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THE SUBSTRUCTURE OF THE WEAK BOSONS AND THE WEAK MIXING ANGLE *)

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A B S T R A C T

In models in which the weak intermediate bosons are bound states consisting of spin $\frac{1}{2}$ constituents the effective neutral current mixing angle $\sin^2\theta_W$ is related to the W wave function at the origin. Dynamical constraints for the bound state structure of the W boson follow. Although the observed value of $\sin^2\theta_W$ is rather large, it seems possible to obtain the observed value in a dynamical mixing scheme. The weak bosons are extended objects, whose sizes are of the order of 10^{-16} cm.

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Recently several authors have speculated about the internal structure of leptons, quarks, and the intermediate bosons W and Z. Especially it has been stressed that the observed weak interactions may be nothing but a manifestation of the bound state dynamics of the lepton-quark constituents[1)-3)], just like the nuclear forces, which within QCD are interpreted as indirect manifestations of the quark-gluon dynamics. In this approach, discussed especially in ref. (2), the W- and Z- bosons are composite objects, while the photon is elementary. The experimental data on the neutral current interaction require a mixing between the photon and the W^3 boson (the neutral partner of W^+ and W^-), which in the standard SU(2) x U(1) scheme is caused by the spontaneous symmetry breaking.

Within our approach[2)] this mixing is due the W^3- γ transitions, generated dynamically like the ρ-γ transitions in QCD (for an early discussion, based on vector meson dominance, see ref. (4)). The magnitude of $\sin^2\theta_W$ (θ_W: SU(2) x U(1) mixing angle) is directly related to the strength of the γ-W_3 transition. The latter is determined by the electric charges of the W-constituents and by the W wave function near the origin. In this paper we should like to study the implications of the observed value of $\sin^2\theta_W$ for the dynamics of the W-constituents.

We suppose following ref. (2) that in the absence of electromagnetism (e = 0) the weak interactions are described by an $SU(2)^W$ invariant theory. At low energies the weak forces are dominated by the exchange of the lowest lying vector mesons W^+, W^- and W^3, which form an $SU(2)^W$ triplet. At energies of the order of M_W or larger the vector meson pole approximation is no longer valid; excited states which at sufficiently high energies can be described by a continuum of constituent pairs become relevant.

The elementary constituents are denoted by "haplons"[2)]. The dynamics of haplons is assumed to be given by a confining gauge theory ("QHD"), based on the hypercolor group SU(n) (for simplicity we use SU(n), where n is yet unspecified; the extension to other groups is easily made).

The following two classes of models are considered:

A) The W-bosons consist of two color singlet fermions, transforming as n-representations of $SU(n)_{hypercolor}$, they are denoted by α and β. We have $Q(\alpha) = +1/2$, $Q(\beta) = -1/2$ (Q: electric charge), $W^+ = (\bar{\beta}\alpha)$, $W^- = (\bar{\alpha}\beta)$. The W-bosons are hypercolor singlets. Such a situation can arise e.g. in the models discussed in ref (3) (see also ref. (5)).

B) The W-bosons consist of color triplet fermions α and β, transforming as n under $SU(n)_{hypercolor}$. We have $Q(\alpha) = -1/2$, $Q(\beta) = +1/2$, $W^+ = (\bar{\alpha}\beta)$, $W^- = (\bar{\beta}\alpha)$[2].

Following ref. (2), we suppose that in the absence of electromagnetism the weak interactions are mediated by the triplet (W^+, W^-, W^3), where $M(W^+) = M(W^-) = M(W^3) = 0\ (\Lambda_H)$.

After the introduction of the electromagnetic interaction the photon and the W^3 - boson mix. We denote the strength of this mixing by a parameter λ, following ref. (6, 7), which is related to g (W-fermion coupling constant) and the effective value of $\sin^2\theta_W$ as follows:

$$\sin^2\theta_W = \frac{e}{g} \cdot \lambda \tag{1}$$

Furthermore one has:

$$M_W = g \cdot 123 \text{ GeV}$$

$$M_Z^2 = \frac{M_W^2}{1-\lambda^2} \tag{2}$$

The mixing parameter λ is determined by the decay constant F_W (or f_W) of the W-boson (see Fig. (1)), which we define in analogy to the decay constants of the ρ_0-meson (F_ρ, f_ρ respectively):

$$<o|j_\mu^3|W^3> = <o|\frac{1}{2}(\bar{\alpha}\gamma_\mu\alpha - \bar{\beta}\gamma_\mu\beta)|W^3> = \varepsilon_\mu M_W^2/f_W = \varepsilon_\mu M_W F_W \tag{3}$$

One finds:

$$\lambda = \frac{e}{f_W} = e \cdot \frac{F_W}{M_W} \tag{4}$$

In the table we have displayed the numerical values for F_W as a function of g. F_W is a rather sensitive function of g; for g = 0.75 one finds F_W = 166 GeV; for g = 1.2 one obtains F_W = 425 GeV.

In order to study the physics of the bound state structure of the weak bosons in more detail we express the decays constant F_W in terms of the bound state wave function of the weak bosons. We use a non-relativistic wave function, which, of course, cannot be but a very crude approximation.

One has, leaving out irrelevant Lorentz indices:

$$\text{A:} \quad |W^3> = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{n}} \sum_{i=1}^{n} (\bar{\alpha}_i \alpha_i - \bar{\beta}_i \beta_i) \phi\ (x) \tag{5}$$

(ϕ (x) coordinate space wave function, i: hypercolor index, n: number of hypercolors)

$$\text{B:} \quad |W^3> = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{n}} \frac{1}{\sqrt{3}} \sum_{i=1}^{n} \sum_{j=1}^{3} (\bar{\alpha}_{i,j} \alpha_{i,j} - \bar{\beta}_{i,j} \beta_{i,j})\ \phi\ (x) \tag{6}$$

(j: color index).

The matrix element (3) can be written as

$$<o|j_\mu{}^3|W^3> = \varepsilon_\mu \sqrt{n} \cdot \gamma \sqrt{2M_W} \cdot \phi\ (o) = \varepsilon_\mu \cdot M_W F_W \tag{7}$$

$$F_W = \sqrt{n} \cdot \gamma \cdot \sqrt{2/M_W}\ \phi\ (o)$$

where γ = 1 in case A and $\gamma = \sqrt{3} = \sqrt{n_c}$ in case B. Using eqs. (1), (4) and (7), one finds

$$\sin^2\theta_W = \frac{e^2}{g} \sqrt{n} \cdot \gamma \sqrt{2/M_W{}^3}\ \phi\ (o), \tag{8}$$

$$= e^2/g \cdot F_W / M_W.$$

e.g. $\sin^2\theta_W$ is proportional to the coordinate space wave function of the W-boson at the origin. Taking for example a constant coordinate space wave function, the inverse radius Λ_W of the W-boson and $\phi(o)$ are related: $|\phi(o)|^2 = (4/3\pi)^{-1} \cdot \Lambda_W^3$.

It is useful to introduce the dimensionless quantity $x = (\Lambda_W / M_W)$, and one obtains

$$\sin^2\theta_W = e^2/g \cdot \sqrt{2n} \quad \gamma(\tfrac{4}{3}\pi)^{-1/2} \quad x^{3/2} . \tag{9}$$

For example for $g = 1$, $\gamma = \sqrt{3}$ (case B) and $n = 4$ one finds $x = 1.0$, i.e. $\Lambda_W \approx M_W$. Unless n is very large ($n > 10$), x is not much smaller than one.

It is instructive to compare the structure of the W-bosons with the one of the ρ-meson. In the ρ-case one has

$$\lambda_\rho = \frac{e}{f_\rho} = 0.054, \tag{10}$$

taking $f_\rho = 5.6$. This leads to

$$x_\rho = \Lambda_\rho / m_\rho \approx 0.28. \tag{11}$$

Thus the structure of the ρ-meson differs substantially from the structure of the W-boson. Using eq. (1) and $g = 1$, one would find $\sin^2\theta_W = 0.016$.

We conclude: if the bound state dynamics of the W-bosons were similar to the quark dynamics inside the ρ-meson, the effective SU(2) x U(1) mixing parameter $\sin^2\theta_W$ would be more than an order of magnitude smaller than observed. In order to accommodate the observed value of $\sin^2\theta_W$, the decay constant F_W and the inverse size Λ_W must be of the same order as the mass of the W-boson. This suggests a similarity of the W bound state dynamics to the bound state dynamics of the π-meson in QCD. In the case of the π-meson one has $F_\pi \approx M_\pi$; the inverse radius of the π-meson, defined analogously to Λ_W, and the pion mass are of the same order, while in the case of the ρ-meson the radius is about four times larger than the inverse mass. One may conclude that the mass of the W-boson is anomalously small, compared to its inverse size.

The pion decay constant is about equal to the Λ-parameter of QCD. It may be that the corresponding parameter in the hypercolor dynamics Λ_H is of the same order as the W-mass, i.e. 100 ... 160 GeV.

Thus far we have assumed that the W-bosons consist of spin 1/2 constituents. In some models spin zero constituents are used as W-constituents (see e.g. ref. (1)). We should like to argue that such models can be ruled out, on the basis of the following argument. According to eq. (8) the effective SU(2) x U(1) mixing parameter $\sin^2\theta_W$ is proportional to the wave function at the origin $\phi(o)$. If the W-bosons consist of spin zero constituents, the coordinate space wave function must be a p-wave, i.e. $\phi(o) = 0$. Thus in a nonrelativistic approach one finds $\sin^2\theta_W = 0$. Even if we allow for fairly large relativistic corrections, it seems impossible to us to obtain the observed large value of $\sin^2\theta_W$ in such models.

Conclusions: We conclude that it seems possible to obtain the observed, relatively large value of $\sin^2\theta_W$ in a dynamical mixing scheme, based on models in which the W-bosons consist of spin 1/2 constituents. (In the case of spin zero constituents serious problems arise.) However it is required that the mass and the inverse size of the W are about of the same order of magnitude, unlike the situation in QCD, where the inverse size of the ρ-meson is smaller than the mass. Thus the spatial extension of the W-bosons is given by $\Lambda_W^{-1} \approx 10^{-16}$ cm. The sizes of the leptons and quarks are expected to be roughly of the same order of magnitude, i.e. in the range between 10^{-16} and 10^{-17} cm. If the extensions of leptons and quarks are indeed in the range given above, one expects to find manifestations of the lepton-quark substructure in the experiments carried out in the near future, e.g. at the CERN collider.

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g	0.75	0.9	1	1.1	1.2
λ	0.54	0.65	0.73	0.80	0.87
M_W	92	111	123	135	148
M_Z	110	146	179	225	300
F_W	166	239	295	357	425
Λ_W	76	103	123	144	166
x_W	0.82	0.93	1.0	1.06	1.12

Table - All masses and energies in GeV; we have used $\sin^2\theta_W = 0.22$ and $n = 4$.

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Figure caption

Fig. (1). The transition between a photon and the W^3. The haplon constituents of the W^3 annihilate into a photon. This annihilation is determined by the wave function at the origin.

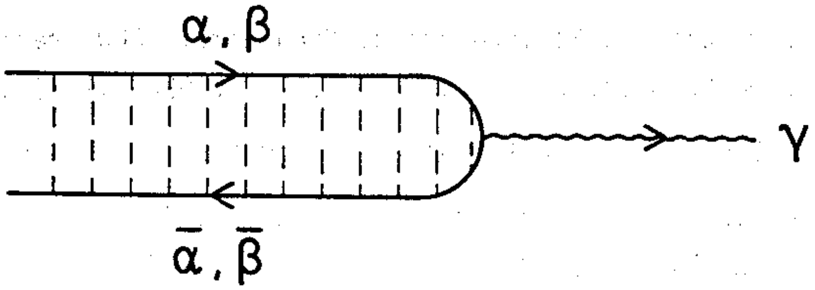

FIG. (1)