we note that solving the ${}^6\text{Li}$ problem would use up only a small fraction of the ${}^7\text{Li}$ whose destruction would be needed to solve the ${}^7\text{Li}$ problem, leading one to consider separate solutions for the two lithium problems. It is also possible that the ${}^6\text{Li}$ problem might be explained by nucleosynthesis due to cosmological cosmic rays produced at the epoch of structure formation [8,68]. We have previously demonstrated that late-decaying massive gravitinos might resolve the ${}^7\text{Li}$ problem within the CMSSM framework [46]. Here we show that the ${}^6\text{Li}$ problem might, independently and in parallel, have at least a partial supersymmetric solution, via the late annihilations of neutralino LSPs.

III. RESIDUAL LATE-TIME NEUTRALINO ANNIHILATIONS

Assuming that the lightest neutralino χ is the LSP, and that R -parity is conserved, the relic neutralino density is essentially fixed at a freeze-out temperature $T_f \sim m_\chi/20$. At lower temperatures, the local density of neutralinos, n_χ , decreases as the Universe expands (presumably) adiabatically, and subsequent annihilations have very little effect on the dark matter density, but may have important effects on the light-element abundances [25,35,36,48].

The rate per volume of annihilation *events* is

$$q_{\text{ann}} = \frac{1}{2} n_\chi^2 \langle \sigma v \rangle_{\text{ann}}, \quad (4)$$

and so the annihilation event rate *per* χ is

$$\Gamma_{\text{ann}} = \frac{q_{\text{ann}}}{n_\chi} = \frac{1}{2} \langle \sigma v \rangle_{\text{ann}} n_\chi = \frac{1}{2} \langle \sigma v \rangle_{\text{ann}} Y_\chi n_b, \quad (5)$$

and thus the annihilation event rate *per baryon* is

$$\frac{q_{\text{ann}}}{n_b} = \Gamma_{\text{ann}} Y_\chi = \frac{1}{2} \langle \sigma v \rangle_{\text{ann}} Y_\chi^2 n_b, \quad (6)$$

where the χ abundance is

$$Y_\chi = \frac{n_\chi}{n_b} = \frac{m_b}{m_\chi} \frac{\Omega_\chi}{\Omega_b}. \quad (7)$$

At the temperatures of interest here, $T_{\text{BBN}} \lesssim 1 \text{ MeV} \ll m_\chi$, the annihilation rate coefficient $\langle \sigma v \rangle_{\text{ann}}$ is very well approximated as a constant, the value of which depends on the specific underlying supersymmetry model. In (4), we are interested in only the s -wave part of the cross section whereas a combination of s - and (mainly) p -wave cross sections is constrained by the requirement of reproducing the present dark matter density within errors.

The annihilations inject nonthermal standard model particles, including both electromagnetic as well as hadronic species. For electromagnetic products we need only track the total energy injected per annihilation. For nonthermal hadrons (nucleons) $h = n, p$, we calculate the spectrum $Q_h^{\text{ann}}(\epsilon)$ of annihilation products, normalized such that $\int Q_h^{\text{ann}}(\epsilon) d\epsilon = B_h$, the expected number of h created per annihilation. Then the injection/source rate of h due to annihilations, per unit volume, per unit time, and per unit kinetic energy ϵ , is

$$\frac{d\mathcal{N}_{h,\text{inj}}^{\text{ann}}}{dV dt d\epsilon} = q_{\text{ann}} Q_h^{\text{ann}}(\epsilon). \quad (8)$$

These particles then lose energy as they propagate in the cosmic plasma. The propagated spectrum of nonthermal particles must be calculated, and this produces the non-thermal reactions on ambient thermal light nuclides that perturb BBN.

The effect of nonthermal particle injection in BBN has been well studied in the case of decays of some unstable particle X . Much of the physics carries over here, once one makes the appropriate substitution of abundances $n_X \rightarrow n_\chi$ and of annihilation rate for decay rate: $\Gamma_X = \tau_X^{-1} \rightarrow \Gamma_{\text{ann}}$. After injection, the nonthermal particle propagation remains the same, and we treat this as in [45]. We also adopt the same set of nonthermal BBN reactions which we include in the same manner, making the appropriate substitution of annihilations for decays.

A. Order-of-magnitude calculation

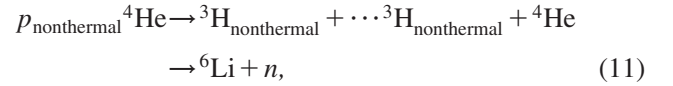
Before turning to our numerical results, we first present an order-of-magnitude calculation that illustrates the basic physics in play, and also serves as a check on our numerical results. The total number of annihilation events per baryon occurring after a given time t_i is the time integral of Eq. (6):

$$\mathcal{N}_{\text{ann}} = \int_{t_i} \Gamma_{\text{ann}} Y_\chi dt \sim Y_\chi^2 \langle \sigma v \rangle_{\text{ann}} n_b(t_i) t_i \quad (9)$$

$$= 5 \times 10^{-9} \text{ events/baryon} \left(\frac{\langle \sigma v \rangle_{\text{ann}}}{10^{-26} \text{ cm}^3 \text{ s}^{-1}} \right) \left(\frac{300 \text{ GeV}}{m_\chi} \right)^2. \quad (10)$$

Our fiducial values correspond to $t_i \sim 100$ sec and $T_i \sim 100$ keV, since this marks the epoch when the ${}^4\text{He}$ abundance becomes large.

Given this number of annihilations per baryon, we now need the branching for ${}^6\text{Li}$ production per annihilation. As discussed earlier, nonthermal particles from annihilations or decays produce ${}^6\text{Li}$ as a secondary by-product of ${}^4\text{He}$ spallation,



and similarly with nonthermal ${}^3\text{He}$; nonthermal D also contributes but is subdominant. As discussed for the late-decay case in [45], each late annihilation produces a mass-3 abundance increment $\Delta Y(^3A)$ which is given in the thin target limit by

$$\Delta Y(^3A) \sim \mathcal{N}_{\text{ann}} B_N \frac{\sigma(N\alpha \rightarrow ^3A + \dots)}{\sigma(N\alpha \rightarrow \text{inelastic})}, \quad (12)$$

where $B_N \sim 0.4$ is the number of nucleons per annihilation. Typically this increases the mass-3 abundance by an amount $\Delta Y(^3A) \sim 10^{-9} \ll Y_{\text{BBN}}(^3A) \sim 10^{-5}$, i.e., much smaller than the standard primordial abundance, and thus we do not expect substantial perturbations to mass-3 nuclides, or to D, which has similar cross sections, or to ${}^4\text{He}$.

The energetic $A = 3$ particles are slowed in the cosmic plasma by ionization and related losses, with a range $R_3 = \int (dE/dX)^{-1} dE$, where dE/dX is the loss rate per thickness $dX = \rho_b dx$ in $[\text{g}/\text{cm}^2]$. Hence the stopping length is R_3/ρ_b . The fraction of mass-3 nuclides which produce ${}^6\text{Li}$ before stopping is this stopping length divided by the mean free path for ${}^6\text{Li}$ production, namely,

$$f(^3A \rightarrow {}^6\text{Li}) \sim n_\alpha \sigma(^3A\alpha \rightarrow {}^6\text{Li}) \frac{R_3}{\rho_b} \sim Y_\alpha \frac{\sigma(^3A\alpha \rightarrow {}^6\text{Li}) R_3}{m_b} \sim 7 \times 10^{-4}. \quad (13)$$

Collecting these results, the residual late-time annihilation contribution to the ${}^6\text{Li}$ abundance per baryon is

$$\Delta Y({}^6\text{Li}) = \Delta Y(^3A) f(^3A \rightarrow {}^6\text{Li}) \quad (14)$$

$$\sim B_N Y_\chi^2 \langle \sigma v \rangle_{\text{ann}} Y_\alpha \frac{\sigma(N\alpha \rightarrow ^3A + \dots)}{\sigma(N\alpha \rightarrow \text{inelastic})} \times \frac{\sigma(^3A\alpha \rightarrow {}^6\text{Li}) R_3}{m_b} n_b(t_i) t_i \quad (15)$$

$$= 7 \times 10^{-13} \left(\frac{\langle \sigma v \rangle_{\text{ann}}}{10^{-26} \text{ cm}^3 \text{ s}^{-1}} \right) \left(\frac{300 \text{ GeV}}{m_\chi} \right)^2. \quad (16)$$

The numerical results given above are evaluated for $t_i = 100$ sec, and we also take $R_3 = 1 \text{ g/cm}^2$ and $\sigma({}^3\text{A}\alpha \rightarrow {}^6\text{Li}) = 30$ mb. This formula gives the scaling $\Delta Y({}^6\text{Li}) \propto B_N \langle \sigma v \rangle_{\text{ann}} (\Omega_\chi / m_\chi)^2$ which we verify with our full numerical results, and the normalization agrees to within a factor ~ 2 . This agreement lends confidence in our code and our understanding of the physics.

B. Numerical results

We turn now to our full numerical results. In order to establish the context for our subsequent analysis of the possible annihilation effects along the strips in CMSSM parameter space that are compatible with WMAP and other constraints on the present-day dark matter density, we first discuss the full CMSSM $(m_{1/2}, m_0)$ planes shown in Fig. 1. The light blue lines are contours of the ${}^6\text{Li}$ abundance, and the relic density is WMAP-compatible [1] along the dark blue strips, assuming that the lightest neutralino χ is the LSP and is stable, as in R -conserving models. There would be no consistent electroweak vacuum in the pink shaded region at small $m_{1/2}$ and large m_0 , the lighter $\tilde{\tau}$ would be the LSP in the brown shaded region in the lower right corner of the figure, and the green shaded region at relatively low $m_{1/2}$ is excluded by $b \rightarrow s\gamma$ decay [69].³ Regions to the left of the red dash-dotted (black dashed) (purple) line are excluded by searches for the Higgs boson at LEP (charginos) (LHC searches for sparticles) [59]. In the paler pink region the supersymmetric contribution remedies the discrepancy between the experimental measurement of $g_\mu - 2$ and theoretical calculation within the standard model [70,71] using low-energy e^+e^- data, with $1-(2-)\sigma$ [71] consistency being indicated by the dashed (solid) black lines.

We see in the left panel of Fig. 1 showing the $(m_{1/2}, m_0)$ plane for $\tan\beta = 10$ that most of the lower (coannihilation) WMAP strip has ${}^6\text{Li}/\text{H} < 10^{-13}$, whereas the upper (focus-point) strip may have ${}^6\text{Li}/\text{H}$ as large as 10^{-12} . There is a region where ${}^6\text{Li}/\text{H}$ seems able to exceed 10^{-11} , but this is well inside the region between the WMAP strips, where the relic χ is overdense according to conventional big bang cosmology. In the right panel of Fig. 1 for $\tan\beta = 55$, we see that ${}^6\text{Li}/\text{H} \sim 10^{-13}$ along the coannihilation strip and in the funnel region at large $m_{1/2}$ and m_0 where the relic density is brought into the WMAP range by rapid annihilations through direct-channel H/A resonances, though somewhat larger values of ${}^6\text{Li}/\text{H}$ are

³According to conventional big bang cosmology and in the absence of R violation, the LSP χ would be overdense in the regions between the WMAP strips. It would be underdense in the regions between these strips and the pink and brown shaded regions.

possible at small $m_{1/2}$. Along the focus-point strip, we see that values of ${}^6\text{Li}/\text{H} \sim 10^{-12}$ are also possible at small $m_{1/2}$, falling to $\sim 10^{-13}$ at large $m_{1/2}$. The range ${}^6\text{Li}/\text{H} \sim 10^{-11}$ is never attained for $\tan\beta = 55$, even in the overdense region of the $(m_{1/2}, m_0)$ plane.

We now focus on the WMAP strips in Fig. 1. The left panel of Fig. 2 displays the figure-of-merit combination $\langle \sigma v \rangle_{\text{ann}} (\Omega_\chi h^2 / m_\chi)^2$ as a function of $m_{1/2}$ along the WMAP strips in the CMSSM for $\tan\beta = 10$ and 55. We see that $\langle \sigma v \rangle_{\text{ann}} (\Omega_\chi h^2 / m_\chi)^2$ along the coannihilation strip for $\tan\beta = 10$ is much smaller than along the other strips. This can be understood from the fact that along this strip several coannihilation processes involving sleptons contribute to reducing the relic χ density into the WMAP range, and that their relative contributions become more important as $m_{1/2}$ increases. In addition, along this strip the s -wave cross section relevant during BBN is significantly smaller than the p -wave cross section that dominates during freeze-out. These coannihilation processes are less important along the corresponding strip for $\tan\beta = 55$, and unimportant in the funnel region and along the focus-point strips, where the s -wave cross section becomes comparable to the total cross section. Hence, along these strips $\langle \sigma v \rangle_{\text{ann}}$ must be larger, in order to bring the relic density down into the WMAP range unaided. Note that the precipitous drop in the cross section at low $m_{1/2}$ for the $\tan\beta = 10$ focus-point strip occurs as we pass below the W^+W^- threshold.

The $\chi\chi$ annihilations feed many different particle species into the cosmological background, initially with non-thermal spectra that we model using PYTHIA [72]. The only species that survive long enough to interact significantly with background nuclei are protons and neutrons (and their antiparticles) and photons. The former are far more important for the nuclear reactions of interest here, so we focus on their numbers and spectra. The right panel of Fig. 2 displays the numbers of protons (solid or dotted lines) and neutrons (dashed or dash-dotted lines) produced per annihilation event, again along the WMAP strips for $\tan\beta = 10, 55$ discussed previously. We see that in general the numbers of protons and neutrons increase significantly as $m_{1/2}$ increases, with some bumps as new annihilation thresholds are crossed.

Figure 3 of [73] displays the most important branching fractions for final states in $\chi\chi$ annihilations as functions of $m_{1/2}$ along the WMAP strips for $\tan\beta = 10$ and 55, which include the final states $\tau^+\tau^-$, $b\bar{b}$, W^+W^- , $t\bar{t}$, Zh , and ZZ . Of these, the $\tau^+\tau^-$ final state clearly yields no baryons, while the numbers of baryons yielded by the final states W^+W^- , Zh , and ZZ are all independent of the annihilation center-of-mass energy $2m_\chi$. Only the $b\bar{b}$ and $t\bar{t}$ final states yield numbers of baryons that increase with the annihilation center-of-mass energy.

Figure 3 displays the spectra of protons (upper panel) and neutrons (lower panel) for the W^+W^- , Zh , and ZZ

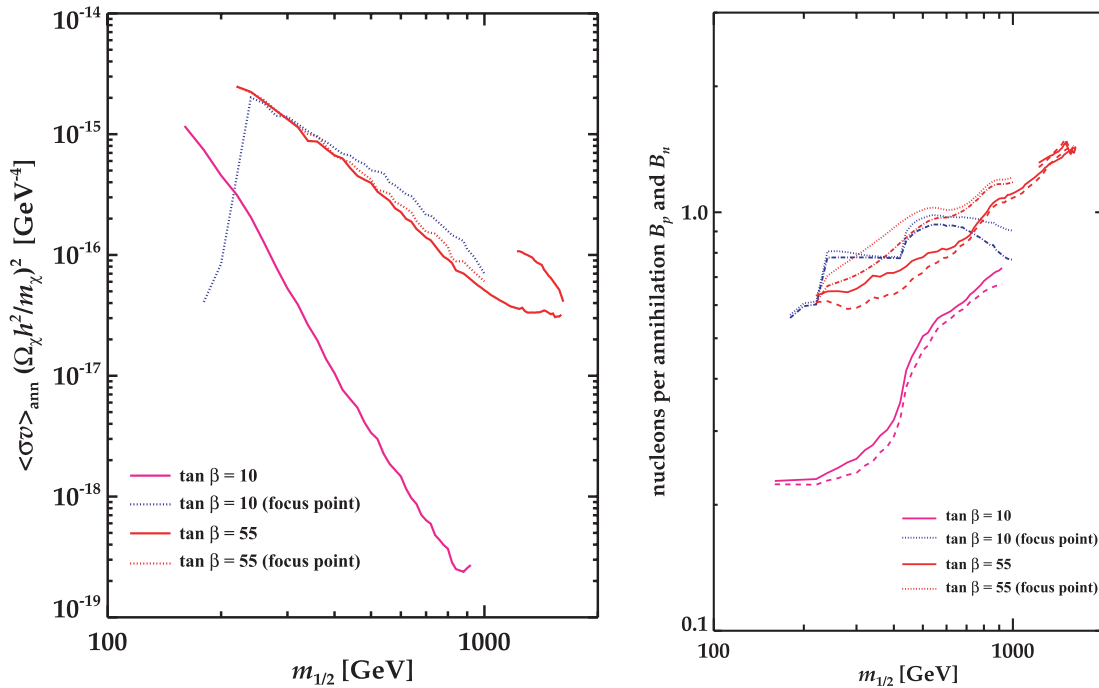


FIG. 2 (color online). Left: The figure of merit for the late-time annihilation rate, namely, the velocity-averaged $\chi\chi$ annihilation cross section divided by the square of the neutralino mass, $\langle\sigma v\rangle_{\text{ann}}(\Omega_\chi h^2/m_\chi)^2$, along the WMAP strips in the coannihilation, focus-point, and funnel regions for $\tan\beta = 10, 55$, $A_0 = 0$, and $\mu > 0$, as functions of $m_{1/2}$. We see that the figure of merit along the $\tan\beta = 10$ coannihilation strip is much smaller than along the other strips, and that all decrease rapidly as $m_{1/2}$ increases. Right: The numbers $B_{p,n}$ of protons (solid or dotted lines) and neutrons (dashed or dash-dotted lines) produced per $\chi\chi$ annihilation event, as calculated using PYTHIA, along the WMAP strips in the coannihilation, focus-point, and funnel regions for $\tan\beta = 10, 55$, $A_0 = 0$, and $\mu > 0$, as functions of $m_{1/2}$. We see that in general the numbers increase significantly as $m_{1/2}$ increases.

final states for $m_\chi = 250$ GeV, and for the $b\bar{b}$ final state for $m_\chi = 100, 250$ GeV, all calculated using PYTHIA. We display the number of protons or neutrons per unit of the parameter $x \equiv \sqrt{E_i^2 - m_i^2}/m_\chi$, where $i = p, n$. The proton and neutron spectra are almost identical. They differ in the small x region primarily because of the difference of m_p and m_n . We also see that the W^+W^- , Zh , and ZZ final states yield rather similar spectra, with the spectrum from the Zh final state rising slightly higher. The spectra of baryons from $b\bar{b}$ final states rise from being lower at $m_\chi = 100$ GeV to being higher at $m_\chi = 250$ GeV.

Together with Fig. 3 of [73], Fig. 3 enables us to understand the salient features of the baryon production rates shown in the right panel of Fig. 2. The large branching fraction for $\tau^+\tau^-$ suppresses baryon injection along the coannihilation strip for $\tan\beta = 10$, particularly for small $m_{1/2}$ but less so for large $m_{1/2}$ where the W^+W^- branching fraction grows.

Following their injection into the primordial plasma, some of the nucleons cause spallation of ${}^4\text{He}$, yielding $A = 3$ nuclei as discussed above. These are produced with large, nonthermal energies and subsequently thermalize, but may previously induce $t(\alpha, n){}^6\text{Li}$ or ${}^3\text{He}(\alpha, p){}^6\text{Li}$

reactions. Figure 4 displays the enhancement of the cosmological ${}^6\text{Li}$ abundance that we find along the WMAP strips discussed above. The homogeneous BBN value $\sim 10^{-14}$ is attained at large $m_{1/2}$ along the WMAP coannihilation strip for $\tan\beta = 10$, but much larger values are possible along the other WMAP strips,⁴ where we find ${}^6\text{Li}/\text{H} \sim 10^{-13}$ at large $m_{1/2}$ to 10^{-12} at small $m_{1/2}$. We have found that the enhancement of ${}^6\text{Li}$ scales very closely with the combination $B_N\langle\sigma v\rangle_{\text{ann}}/m_\chi^2$, as was to be expected.

We recall that the enhancement of ${}^6\text{Li}/\text{H}$ that would be required for consistency with (3) is by a factor ~ 1000 , rather than the factor of up to ~ 100 that we find here. However, as we have noted there remains a question as to whether or not the plateau ratio of 0.05 should be attributed to ${}^6\text{Li}$. The abundance of ${}^6\text{Li}$ we find here is potentially observable and would in fact be seen as a plateau extending to low metallicities. Optimistically, we could envision ${}^6\text{Li}$ observations playing a role in discerning between supersymmetric models. In any case, we regard the enhancement

⁴The region of enhanced ${}^6\text{Li}$ along the WMAP coannihilation strip for $\tan\beta = 10$ with $m_{1/2} < 400$ GeV is now excluded by the unsuccessful LHC searches for supersymmetry [53,59].

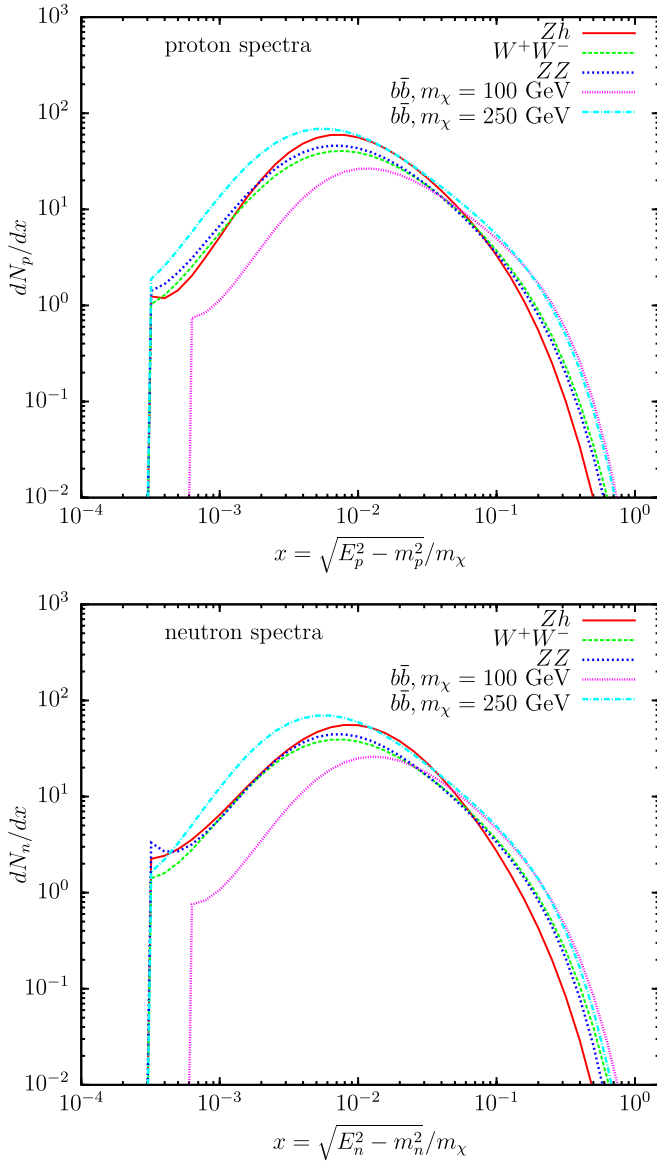


FIG. 3 (color online). The spectra of protons (upper panel) and neutrons (lower panel) injected by $\chi\chi$ annihilations into the Zh , W^+W^- , ZZ , and $b\bar{b}$ (for $m_\chi = 100$ and 250 GeV) final states, as calculated using PYTHIA.

we find as already an interesting contribution to the analysis of the ${}^6\text{Li}$ problem.

C. Exploration of nonuniversal Higgs models

It is quite possible that some modifications of the CMSSM might yield even greater enhancements of the ${}^6\text{Li}$ abundance. To be successful in this respect, it is apparent from (16) that such a model would require a relatively large annihilation cross section $\langle\sigma v\rangle_{\text{ann}}$ combined with a small value of m_χ , as in the focus-point region of the CMSSM. There, the relatively large value of $\langle\sigma v\rangle_{\text{ann}}$ is made possible by the admixture of a Higgsino component

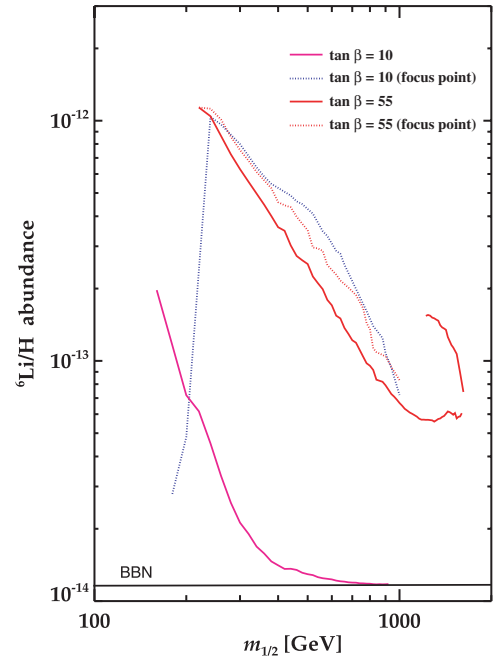


FIG. 4 (color online). The enhancement of the cosmological ${}^6\text{Li}$ abundance as a function of $m_{1/2}$ along the WMAP strips discussed in the text. The standard homogeneous BBN value $\sim 10^{-14}$ is attained at large $m_{1/2}$ along the WMAP coannihilation strip for $\tan\beta = 10$.

in the χ , and along this strip the low value of m_χ is consistent with the LHC and other constraints [53].

In an initial probe of other possibilities for a large enhancement of the ${}^6\text{Li}$ abundance, we have explored the NUHM1 model, in which the soft supersymmetry-breaking contributions to the Higgs masses have a common value that differs from m_0 . It is known that in this model the Higgsino component in the LSP χ may be enhanced at values of $m_{1/2}$ and m_0 away from the focus-point region, thanks to a level-crossing transition at particular values of $\mu/m_{1/2}$ [55]. In the CMSSM, the value of μ is generally fixed by applying the conditions for a consistent electroweak vacuum. However, in the NUHM1 the value of $\mu/m_{1/2}$ can be adjusted by varying the degree of nonuniversality in the soft supersymmetry-breaking Higgs masses, enabling a WMAP-compatible relic density to be found in models with values of $(m_{1/2}, m_0)$ different from those allowed in the CMSSM.

We have explored the conditions under which such transition regions in the NUHM1 may yield an enhancement of ${}^6\text{Li}/\text{H}$ similar to or (possibly) greater than the value $\sim 10^{-12}$ attainable in the CMSSM in the focus-point region. To this end, we have studied over a dozen NUHM1 parameter planes. In no case did we find enhancements of ${}^6\text{Li}/\text{H}$ significantly larger than in the CMSSM (and this is also the case in some planes we explored in the NUHM2, in which both Higgs soft supersymmetry-breaking masses are treated as free, nonuniversal parameters).

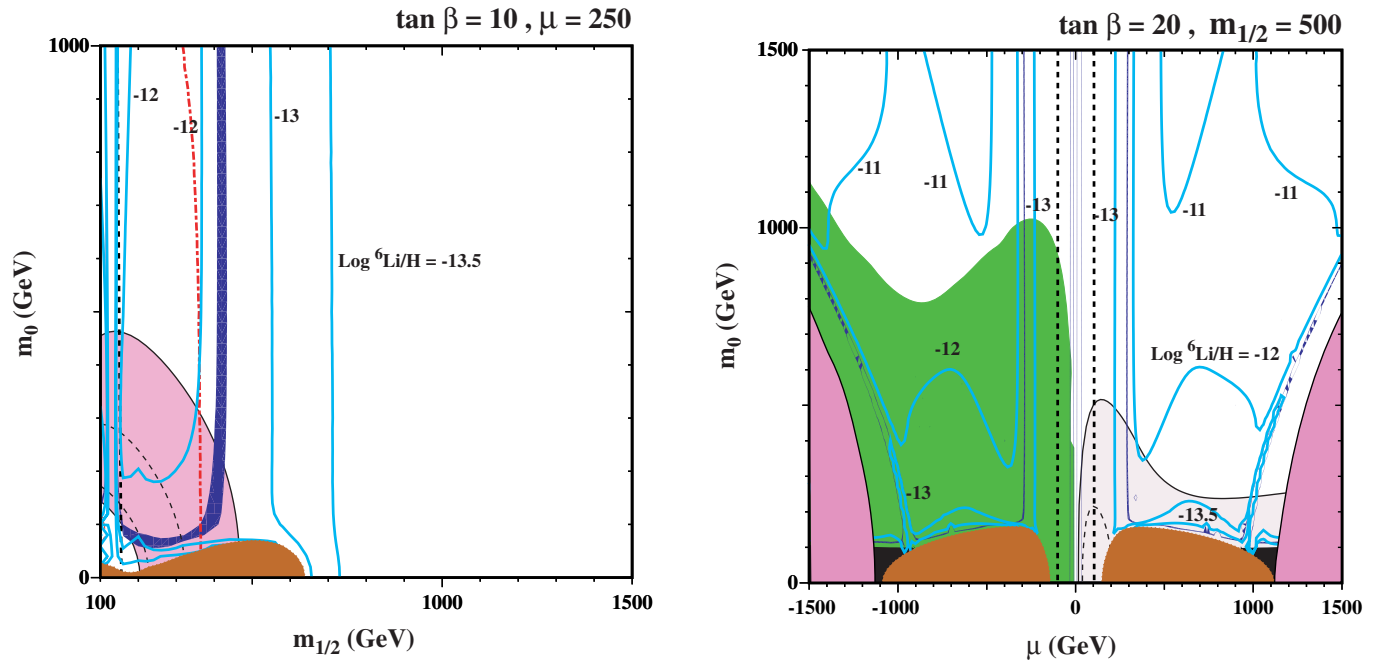


FIG. 5 (color online). Left: The NUHM1 $(m_{1/2}, m_0)$ plane for $\mu = 250$ GeV, $A_0 = 0$, and $\tan\beta = 20$. Right: The NUHM1 (μ, m_0) plane for $m_{1/2} = 500$ GeV, $A_0 = 0$, and $\tan\beta = 20$, displaying contours of the ${}^6\text{Li}$ abundance including the effects of late-time $\chi\chi$ annihilations. Contours of the ${}^6\text{Li}$ abundance are colored light blue, and the WMAP-compatible strips of parameter space are shaded dark blue. The other shaded regions and lines have the same meanings as in Fig. 1.

Figure 5 shows results in a couple of selected NUHM1 parameter planes. The left panel shows an $(m_{1/2}, m_0)$ plane for $\tan\beta = 10$, $A_0 = 0$, and fixed $\mu = 250$ GeV. In this case, there is a near-vertical WMAP-compatible strip in a transition region at $m_{1/2} \sim 400$ GeV. This transition strip is compatible with the LEP Higgs constraint, and the upper part of the strip above $m_0 \sim 700$ GeV is compatible with the constraints imposed by LHC searches for sparticles. We see that the ${}^6\text{Li}/\text{H}$ ratio is remarkably constant at $\sim 5 \times 10^{-13}$ along this strip. It would be possible to increase ${}^6\text{Li}/\text{H}$ to $\sim 10^{-12}$ by choosing μ somewhat smaller, in which case the WMAP-compatible strip would be at smaller $m_{1/2}$. In that case, the LHC would enforce a stronger lower limit on m_0 , closer to the CMSSM focus-point strip. On the other hand, larger values of μ yield small values of the ${}^6\text{Li}$ abundance, and we find no increase in the ${}^6\text{Li}$ abundance for larger $\tan\beta$.

The right panel of Fig. 5 displays a (μ, m_0) plane in the NUHM1 for $\tan\beta = 20$, $A_0 = 0$, and fixed $m_{1/2} = 500$ GeV, at the lower end of the range allowed by the LHC and other data for $m_0 < 1000$ GeV. In this case, there are near-vertical WMAP-compatible strips in the transition region at $|\mu| \sim 300$ GeV, where ${}^6\text{Li}/\text{H}$ approaches 10^{-12} . There is also a WMAP-compatible strip near $\mu \sim 1000$ GeV that parallels the region without a consistent electroweak vacuum (here caused by $m_A^2 < 0$), where ${}^6\text{Li}/\text{H}$ is again somewhat below 10^{-12} . Connecting these two regions is a coannihilation segment at $m_0 \sim 100$ – 200 GeV where the ${}^6\text{Li}$ abundance is relatively small.

We have explored several other NUHM1 (μ, m_0) planes, finding that increasing $m_{1/2}$ decreases the attainable value of ${}^6\text{Li}/\text{H}$. We have also explored several other projections of the NUHM1 and NUHM2, including $(m_A, m_{1/2})$, (m_A, m_0) , (μ, m_A) , and (m_1, m_2) planes, without finding values of ${}^6\text{Li}/\text{H}$ above 10^{-12} .

IV. SUMMARY AND CONCLUSIONS

We have demonstrated in this paper that in both the CMSSM and the NUHM1 it is possible that late neutralino LSP annihilations may enhance significantly the cosmological ${}^6\text{Li}$ abundance, without affecting significantly the BBN abundances of the other light element deuterium, ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$ [36]. This enhancement may be up to 2 orders of magnitude, yielding ${}^6\text{Li}/\text{H} \sim 10^{-12}$ compared to the BBN value $\sim 10^{-14}$.

As we have shown, this enhancement occurs typically when the neutralino LSP is relatively light and has a large annihilation cross section, as occurs when the LSP contains a strong Higgsino admixture. This phenomenon appears, in particular, in the focus-point region of the CMSSM and in transition regions of the NUHM1.

While interesting for the debates on the astrophysical lithium abundances, this enhancement falls short of resolving by itself the cosmological ${}^6\text{Li}$ problem. Further work could include a more exhaustive study of other supersymmetric models, to see whether they could reconcile a larger enhancement with the available theoretical, phenomenological, experimental, and cosmological constraints.

Alternatively, is it possible that the height of the ${}^6\text{Li}$ plateau may receive contributions from other sources such as an early generation of stars, or might the height of the ${}^6\text{Li}$ plateau be overestimated, perhaps because of convective processes involving ${}^7\text{Li}$ [20]? It is clearly desirable to pin down more definitively the magnitude of the ${}^6\text{Li}$ problem by establishing more solidly the existence and height of the inferred ${}^6\text{Li}$ plateau in halo stars. However, it already seems that a substantial enhancement of the standard homogeneous BBN prediction for ${}^6\text{Li}/\text{H}$ might be an interesting signature of supersymmetric models.

Finally, our work illustrates in detail the more general point that ${}^6\text{Li}$ production should play a role in—and thus probe—*any* WIMP dark matter scenario involving hadronic annihilation products [31]. Specifically, we have seen that ${}^6\text{Li}$ production is essentially guaranteed provided there are nonthermal nucleons injected with kinetic energies \gtrsim few MeV. We also find that the level of ${}^6\text{Li}$ abundance due to residual annihilations is model-dependent, in our case spanning a range from 100 times the standard yield down to an unobservable perturbation to this level. The lessons for WIMP modelers

would seem to be that ${}^6\text{Li}$ observations already provide important constraints which one must test against, and that a confirmed detection of primordial ${}^6\text{Li}$ —particularly if it is above the standard level—will likely shed light on the details of the nonstandard physics which produced it.

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