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DETAILED DESCRIPTION OF THE VIOLATIONS OF BJORKEN SCALING IN QCD (INCLUDING HIGHER ORDER CORRECTIONS)

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ABSTRACT

We present a compendium of formulae and parameters required to study the violations of Bjorken scaling, up to and including the subleading (i.e. $0(\alpha_s)$ corrections.

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The violations of Bjorken scaling in deep inelastic lepton-hadron scattering provide a quantitative test of QCD [1),2)], and indeed many experiments have been and are being performed to measure this phenomenon. The leading order predictions for the violations of Bjorken scaling have existed for several years [1),2)], and recently we have calculated the $O(\alpha_s)$ corrections to these leading terms [3),4)]. Since the running coupling constant $\alpha_s(Q^2)$ at $Q^2 \sim 10\ \mathrm{GeV}^2$ is usually taken from phenomenological analyses to be 0.2 to 0.5, which is not very small, these non-leading corrections are relevant. In this letter we collect all the necessary formulae and parameters required to study deep inelastic lepton-hadron scattering up to and including the subleading corrections [i.e. the $O(\alpha_s)$ corrections]. All the intermediate results presented have been calculated using dimensional regularisation and the minimal subtraction renormalisation prescription; physical results are of course independent of the renormalisation scheme used.

From the Wilson operator expansion, we have the following relation for the Nth moment of a structure function

$$\begin{aligned} F_i^N(Q^2) &\equiv \int_0^1 dx\ x^{N-1}\ F_i(x,Q^2) \\ &= C_{i,NS}^N(Q^2)\ M_{NS} + C_{i,f}^N(Q^2)\, M_f + C_{i,g}^N(Q^2)\, M_g \end{aligned} \tag{1}$$

where F_i may be F_1, F_2/x or F_3, and M_{NS}, M_f and M_g are constants and must be extracted from the data. The labels f,g, and NS stand for fermion, gluon and non-singlet, respectively. The Callan-Symanzik equation enables us to calculate the Q^2 behaviour of the coefficient functions C_i^N. The functions F_1, F_2, F_3 are defined here in the following way,

$$F_1 = \frac{1}{2\pi}\,\mathrm{Im}\,T_1\ ;\quad F_2 = \frac{1}{2\pi}\frac{\nu}{m_H^2}\,\mathrm{Im}\,T_2\ ;\quad F_3 = \frac{1}{2\pi}\frac{\nu}{m_H^2}\,\mathrm{Im}\,T_3$$

where M_H is the mass of the target $\nu = p\cdot q$, and the T_i's are given by

$$\begin{aligned} &i\int d^4x\ e^{iq\cdot x}\ \langle p|\,T(J_\mu^+(x)\ J_\nu(0))\,|p\rangle \\ &= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)T_1(q^2,x) + \frac{1}{m_H^2}\left(p_\mu - \frac{\nu q_\mu}{q^2}\right)\left(p_\nu - \frac{\nu q_\nu}{q^2}\right)T_2(q^2,x) \\ &\qquad - i\,\frac{\varepsilon_{\mu\nu\alpha\beta}\,p^\alpha q^\beta}{m_H^2}\,T_3(q^2,x) \end{aligned}$$

a) Non-Singlet Combinations of Structure Functions

It is possible experimentally to extract combinations of structure functions whose q^2 behaviour is determined only by the non-singlet operators, so that only the first term on the right-hand side of (1) survives. Examples of such combinations which have recently been studied experimentally are $F_3^{\nu A}$, where A is an isoscalar target [5] and $F_2^{\mu p} - F_2^{\mu n}$ [6]. For non-singlet combinations the solution to the Callan-Symanzik equation gives:

$$F^N_{i,NS}(Q^2) = F^N_{i,NS}(Q_0^2)\left[1 + A^N_{i,NS}\,\frac{\bar{g}^2(Q^2)-\bar{g}^2(Q_0^2)}{16\pi^2}\right]\left(\frac{\bar{g}^2(Q^2)}{\bar{g}^2(Q_0^2)}\right)^{\gamma_0^N/2\beta_0} \qquad (2)$$

where N takes even (odd) values for structure functions or combinations of structure functions which are odd (even) in x at fixed q^2, e.g. $F_1, F_2/x$ for electro-production or muon-production (e.g. $F_3^{\nu A}$).

$$\gamma_0^N = 2C_F\left[1 - \frac{2}{N(N+1)} + 4\sum_{j=2}^{N}\frac{1}{j}\right] \qquad (3)$$

and

$$\beta_0 = \frac{1}{3}\left[11C_A - \frac{4}{3}T_R\right] \qquad (4)$$

C_F, C_A and T_R are eigenvalues of the Casimir operators of the gauge group are defined by

$$\delta_{ij}C_F = T^a_{ik}T^a_{kj} \qquad (5a)$$

$$\delta_{cd}C_A = f^{abc}f^{abd} \qquad (5b)$$

and

$$\delta_{ab}T_R = T^a_{ij}T^b_{ji} \qquad (5c)$$

where T^a_{ij} are the generators of the gauge group in the same representation as the quarks and f^{abc} are the structure constants. If the colour group is SU(3) and the quarks and gluons are in the fundamental and adjoint representations respectively, then CF= 4/3, CA= 3 and T_R= 1/2 the number of flavours.

$$A^N_{i,NS} = \varepsilon^N_{i,q} + \frac{\gamma_1^N}{2\beta_0} - \frac{\gamma_0^N \beta_1}{2\beta_0^2} \tag{6}$$

where [7]

$$\beta_1 = \frac{34}{3} C_A^2 - \frac{20}{3} C_A T_R - 4 C_F T_R \tag{7}$$

γ_1^N has been calculated in Ref. 3) and the result has recently been rewritten in the following relatively simple form [8]:

$$\begin{aligned}
\gamma_1^N = {} & \left(C_F^2 - \frac{C_F C_A}{2}\right)\Big\{16\, S_1(N)\, \frac{2N+1}{N^2(N+1)^2} + 16\left[2S_1(N) - \frac{1}{N(N+1)}\right]\cdot \\
& \cdot\left[S_2(N) - S_2'\left(\tfrac{N}{2}\right)\right] + 64\, \tilde{S}(N) + 24 S_2(N) - 3 - 8\, S_3'\left(\tfrac{N}{2}\right) - 8\, \frac{3N^3+N^2-1}{N^3(N+1)^3} \\
& - 16(-1)^N\, \frac{2N^2+2N+1}{N^3\,(N+1)^3}\Big\} + C_F C_A\Big\{S_1(N)\left[\frac{536}{9} + 8\,\frac{2N+1}{N^2(N+1)^2}\right] - 16\, S_1(N)\, S_2(N) \\
& + S_2(N)\left[-\frac{52}{3} + \frac{8}{N(N+1)}\right] - \frac{43}{6} - 4\, \frac{151N^4 + 263N^3 + 97N^2 + 3N + 9}{9N^3\,(N+1)^3}\Big\} + \\
& C_F T_R\Big\{-\frac{160}{9} S_1(N) + \frac{32}{3} S_2(N) + \frac{4}{3} + 16\, \frac{11N^2 + 5N - 3}{9N^2\,(N+1)^2}\Big\}
\end{aligned} \tag{8}$$

where

$$S_i(N) = \sum_{j=1}^{N} \frac{1}{j^i} \qquad ; \qquad \tilde{S}(N) = \sum_{j=1}^{N} \frac{(-1)^j S_1(j)}{j^2}$$

and

$$S_i'\left(\frac{N}{2}\right) = \frac{1+(-1)^N}{2} S_i\left(\frac{N}{2}\right) + \frac{1-(-1)^N}{2} S_i\left(\frac{N-1}{2}\right) \qquad (9)$$

The $\varepsilon_{i,f}$ are given by [4),9)]

$$\varepsilon_{2,f} = C_F \left\{ 3S_1(N) - 4S_2(N) - \frac{2}{N(N+1)} S_1(N) + 4\sum_{j=1}^{N} \frac{S_1(j)}{j} + \frac{3}{N} + \frac{4}{N+1} + \frac{2}{N^2} - 9 + (\log 4\pi - \gamma)\left(4S_1(N) - 3 - \frac{2}{N(N+1)}\right)\right\} \qquad (10a)$$

and

$$\varepsilon_{1,f} = \varepsilon_{2,f} - \frac{4C_F}{(N+1)} \qquad (10b)$$

$$\varepsilon_{3,f} = \varepsilon_{2,f} - \left(\frac{2}{N} + \frac{2}{N+1}\right) C_F \qquad (10c)$$

where γ is Euler's constant $\cong$ 0.577216. There still remains some freedom to absorb part of the $E_{i,f}$ (in particular the terms where only N dependant factors are γ_o^N) in a rescaling of Λ [9)]. This freedom corresponds to our ignorance of the next order corrections.

Finally we need to know $\bar{g}^2(Q^2)$ in terms of Q^2, and the first two terms in the asymptotic expansion are given by

$$\bar{g}^2(Q^2) = \frac{16\pi^2}{\beta_0 \log Q^2/\Lambda^2} \left(1 - \frac{\beta_1 \log\left(\log\left(\frac{Q^2}{\Lambda^2}\right)\right)}{\beta_0 \log \frac{Q^2}{\Lambda^2}}\right) \qquad (11)$$

where Λ is a parameter which has to be determined from the data.

In order to facilitate phenomenological applications of formula (2), we tabulate the values of $A_{2,NS}$ for three to six flavours and for values of N from 1 to 20 in Table 1. Values of $A_{1,NS}$ and $A_{3,NS}$ can be easily obtained from Table 1 by using equations (6), (10b) and (10c).

Since γ_1^N for even N is not the analytic continuation of that for odd N [10], in principle one should only take the values of $A_{i,NS}$, using Table 1, for even or odd N (depending on whether the combination of structure functions being studied is even or odd in x). However since numerically the difference between analytically continuing $A_{i,NS}$ from even (odd) N to odd (even) N and the values of $A_{i,NS}$ themselves for odd (even) N turns out to be small [10], in practice it is reasonable to use the values of $A_{i,NS}$ from Table 1 for both even and odd N in equation (2) for any crossing symmetric or antisymmetric combination of non-singlet structure functions (i.e. even or odd x).

b) The Singlet Contribution

In addition to the non-singlet contribution discussed above, in general a structure function will also have a contribution from the singlet operators [i.e. the second and third terms on the right-hand side of equation (1)]. We denote this singlet contribution by $Q(x,Q^2)$, and like to think of it as the total quark distribution. Because of the mixing of the singlet operators in higher orders $Q(x,Q^2)$ depends not only on its value at some fixed $Q^2=Q_o^2$ but also on the gluon distribution $G(x,Q_o^2)$ *) at this value of Q^2. We have for Nth moment of $Q(x,Q^2)$ [4]

$$\begin{aligned} Q_i^N(Q^2) = \Bigg\{ & \left[A_{11}^N + A_{12}^{(i)N}\frac{\bar g^2(Q^2)}{16\pi^2} + A_{13}^{(i)N}\frac{\bar g^2(Q_0^2)}{16\pi^2}\right]\left[\frac{\bar g^2(Q^2)}{\bar g^2(Q_0^2)}\right]^{\gamma_+^N/2\beta_0} \\ & + \left[A_{21}^N + A_{22}^{(i)N}\frac{\bar g^2(Q^2)}{16\pi^2} + A_{23}^{(i)N}\frac{\bar g^2(Q_0^2)}{16\pi^2}\right]\left[\frac{\bar g^2(Q^2)}{\bar g^2(Q_0^2)}\right]^{\gamma_-^N/2\beta_0}\Bigg\} Q_i^N(Q_0^2) \\ - \Bigg\{ & \left[B_{11}^N + B_{12}^{(i)N}\frac{\bar g^2(Q^2)}{16\pi^2} + B_{13}^{(i)N}\frac{\bar g^2(Q_0^2)}{16\pi^2}\right]\left[\frac{\bar g^2(Q^2)}{\bar g^2(Q_0^2)}\right]^{\gamma_+^N/2\beta_0} \\ & + \left[B_{21}^N + B_{22}^{(i)N}\frac{\bar g^2(Q^2)}{16\pi^2} + B_{23}^{(i)N}\frac{\bar g^2(Q_0^2)}{16\pi^2}\right]\left[\frac{\bar g^2(Q^2)}{\bar g^2(Q_0^2)}\right]^{\gamma_-^N/2\beta_0}\Bigg\} G_i^N(Q_0^2) \end{aligned} \qquad (12)$$

*) There is some freedom in the definition of the gluon distribution. Equation (12) together with tables 2 to 5 correspond to a consistent definition.

where the label i takes the values of 1 or 2 depending on whether one is considering the structure function F_1 or F_2. The coefficients A_{ij}, B_{ij} depend on the one and two loop anomalous dimensions of the singlet operators, β_o and β_1, and the one loop contributions to the coefficient functions, in a way which is explained in reference 4. In tables 2,3,4 and 5 we tabulate their values for N = 2 to 20 (gluon operator matrix elements are non-zero for even N only, by charge conjugation - for odd N the singlet and non-singlet operators have identical Q^2 dependence) for three, four, five and six flavours, respectively.

For any structure function or combination of structure functions we can rewrite equation (1) as

$$F_i^N(Q^2) = F_{i,NS}^N(Q^2) + Q_i^N(Q^2) \tag{13}$$

where the Q^2 behaviour of the $F^N_{i,NS}$ is given by equation (2) and that of the Q_i^N by equation (12). For each N there are still three constants to be determined from the data, one to normalise $F^N_{i,NS}$ and two to normalise Q_i^N.

Asymptotically, as $Q^2 \to \infty$, the scale breaking effects described above are the dominant ones, however at values of Q^2 accessible to present day accelerators it is probable that scale breaking effects due to the mass of the target are not completely negligible. These can be included by taking Nachtmann moments [11] instead of the Cornwall-Norton moments [defined by equation (1)]*). The Q^2 behaviour of the Nachtmann moments is precisely that given by equations (2) and (12). The definition of these moments for all the structure functions measured in deep inelastic leptoproduction is presented in ref. 12).

*) Scale breaking effects arising from the quark masses, and from operators in the Wilson expansion which are of twist 4 or higher are still neglected.

N \ f	3	4	5	6
1	0	0	0	0
2	8.899	9.045	9.295	9.700
3	15.85	16.02	16.33	16.86
4	21.59	21.75	22.09	22.71
5	26.49	26.64	27.00	27.68
6	30.78	30.93	31.30	32.03
7	34.61	34.75	35.15	35.90
8	38.07	38.22	38.61	39.40
9	41.24	41.38	41.78	42.60
10	44.16	44.29	44.71	45.55
11	46.87	47.00	47.42	48.28
12	49.40	49.53	49.95	50.84
13	51.77	51.90	52.34	53.24
14	54.02	54.14	54.58	55.50
15	56.13	56.26	56.71	57.64
16	58.15	58.27	58.72	59.68
17	60.07	60.19	60.64	61.61
18	61.90	62.02	62.47	63.46
19	63.65	63.77	64.23	65.23
20	65.33	65.49	65.91	66.92

Table 1

Values of $A_{2,NS}$ for three to six flavours (f) and for for values of N from 1 to 20.

N	2	4	6	8	10	12	14	16	18	20
A_{11}	0.6400	0.0175	0.0037	0.0015	0.0007	0.0004	0.0003	0.0002	0.0001	0.0001
$A^{(1)}_{12}$	6.879	0.1907	0.0334	0.0115	0.0054	0.0030	0.0019	0.0013	0.0010	0.0007
$A^{(2)}_{12}$	6.778	0.1794	0.0320	0.0112	0.0053	0.0030	0.0019	0.0013	0.0010	0.0007
$A^{(1)}_{13}$	-2.292	1.453	-0.2363	-0.0517	-0.0214	-0.0115	-0.0071	-0.0049	-0.0035	-0.0026
$A^{(2)}_{13}$	-3.472	1.366	-0.2570	-0.0595	-0.0251	-0.0135	-0.0084	-0.0056	-0.0040	-0.0030
A_{21}	0.3600	0.9824	0.9962	0.9985	0.9993	0.9996	0.9997	0.9998	0.9999	0.9999
$A^{(1)}_{22}$	1.334	18.41	30.10	37.47	43.66	48.98	53.66	57.83	61.62	65.07
$A^{(2)}_{22}$	0.3922	19.55	30.88	38.07	44.15	49.39	54.01	58.15	61.90	65.33
$A^{(1)}_{23}$	-5.920	-20.06	-29.90	-37.43	-43 65	-48.97	-53.65	-57.83	-61.61	-65.07
$A^{(2)}_{23}$	-3.699	-21.10	-30.66	-38.02	-44.13	-49.38	-54.01	-58.14	-61.89	-65.32
B_{11}	0.3600	0.1393	0.0723	0.0480	0.0355	0.0279	0.0228	0.0192	0.0165	0.0145
$B^{(1)}_{12}$	3.869	1.514	0.6462	0.3768	0.2567	0.1925	0.1539	0.1287	0.1113	0.0986
$B^{(2)}_{12}$	3.812	1.424	0.6193	0.3654	0.2509	0.1892	0.1518	0.1273	0.1103	0.0979
$B^{(1)}_{13}$	-6.030	3.998	3.795	3.273	2.867	2.550	2.297	2.091	1.920	1.776
$B^{(2)}_{13}$	-5.414	3.315	3.397	3.018	2.692	2.422	2.199	2.014	1.858	1.725
B_{21}	-0.3600	-0.1393	-0.0723	-0.0480	-0.0355	-0.0279	-0.0228	-0.0192	-0.0165	-0.0145
$B^{(1)}_{22}$	-1.334	-2.610	-2.185	-1.801	-1.550	-1.367	-1.225	-1.112	-1.019	-0.9417
$B^{(2)}_{22}$	-0.3922	-2.771	-2.242	-1.830	-1.568	-1.378	-1.233	-1.118	-1.024	-0.9454
$B^{(1)}_{23}$	3.494	-2.901	-2.256	-1.848	-1.574	-1.376	-1.226	-1.108	-1.012	-0.9333
$B^{(2)}_{23}$	1.993	-1.968	-1.774	-1.554	-1.375	-1.233	-1.118	-1.024	-0.9446	-0.8777
$\gamma^{-}/2\beta_0$	0	0.7599	0.9958	1.160	1.287	1.392	1.480	1.557	1.626	1.687
$\gamma^{+}/2\beta_0$	0.6173	1.638	2.203	2.587	2.882	3.121	3.323	3.498	3.653	3.791

__Table 2__

Quantities occurring in Eq. (12) in the case of three flavours

N	2	4	6	8	10	12	14	16	18	20
A_{11}	0.5714	0.0197	0.0044	0.0018	0.0009	0.0005	0.0004	0.0002	0.0002	0.0001
$A_{12}^{(1)}$	7.253	0.2331	0.0408	0.0137	0.0062	0.0034	0.0021	0.0014	0.0010	0.0008
$A_{12}^{(2)}$	6.893	0.2160	0.0386	0.0132	0.0060	0.0033	0.0021	0.0014	0.0010	0.0007
$A_{13}^{(1)}$	-1.960	-4.038	-0.1389	-0.0515	-0.0284	-0.0185	-0.0131	-0.0098	-0.0077	-0.0062
$A_{13}^{(2)}$	-3.125	-4.142	-0.1643	-0.0613	-0.0330	-0.0210	-0.0147	-0.0108	-0.0084	-0.0067
A_{21}	0.4286	0.9803	0.9956	0.9982	0.9991	0.9995	0.9996	0.9998	0.9998	0.9999
$A_{22}^{(1)}$	1.943	23.95	30.13	37.60	43.80	49.12	53.79	57.96	61.74	65.19
$A_{22}^{(2)}$	0.5714	25.11	30.91	38.21	44.29	49.53	54.15	58.27	62.02	65.45
$A_{23}^{(1)}$	-7.235	-20.15	-30.03	-37.57	-43.78	-49.10	-53.78	-57.95	-61.73	-65.19
$A_{23}^{(2)}$	-4.340	-21.18	-30.79	-38.16	-44.26	-49.51	-54.13	-58.26	-62.01	-65.44
B_{11}	0.4286	0.1700	0.0907	0.0608	0.0452	0.0357	0.0293	0.0247	0.0213	0.0186
$B_{12}^{(1)}$	5.439	2.016	0.8404	0.4730	0.3108	0.2255	0.1750	0.1428	0.1208	0.1053
$B_{12}^{(2)}$	5.170	1.868	0.7951	0.4536	0.3009	0.2197	0.1714	0.1404	0.1192	0.1041
$B_{13}^{(1)}$	-7.114	6.095	4.978	4.335	3.813	3.399	3.066	2.794	2.567	2.376
$B_{13}^{(2)}$	-6.464	5.218	4.457	4.000	3.581	3.229	2.937	2.692	2.485	2.308
B_{21}	-0.4286	-0.1700	-0.0907	-0.0608	-0.0452	-0.0357	-0.0293	-0.0247	-0.0213	-0.0186
$B_{22}^{(1)}$	-1.943	-4.153	-2.745	-2.290	-1.981	-1.752	-1.574	-1.431	-1.314	-1.215
$B_{22}^{(2)}$	-0.5714	-4.354	-2.816	-2.327	-2.003	-1.767	-1.585	-1.439	-1.320	-1.220
$B_{23}^{(1)}$	3.618	-3.959	-3.074	-2.518	-2.143	-1.872	-1.667	-1.505	-1.375	-1.266
$B_{23}^{(2)}$	1.865	-2.733	-2.436	-2.127	-1.879	-1.682	-1.523	-1.393	-1.284	-1.192
$\gamma^{-}/2\beta_0$	0	0.8170	1.074	1.252	1.390	1.503	1.599	1.682	1.756	1.821
$\gamma^{+}/2\beta_0$	0.7467	1.852	2.460	2.875	3.192	3.451	3.669	3.858	4.025	4.174

Table 3

Quantities occurring in Eq. (12) in the case of four flavours.

N	2	4	6	8	10	12	14	16	18	20
A_{11}	0.5161	0.0210	0.0049	0.0020	0.0010	0.0006	0.0004	0.0003	0.0002	0.0002
$A_{12}^{(1)}$	7.695	0.2812	0.0505	0.0169	0.0076	0.0041	0.0025	0.0016	0.0012	0.0009
$A_{12}^{(2)}$	7.128	0.2582	0.0474	0.0161	0.0073	0.0040	0.0024	0.0016	0.0011	0.0009
$A_{13}^{(1)}$	-1.906	- 0.2855	- 0.0809	- 0.0483	- 0.0331	- 0.243	- 0.0186	- 0.0147	- 0.0119	- 0.0098
$A_{13}^{(2)}$	-3.059	- 0.4020	- 0.1105	- 0.0598	-0.0387	- 0.0274	- 0.0205	- 0.0159	- 0.0127	- 0.0104
A_{21}	0.4839	0.9790	0.9951	0.9980	0.9990	0.9994	0.9996	0.9997	0.9998	0.9998
$A_{22}^{(1)}$	2.500	20.41	30.41	37.98	44.21	49.54	54.23	58.41	62.20	65.66
$A_{22}^{(2)}$	0.7354	21.58	31.20	38.58	44.70	49.95	54.59	58.72	62.48	65.91
$A_{23}^{(1)}$	-8.290	-20.40	-30.37	-37.95	-44.18	-49.52	-54.21	-58.40	-62.19	-65.65
$A_{23}^{(2)}$	-4.804	-21.44	-31.13	-38.54	-44.67	-49.93	-54.57	-58.71	-62.47	-65.90
B_{11}	0.4839	0.1961	0.1070	0.0724	0.0541	0.0428	0.0352	0.0298	0.0257	0.0225
$B_{12}^{(1)}$	7.214	2.632	1.100	0.6123	0.3962	0.2825	0.2156	0.1730	0.1443	0.1240
$B_{12}^{(2)}$	6.682	2.416	1.033	0.5833	0.3813	0.2739	0.2102	0.1694	0.1417	0.1222
$B_{13}^{(1)}$	-8.159	6.519	6.090	5.351	4.726	4.223	3.815	3.480	3.200	2.963
$B_{13}^{(2)}$	-7.520	5.457	5.450	4.937	4.439	4.012	3.654	3.353	3.097	2.879
B_{21}	-0.4839	- 0.1961	- 0.1070	- 0.0724	- 0.0541	- 0.0428	- 0.0352	- 0.0298	- 0.0257	- 0.0225
$B_{22}^{(1)}$	-2.500	- 4.088	- 3.270	- 2.755	- 2.393	- 2.122	- 1.910	- 1.739	- 1.598	-1.479
$B_{22}^{(2)}$	-0.7354	-4.324	- 3.355	- 2.799	- 2.419	- 2.139	- 1.923	- 1.748	- 1.605	- 1.485
$B_{23}^{(1)}$	3.445	- 5.063	- 3.920	- 3.209	- 2.729	- 2.383	- 2.121	- 1.914	- 1.746	- 1.608
$B_{23}^{(2)}$	1.573	- 3.550	- 3.128	- 2.722	- 2.400	- 2.146	- 1.941	- 1.774	- 1.634	- 1.516
$\gamma^-/2\beta_0$	0	0.8846	1.166	1.360	1.511	1.633	1.738	1.828	1.908	1.980
$\gamma^+/2\beta_0$	0.8986	2.104	2.762	3.212	3.557	3.838	4.075	4.281	4.462	4.624

Table 4

Quantities occurring in Eq. (12) in the case of five flavours

N	2	4	6	8	10	12	14	16	18	20
A_{11}	0.4706	0.0217	0.0053	0.0022	0.0011	0.0007	0.0005	0.0003	0.0002	0.0002
$A_{12}^{(1)}$	8.310	0.3415	0.0644	0.0222	0.0102	0.0055	0.0034	0.0023	0.0016	0.0012
$A_{12}^{(2)}$	7.570	0.3126	0.0604	0.0211	0.0098	0.0054	0.0033	0.0022	0.0016	0.0012
$A_{13}^{(1)}$	-2.158	0.1380	-0.0389	-0.0443	-0.0367	-0.0295	-0.0238	-0.0194	-0.0161	-0.0135
$A_{13}^{(2)}$	-3.301	0.0113	-0.0719	-0.0572	-0.0430	-0.0330	-0.0259	-0.0208	-0.0171	-0.0142
A_{21}	0.5294	0.9783	0.9947	0.9978	0.9989	0.9993	0.9995	0.9997	0.9998	0.9998
$A_{22}^{(1)}$	2.990	20.43	31.03	38.74	45.04	50.43	55.15	59.37	63.19	66.68
$A_{22}^{(2)}$	0.8793	21.63	31.83	39.35	45.53	50.84	55.51	59.68	63.47	66.93
$A_{23}^{(1)}$	-9.141	-20.91	-31.06	-38.72	-45.02	-50.40	-55.13	-59.35	-63.17	-66.66
$A_{23}^{(2)}$	-5.148	-21.95	-31.82	-39.31	-45.50	-50.81	-55.49	-55.67	-63.45	-66.92
B_{11}	0.5294	0.2187	0.1216	0.0829	0.0622	0.0494	0.0407	0.0345	0.0298	0.0262
$B_{12}^{(1)}$	9.348	3.437	1.481	0.8398	0.5506	0.3962	0.3042	0.2450	0.2045	0.1758
$B_{12}^{(2)}$	8.516	3.146	1.389	0.7998	0.5299	0.3842	0.2966	0.2399	0.2010	0.1733
$B_{13}^{(1)}$	-9.400	7.298	7.077	6.277	5.568	4.988	4.515	4.123	3.796	3.517
$B_{13}^{(2)}$	-8.803	6.058	6.321	5.786	5.226	4.737	4.323	3.972	3.673	3.416
B_{21}	-0.5294	-0.2187	-0.1216	-0.0829	-0.0622	-0.0494	-0.0407	-0.0345	-0.0298	-0.0262
$B_{22}^{(1)}$	-2.990	-4.568	-3.795	-3.220	-2.806	-2.493	-2.247	-2.048	-1.884	-1.745
$B_{22}^{(2)}$	-0.8793	-4.834	-3.892	-3.270	-2.836	-2.513	-2.262	-2.059	-1.892	-1.752
$B_{23}^{(1)}$	3.041	-6.167	-4.764	-3.897	-3.313	-2.892	-2.572	-2.320	-2.116	-1.948
$B_{23}^{(2)}$	1.166	-4.370	-3.819	-3.316	-2.920	-2.608	-2.357	-2.152	-1.982	-1.837
$\gamma^-/2\beta_0$	0	0.9656	1.276	1.489	1.654	1.789	1.903	2.002	2.090	2.168
$\gamma^+/2\beta_0$	1.079	2.403	3.122	3.614	3.992	4.299	4.559	4.784	4.982	5.160

Table 5

Quantities occurring in Eq. (12) in the case of six flavours.

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