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Ω^- NON-LEPTONIC DECAYS

J. Finjord
CERN - Geneva

A B S T R A C T

Non-leptonic decay rates for Ω^- are calculated in a model where strong interactions introduce new $\Delta I = \frac{1}{2}$ operators in the effective Hamiltonian. Both $\Omega^- \to \Xi^0 \pi^0$ and $\Omega^- \to \Lambda^0 K^-$ are predicted to be nearly parity-conserving. $\Delta I = \frac{1}{2}$ contributions are found to dominate the sum of the pionic rates, while $\Delta I = \frac{3}{2}$ contributions are non-negligible in each of them. Rough agreement in magnitude with experimental data is obtained.

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1. - MOTIVATION

Gluon exchange modification of the non-leptonic Hamiltonian provides a qualitative explanation of the $\Delta I = \frac{1}{2}$ rule in strange particle decay by dynamical enhancement vs. suppression of different terms [1),2)]. The lowest order exchange also gives rise to diagrams of the type of Fig. 1, thus introducing new $\Delta I = \frac{1}{2}$ operators in the effective Hamiltonian [3),4)]. These operators are subject to a matrix element enhancement since they annihilate both left- and right-handed quarks, and their contributions to $\Delta I = \frac{1}{2}$ processes have been shown to be dominant although they have small coefficients [3),5)]. Roughly within a factor 1.5 in amplitude they have been shown to give a unified description of kaon and $\underline{8}$ hyperon decays both for the $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ parts, except for some P wave hyperon decays where the data are still inconclusive [5)]. This is an impressive result, considering the approximations involved : the constituent quark model, and the extrapolation of a result given by asymptotic freedom up to the value $g^2/4\pi = 1$ for the gluon-quark coupling constant.

We consider here the predictions for Ω^- decay. The calculated lifetime agrees in magnitude with experimental data [6),7)], while data on the branching ratios are still inconclusive. Future experiments should, however, make possible a test of the model's applicability also to $\underline{10} \rightarrow \underline{8}$ hyperon transitions.

2. - EMBEDDING OF THE NON-LEPTONIC HAMILTONIAN IN Ω^- DECAY

The $\Delta S = 1$, $\Delta C = 0$ part of the effective non-leptonic Hamiltonian, except for a part which involves induced V+A currents and which has been shown by Shifman, Vainshtein and Zakharov (SVZ) to be unimportant in strange particle decay [5)], is

$$H_{eff}^{\Delta S=1} = \sqrt{2}\, G \sin\theta_c \cos\theta_c \sum_{i=1}^{6} c_i O_i \qquad (2.1)$$

where c_i are coefficients and O_i are the following operators, the parentheses indicating their $SU(3)_{flavour}$ and isospin properties :

$$
\begin{aligned}
O_1 &= \bar{d}_L s_L \bar{u}_L u_L - \bar{d}_L u_L \bar{u}_L s_L && (\underset{\sim}{8}_f\,,\ \Delta I = \tfrac{1}{2}) \\
O_2 &= \bar{d}_L s_L \bar{u}_L u_L + \bar{d}_L u_L \bar{u}_L s_L + 2\bar{d}_L s_L \bar{d}_L d_L \\
&\qquad + 2\bar{d}_L s_L \bar{s}_L s_L && (\underset{\sim}{8}_d\,,\ \Delta I = \tfrac{1}{2}) \\
O_3 &= \bar{d}_L s_L \bar{u}_L u_L + \bar{d}_L u_L \bar{u}_L s_L + 2\bar{d}_L s_L \bar{d}_L d_L \\
&\qquad - 3\bar{d}_L s_L \bar{s}_L s_L && (\underset{\sim}{27}\,,\ \Delta I = \tfrac{1}{2}) \\
O_4 &= \bar{d}_L s_L \bar{u}_L u_L + \bar{d}_L u_L \bar{u}_L s_L - \bar{d}_L s_L \bar{d}_L d_L && (\underset{\sim}{27}\,,\ \Delta I = \tfrac{3}{2}) \\
O_5 &= \bar{d}_L \lambda^a s_L (\bar{u}_R \lambda^a u_R + \bar{d}_R \lambda^a d_R + \bar{s}_R \lambda^a s_R) && (\underset{\sim}{8}\,,\ \Delta I = \tfrac{1}{2}) \\
O_6 &= \bar{d}_L s_L (\bar{u}_R u_R + \bar{d}_R d_R + \bar{s}_R s_R) && (\underset{\sim}{8}\,,\ \Delta I = \tfrac{1}{2})
\end{aligned}
\tag{2.2}
$$

where f.i. $\bar{d}_L s_L \bar{u}_L u_L$ is shorthand for $\bar{d}^i_L \gamma_\mu s^i_L \bar{u}^k_L \gamma^\mu u^k_L$ and $\bar{d}_L \lambda^a s_L \bar{u}_R \lambda^a u_R$ for $\bar{d}^i_L \gamma_\mu \lambda^a s^i_L \bar{u}^k_R \gamma^\mu \lambda^a u^k_R$; the λ matrices and upper latin indices reflect the $SU(3)_{colour}$ group, and $\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\psi$ are left- and right-handed fermion components. O_5 and O_6 are the operators originating from the diagram of Fig. 1. Sets of coefficients have been computed [5] with $g^2/4\pi = 1$ at the π mass and the ρ mass, respectively *),

$$
c_1 = -2.8,\ c_2 = .06,\ c_3 = .08,\ c_4 = .4,\ c_5 = -.14,\ c_6 = -.05 \quad (\pi) \tag{2.3}
$$

$$
c_1 = -2.5,\ c_2 = .09,\ c_3 = .08,\ c_4 = .4,\ c_5 = -.06,\ c_6 = -.01 \quad (\rho) \tag{2.4}
$$

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*) Deep inelastic data seem to be consistent with this mass being in the range 600-1000 MeV [8].

The set (2.3) is favoured by $\underset{\sim}{8}$ decays. The best fits [5] to K^+, Λ^0_- and Σ^+_0 decays were, however, obtained for $c_4 = 0.25$ and $c_5 + 3/10\ c_6 = -0.24$; the magnitude of this discrepancy gives a measure of the validity of the approximations involved in the SVZ scheme.

We restrict ourselves to the three decay modes seen experimentally. For $\Omega^- \to \Xi^0 \pi^-$, application of the Hamiltonian (2.1) gives rise to the quark diagrams shown in Fig. 2. The diagrams for $\Omega^- \to \Xi^- \pi^0$ are similar. For $\Omega^- \to \Lambda^0 K^-$ the diagrams are shown in Fig. 3. The first two terms of the operators O_2, O_3 and O_4 are symmetric in the interchange of colour indices of $\bar{u}_L$ and $\bar{d}_L$ or s_L and u_L ; with colour antisymmetric baryon wave functions they do not contribute to diagrams where two quarks in a baryon are annihilated and/or created by the Hamiltonian. In this type of diagrams O_5 and O_6 have no matrix element enhancement [5], making eventual O_1 contributions dominant. Still, these O_1 contributions seem to be smaller than, though comparable in magnitude to, those for O_1, O_5 and O_6 in diagrams with only one baryon quark taking part [5].

Different pictures thus emerge for the dominant contributions to pionic and kaonic Ω^- decays. In the former the matrix elements can be written as products of leptonic matrix elements, the $\Delta I = \frac{1}{2}$ rule being a consequence of the smallness of coefficient c_4 and matrix element enhancement of O_5 and O_6. The latter can be represented by a pole diagram as in Fig. 4, with the matrix element given by the baryon wave function at zero interquark distance. This transition will proceed in a P wave because the non-leptonic vertex conserves spin and parity. Only the lowest spin $\frac{1}{2}$ Ξ^0 state will contribute considerably, since radial excitations imply $|\psi(0)|$ suppression, and the propagator favours the lowest-mass state.

3. - THE $\Omega^- \to \Xi^{\underline{0}}\ \pi^{\overline{0}}$ MODES

The partial matrix element for one operator in $\Omega^- \to \Xi^0\ \pi^-$ can be written

$$M_j(\Omega^- \to \Xi^0 \pi^-) = \sqrt{2}\, G \sin\theta_c \cos\theta_c\ c_j \langle \Xi^0 \pi^- | O_j | \Omega^- \rangle \qquad (3.1)$$

where the matrix element of O_j can be expressed as

$$F_j \langle \pi^- | \bar{d}^i_L \gamma_\mu u^i_L | 0 \rangle \langle \Xi^0 | \bar{u}^k_L \gamma^\mu s^k_L | \Omega^- \rangle$$
$$= -\tfrac{1}{4} i f_\pi q_\mu F_j \langle \Xi^0 | \bar{u}^k \gamma^\mu (1-\gamma_5) s^k | \Omega^- \rangle \tag{3.2}$$

where f_π and q_μ are the constant of the $\pi \to \mu\nu$ decay and the pion momentum, respectively. For $j = 1,\ldots,4$ the constant F_j takes care of the interplay between the two contributing terms of each operator ; it is, by a Fierz transformation and the colour-singlet property, equal to $-\frac{2}{3}$, $\frac{4}{3}$, $\frac{4}{3}$ and $\frac{4}{3}$, respectively. For $j = 5, 6$ F_j contains the matrix element enhancement. Between colour singlets one can make the substitutions

$$O_5 \to \tfrac{16}{3} O_6 \tag{3.3}$$

$$\bar{d}^i_L \gamma_\mu s^i_L \bar{u}^k_R \gamma^\mu u^k_R \to -\frac{2}{3} \frac{1}{2m_u(m_s+m_u)} \partial_\mu \bar{d}^i_L \gamma^\mu u^i_L \partial_\nu \bar{u}^k_L \gamma^\nu s^k_L \tag{3.4}$$

A Fierz transformation and the Dirac equation is used for the last one, which, however, is valid only for matrix elements where the vector terms in each current do not contribute, as is the case for those which concern us. Thus one finds the values $-16/9\ (m_\pi^2/m_u m_s)$ and $-1/3\ (m_\pi^2/m_u m_s)$ for F_5 and F_6, respectively. For the quark masses we take [9]

$$m_u \cong m_d \cong 5.4\ \text{MeV}, \quad m_s \cong 150\ \text{MeV} \tag{3.5}$$

The effect of the coefficients of all $\Delta I = \frac{1}{2}$ operators can now be taken care of by replacing them by c_1^{eff}, using (2.3) or (2.4) for the coefficients,

$$c_1^{\text{eff}} \cong \begin{cases} -6.5 & (\rho \text{ mass}) \\ -12.0 & (\pi \text{ mass}) \end{cases} \tag{3.6}$$

which shows the dominance of O_5 contributions.

The $\Delta I = \frac{1}{2}$ property of operators other than O_4 fixes their contributions to $\Omega^- \to \Xi^- \pi^0$ by using (3.1) and (3.2) with $F_j \to -F_j/\sqrt{2}$. For the $\Delta I = \frac{3}{2}$ contribution one finds instead $F_4 \to \sqrt{2}\, F_4$. Thus

$$\frac{|M(\Omega^- \to \Xi^0 \pi^-)|^2 - 2|M(\Omega^- \to \Xi^- \pi^0)|^2}{|M(\Omega^- \to \Xi^0 \pi^-)|^2 + 2|M(\Omega^- \to \Xi^- \pi^0)|^2} \cong -6 \frac{c_4}{c_1} \sim \begin{cases} 1/5 & (\rho) \\ 1/8 & (\pi) \end{cases} \tag{3.7}$$

where the fit $c_4 = 0.25$ is used. Despite the smallness of c_4, $\Delta I = \frac{3}{2}$ effects are non-negligible in the rates.

The baryonic matrix element of $2\bar{u}_L \gamma^\mu s_L$ in Eq. (3.2) remains to be found. We compute it between SU(6) wave functions,

$$\frac{1}{\sqrt{2}} (\varphi_1 \chi_1 + \varphi_2 \chi_2) \quad \text{and} \quad \varphi_s \chi_{3,4} \tag{3.8}$$

for $\underset{\sim}{8}$ and $\underset{\sim}{10}$, respectively. The decimet SU(3) and spin functions are both symmetric in permutation of particles ; the mixed symmetric SU(3) (spin) functions $\varphi_1(\chi_1)$ and $\varphi_2(\chi_2)$ are symmetric and antisymmetric in permutations of particles 2 and 3 [10]. The colour dependence gives a total factor 1. An immediate observation is that for non-relativistic quark momenta, this matrix element has only axial vector (P wave) contributions ; the quark currents are all of the form (in the rest system of the initial quark)

$$\begin{aligned} & q^\mu \bar{u}_{fs} (\gamma_\mu - \gamma_\mu \gamma_5) u_{ir} \\ & = (m_i - m_f) \sqrt{\frac{E_f + m_f}{2m_f}}\, \chi_s^+ \chi_r + (m_i + m_f) \sqrt{\frac{E_f - m_f}{2m_f}}\, \chi_s \vec{\sigma} \hat{q} \chi_r \end{aligned} \tag{3.9}$$

and taken between orthogonal spin functions the vector term disappears. This is different from $\underset{\sim}{8} \to \underset{\sim}{8}$ transitions where the parity-violating terms are smallest because of the $(m_i \mp m_f)$ factors, but non-vanishing. It is in accord with the form of the matrix element for neutrino production of the Δ resonance [11]. By symmetry considerations and recalling that the matrix element of $\vec{\sigma}_{(1)} \hat{q}$ can be expressed by Clebsch-Gordan and spherical harmonics, one realizes that

$$q_\mu \langle \Xi^0 | \bar{u}\gamma^\mu(1-\gamma_5) s | \Omega^- \rangle$$
$$= -\frac{4}{\sqrt{3}} Z \sqrt{\pi}\,(m_\Omega + m_\Xi)\sqrt{\frac{E_\Xi - m_\Xi}{2m_\Xi}}\,\langle \tfrac{1}{2}\,1\,m'\ell | \tfrac{3}{2}\,m\rangle\, Y_1^\ell(\hat{q}) \tag{3.10}$$

where m and m' are the third components of spin for Ω^- and Ξ^0, respectively, and the sum convention for ℓ is understood. Z is a renormalization factor which will be supposed to have the same value as makes $g_A = 1.25$ in neutron β decay.

The rate for decay into Ξ and a pion is thus

$$\Gamma(\Omega^- \to \Xi^0 \pi^-) = \frac{G^2 m_\pi^4}{8\pi}|\vec{q}|\left(\frac{f_\pi}{m_\pi}\sin\theta_c\cos\theta_c\right)^2$$
$$\times\left(\frac{2Z}{3\sqrt{3}}\,\frac{m_\Omega + m_\Xi}{m_\pi}\,\frac{|\vec{q}|}{\sqrt{m_\Omega(E_\Xi + m_\Xi)}}\right)^2$$
$$\times\begin{cases}(-c_1^{\rm eff} + 2c_4)^2 & (\Xi^0\pi^-)\\ \frac{1}{2}(c_1^{\rm eff} + 4c_4)^2 & (\Xi^-\pi^0)\end{cases} \tag{3.11}$$

and

$$\Gamma^{-1}(\Omega \to \Xi\pi) = 2.25 \times 10^{-8}\,s/(c_1^{\rm eff\,2} + 8c_4^2)$$
$$= \begin{cases}1.6 \times 10^{-10}\ s & (\pi \text{ mass})\\ 5.3 \times 10^{-10}\ s & (\rho \text{ mass})\end{cases} \tag{3.12}$$

for $Z = \frac{3}{4}$. The fit $c_4 = 0.25$ is used. Thus the $\Delta I = \frac{1}{2}$ contributions dominate the total pionic rate.

4. - THE $\Omega^- \to \Lambda^0 K^-$ MODE

The rate will be calculated using data for Δ^- strong decay and a non-leptonic matrix element which gives a reasonable fit in an $\underset{\sim}{8} \to \underset{\sim}{8}$ process. Thus, from Fig. 4,

$$\Gamma(\Omega^- \to \Lambda^0 K^-) = \frac{m_\Lambda}{m_\Xi} \Gamma(\Omega^- \to \Xi^0 K^-) \left(\frac{2 m_\Xi}{m_\Lambda^2 - m_\Xi^2} \right)^2 \times$$
$$\times \left(\sqrt{2}\, G \sin\theta_c \cos\theta_c \, c_1 m_\pi^3 \right)^2 \left| \langle \Lambda^0 | O_1 | \Xi^0 \rangle / m_\pi^3 \right|^2 \qquad (4.1)$$

The expression for the propagator is valid for a spin-conserving non-leptonic vertex. Using PCAC for the unphysical decay $\Omega^- \to \Xi^0 K^-$, its rate can be related to that for $\Delta^- \to N^0 \pi^-$ [12]. Computing the rates as in Ref. 12), and adjusting also the (dipole) axial form factors for symmetry breaking effects by letting them depend on the Q [6] mass and the A_1 mass, respectively, one obtains with $f_K = 1.21\, f_\pi$,

$$\Gamma(\Omega^- \to \Xi^0 K^-) \cong 1.1\, \Gamma(\Delta^- \to N^0 \pi^-) \qquad (4.2)$$

and thus

$$\Gamma^{-1}(\Omega^- \to \Lambda^0 K^-) = \frac{m_\pi^6}{|\langle \Lambda^0 | O_1 | \Xi^0 \rangle|^2} \times 10^{-10} s \times \begin{cases} 2.8 & (\pi) \\ 3.5 & (\rho) \end{cases} \qquad (4.3)$$

Values for both choices of the mass giving $g^2/4\pi = 1$ are again presented.

For non-relativistic quark momenta

$$\langle B_f | O_1 | B_i \rangle = \frac{1}{2} \sum_{j \neq k} \langle B_f | (1 - \vec{\sigma}_j \vec{\sigma}_k)\, \tau_j^- v_k^+ \delta(\vec{r}_j - \vec{r}_k) | B_i \rangle \qquad (4.4)$$

where τ_j^- and v_k^+ are isospin and V spin operators on quarks j and k, respectively. With (4.4) and (3.8) one finds easily

$$\langle \Lambda^0 | O_1 | \Xi^0 \rangle = \sqrt{\tfrac{2}{3}} \langle p | O_1 | \Sigma^+ \rangle = \sqrt{6} \langle \psi^s | \delta(\vec{r}_2 - \vec{r}_3) | \psi^s \rangle \qquad (4.5)$$

where ψ^S is the space-symmetric wave function. SVZ [5] have an estimate for $< p|O_1|\Sigma^+ >$ based on the insertion of one-particle intermediate states, with equal vector and axial-vector form factors given by $F(q^2) = (1 - q^2/\Delta^2)^{-2}$, which gives a reasonable contribution to Σ_0^+ decay for $\Delta = 0.8$ GeV,

$$< p|O_1|\Sigma^+ > \cong 1.23\, m_\pi^3 \tag{4.6}$$

and, by (4.3) and (4.5),

$$\Gamma^{-1}(\Omega^- \to \Lambda^0 K^-) = \begin{cases} 2.8 \times 10^{-10}\ \mathrm{s} & (\pi) \\ 3.5 \times 10^{-10}\ \mathrm{s} & (\rho) \end{cases} \tag{4.7}$$

This inverse rate is strongly parameter-dependent ; it goes roughly as Δ^{-6} [5]. Another ambiguous factor is the symmetry breaking correction in Eq. (4.2) ; however, the approximations involved do certainly not allow one to make a fit to find the "best" form factors.

The value for $< \psi^S|\delta(\vec{r}_2 - \vec{r}_3)|\psi^S > = |\psi(0)|^2$ resulting from (4.5) and (4.6) is $0.41\ m_\pi^3$. Using a harmonic oscillator wave function one finds $|\psi(0)|^2 = (\Omega/12\pi)^{\frac{3}{2}} = 1.8\ m_\pi^3$ with the value of Ω reproducing Regge slopes [12] ; this $|\psi(0)|^2$ value inserted in Eq. (4.3) would bring us out of contact with experiment [6),7)]. Schmid [13] reports finding good agreement with data on Σ_0^+ S wave decays with $|\psi(0)|^2 = (3\sqrt{3}/\pi)m_\pi^3 = 1.65\ m_\pi^3$ and no enhancement factor for O_1, using PCAC. Le Yaouanc et al. [14] use $|\psi(0)|^2 = 4.44\ m_\pi^3$ in pole diagram fits to Λ_-^0 P wave decay. As both reactions in the present scheme have important contributions of the kind dominating $\Omega \to \Xi\pi$ [5], those results are not easily comparable with ours. One observes, however, that they are in strong disagreement with each other, as well as with models predicting a wave function suppression at zero interquark distance.

5. - CONFRONTATION WITH DATA AND CONCLUSION

The Ω lifetime as given by the Tables [6] and Ref. 7) is of the order of $1.3-1.4\times10^{-10}$ s, to which our result $1.0-2.1\times$ $\times\,10^{-10}$ s from Eqs. (3.12) and (4.7) should be compared. Experimental branching ratios are still inconclusive [7]. Our predicted rates would be consistent with about equal branching ratios into pions and kaons roughly within a factor two ; that would be the same level of agreement with data as was obtained by SVZ in octet non-leptonic decays. It remains to see if new data, corrected for detection efficiencies, show this trend, which is suggested by the numbers of events in Ref. 7). SVZ's "best fit" value $c_5+(3/16)c_6=-0.24$ would then give too high a rate for $\Omega\to\Xi\,\pi$. The fitted value $c_4=0.25$ used in the estimate (3.7) of $\Delta I=\frac{3}{2}$ effects may be more reliable since the calculated value of c_4 is quite stable against variation of parameters [5], and the fitted value has been successfully applied to both kaon and hyperon decays. Equations (3.7) and (3.12) should permit one to check both c_4 and c_1^{eff}. With the approximations involved and insufficient data at hand, our results do not allow one to make a case for one subtraction point vs. the other, cf. Eqs. (2.3) and (2.4).

The prediction that both the pionic and the kaonic decay modes of Ω^- are nearly parity-conserving, is, however, an unambiguous consequence of the SVZ model, and a test of this should be used as a first check of the model's validity. This also applies to the prediction of $\Delta I=\frac{1}{2}$ dominance in the sum of the pionic rates, with the non-negligible $\Delta I=\frac{3}{2}$ contributions to each channel cancelling each other.

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REFERENCES

1) M.K. Gaillard and B.W. Lee - Phys.Rev.Letters 33, 108 (1974).

2) G. Altarelli and L. Maiani - Phys.Letters 52B, 351 (1974).

3) M.A. Shifman, A.I. Vainshtein and V.I. Zakharov - Nuclear Phys. B120, 316 (1977).

4) M.A. Shifman, A.I. Vainshtein and V.I. Zakharov - ITEP Preprint ITEP-63 (1976).

5) A.I. Vainshtein, V.I. Zakharov and M.A. Shifman - ITEP Preprint ITEP-64 (1976).

6) Particle Data Group - Revs.Modern Phys. 48, Part II (1976).

7) M. Deutschmann et al. - CERN Preprint CERN/EP/PHYS 77-26 Rev. (1977).

8) M.K. Gaillard - Invited talk at the International Conference on Neutrino Physics and Astrophysics "Neutrino-77", Elbrus, USSR (June 1977).

9) H. Leutwyler - Phys.Letters 48B, 431 (1974) ; Nuclear Phys. B76, 413 (1974).

10) See, for example :

J. Finjord - Nuclear Phys. A274, 495 (1976), Appendix B, for the spin functions.

11) C.H. Llewellyn Smith - Physics Reports 3C, 261 (1972).

12) R.P. Feynman, M. Kislinger and F. Ravndal - Phys.Rev. D3, 2706 (1971).

13) C. Schmid - Phys.Letters 66B, 353 (1977).

14) A. Le Yaouanc, L. Oliver, O. Pène and J.-C. Raynal - Phys.Letters 72B, 53 (1977).

FIGURE CAPTIONS

Figure 1 Diagram with lowest-order gluon exchange giving new terms in non-leptonic effective Hamiltonian.

Figure 2 Diagrams contributing to $\Omega^- \to \Xi^0 \pi^-$.

Figure 3 Diagrams contributing to $\Omega^- \to \Lambda^0 K^-$.

Figure 4 Dominant contribution to $\Omega^- \to \Lambda^0 K^-$ as a pole diagram.

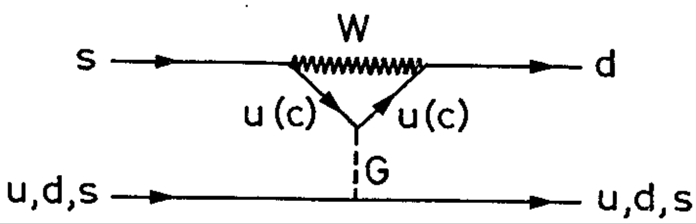

FIG.1

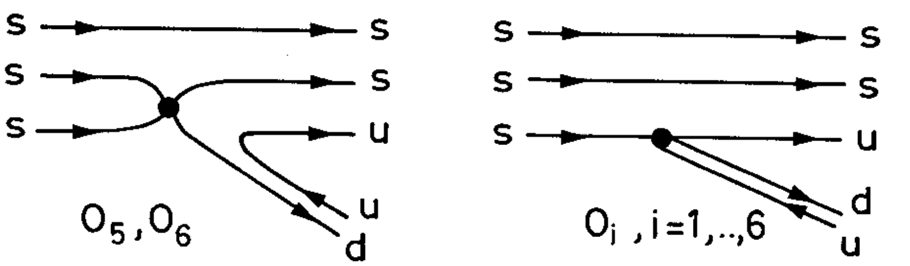

FIG.2

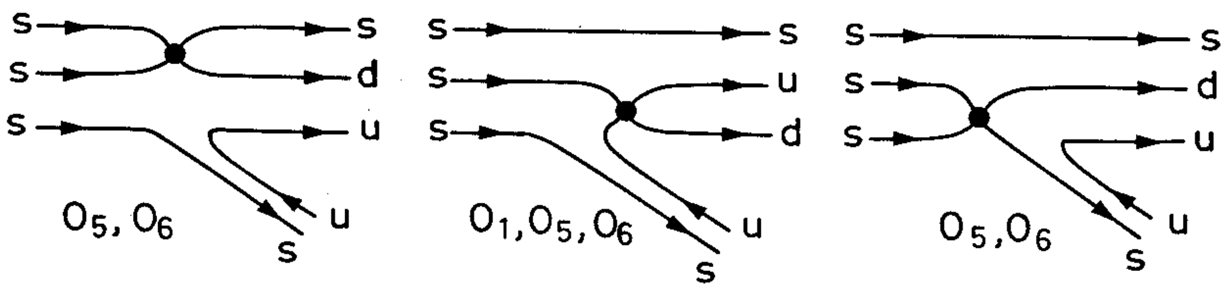

FIG.3

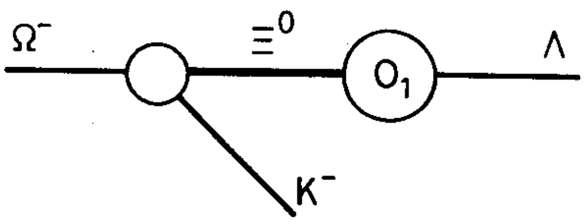

FIG.4