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## Abstract

Since many years the Proton Synchrotron Booster (PSB) high intensity beams have shown head-tail instabilities in all of the four rings at around 100 ms after the injection. In this paper we present the latest observations together with the evaluation of the instability rise time and its dependence on the bunch intensity. The acquired head-tail modes and the growth rates are compared with HEADTAIL numerical simulations, which together with the Sacherer theory points at the resistive wall impedance as a possible source of the instability.



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*Abstract*

Since many years the Proton Synchrotron Booster (PSB) high intensity beams have shown head tail instabilities in all of the four rings at around 100 ms after the injection. In this paper we present the latest observations together with the evaluation of the instability rise time and its dependence on the bunch intensity. The acquired head tail modes and the growth rates are compared with HEADTAIL numerical simulations, which together with the Sacherer theory points at the resistive wall impedance as a possible source of the instability.

## INTRODUCTION

The PSB is made of four stacked similar rings and we reported the last observations of the horizontal instabilities in Ring2 which are observed for the 1.4 GeV cycle (NORMGPS).

We compared the observations of the horizontal bunch profiles with the Sacherer theory. We have also calculated the bunch frequency spectrum for a parabolic bunch. In fact the longitudinal bunch profile seems to be best fitted by a parabola instead of a Gaussian.

In Ring2 from the experimental data we calculated the growth rates of the instability which develops 100 ms after the beam injection: while increasing the bunch intensity we observed that, despite the fact that the bunch length stays the same, the number of nodes decreases. The Sacherer theory foresees an higher number of modes respect to those observed.

## THE INSTABILITIES AND THE PARAMETERS

In this section we report the typical pattern of the losses that can be observed at the PSB while increasing the bunch intensity without the transverse damper. All the following measurements has been taken using only one RF cavity (CO2). The machine parameters are reported in Tab.1.

| | | |
|---|---|---|
| $Q_x/Q_y$ | H/V tune | 4.22/4.4 |
| $R$ | Machine radius [m] | 25 |
| $\alpha$ | Momentum compaction factor | $6.1 \cdot 10^{-2}$ |
| $\xi_x/\xi_y$ | Chromaticity | -0.95/-2.1 |

**Table 1:** PSB parameters for the Ring2 at the time 378 ms during the cycle.

The two instabilities could develop either 100 ms or 200 ms after the injection into the PSB, which approximately correspond to 378 ms and 478 ms in the magnetic cycle. We have only carried out the analysis in the Ring2, after observing the same unstable behavior in all four rings.

The losses might be easily observed while increasing the intensity Fig. 1. In fact it is clear that the instability has a strong dependence on the bunch current. Both the instabilities occur for a current higher that $\approx 2.5 \cdot 10^{12}$ ppb and it has been observed that they might even appear during the same cycle.

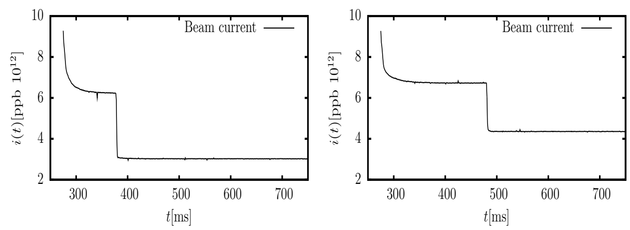



**Figure 1:** Typical pattern of the losses observed in the NORMGPS cycle at 378 ms (left) and 478 ms (right) the injection. The energies are ≃131 MeV and and ≃330 MeV respectively.

The pattern observed from the horizontal pick-up ( both the instabilities start from the horizontal plane [1]) is reported in Fig. 2.

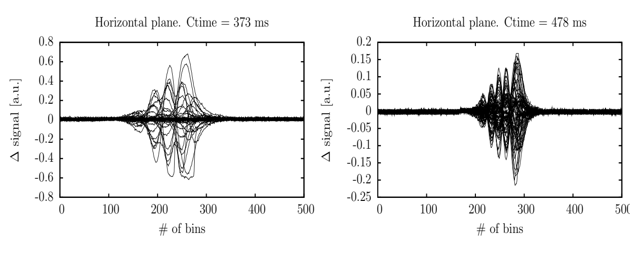



**Figure 2:** Typical pattern from the pick-up signal. Each bin $\Delta t$ is $\Delta t = 2$ns (left) and and $\Delta t = 1$ns (right).

From now on, we will only focus on the first instability (378 ms). In the theory of the head tail instability the chromatic frequency shift indicates which mode will be driven unstable by the wake field. For this purpose we have also measured the horizontal and vertical chromaticity. The curves are reported in Fig. 3.

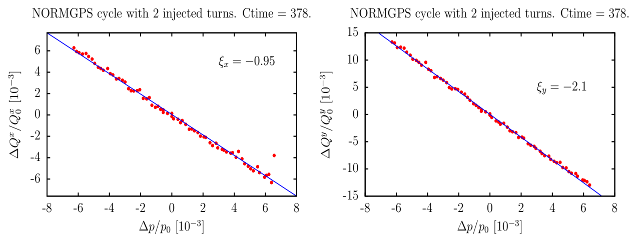



**Figure 3:** Measured horizontal (left) and vertical (right) chromaticities for the NORMGPS cycle.

We have also observed that the bunch length stays almost constant respect to the bunch population Fig. 4.

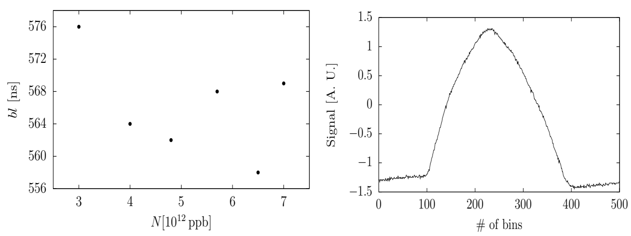



**Figure 4:** Longitudinal bunch length measurements at 378 ms. Left: the acquired bunch length as a function of the bunch intensity. Right: an example of the bunch profile acquired for a bunch intensity of $3 \cdot 10^{12}$ ppb. In the right plot each bin is $\Delta t = 2$ ns.

The longitudinal bunch profile seems to be best fitted by a parabola instead of a Gaussian curve. Since the longitudinal bunch shape plays a fundamental role in the head tail theory we calculated the bunch spectrum for a parabolic bunch.

## THE PARABOLIC BUNCH

In this section we calculate the bunch spectrum for a parabolic distribution in the longitudinal plane. We want to describe the modes with a parabolic longitudinal shape of the bunch instead of a sin/cos shape. As suggested in [2] the pick-up signal has the form

$$f(t;j;n) = -4\frac{(n+1)^2}{bl^2}t^2 + + 4(n+1)\frac{2j+1}{bl}t + 4j^2 - 4j(j+1) \tag{1}$$

where $bl$[s] is the bunch length. Eq. (1) describes the bunch shape for the mode $n$ in the portion of the bunch from $blj/(n+1)$ and $bl(j+1)/(n+1)$. So the full pick-up signal is given at the $k-th$ turn by

$$F(t;n;k) = \sum_{j=0}^{n}(-1)^j\left[\theta(t - bl\frac{j}{n+1}) - \theta(t - bl\frac{j+1}{n+1})\right]\cdot \cdot f(t;j;n)e^{i(\omega_\xi + 2\pi k Q_x)} \tag{2}$$

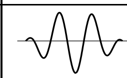
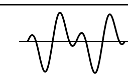
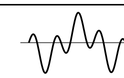
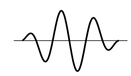
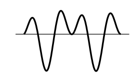
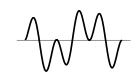
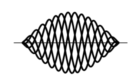
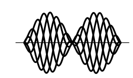
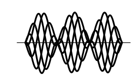
| | $n=0$ | $n=1$ | $n=2$ |
|---|---|---|---|
| $k=0$ |  |  |  |
| $k=4$ |  |  |  |
| $k=0,\cdots,4$ |  |  |  |

**Table 2:** Head tail modes for $\xi = -1$, for a parabolic distribution Eq.(2). We used the parameters of the PSB.

where $Q_x$ stands for the tune and $\omega_\xi = \xi Q_x \beta c/R\eta$. The bunch spectrum is a function of $h(\omega)$ where

$$h(\omega) = \int_{\mathbb{R}} dte^{i\omega t}\sum_{j=0}^{n}(-1)^j\Big[\theta(t - bl\frac{j}{n+1}) - \theta(t - bl\frac{j+1}{n+1})\Big]f(t;j;n). \tag{3}$$

Letting be $A = bl\dfrac{j}{n+1}, B = bl\dfrac{j+1}{n+1}, \alpha = -4\dfrac{(n+1)^2}{bl^2}, \beta = 4(n+1)\dfrac{2j+1}{bl}, \gamma = 4j^2 - 4j(2j+1)$,and introducing the functions

$$\begin{cases} \mathcal{I}_1(\omega;j;n) = \alpha\left[Be^{i\omega B}\left(\frac{B^2}{i\omega} - 2\frac{B}{(i\omega)^2} + \frac{2}{(i\omega)^3}\right) - \right. \\ \qquad \left. -Ae^{i\omega A}\left(\frac{A^2}{i\omega} - 2\frac{A}{(i\omega)^2} + \frac{2}{(i\omega)^3}\right)\right] \\ \mathcal{I}_2(\omega;j;n) = \frac{\beta}{(i\omega)^2}\left[e^{i\omega B}(i\omega B - 1) - e^{i\omega A}(i\omega A - 1)\right] \\ \mathcal{I}_3(\omega;j;n) = \frac{\gamma}{i\omega}\left[e^{i\omega B} - e^{i\omega A}\right] \end{cases} \tag{4}$$

$h(\omega)$ Eq. (3) can be written as

$$h(\omega;n) = \sum_{j=0}^{n}(-1)^j\left[\mathcal{I}_1(\omega;j;n) + \mathcal{I}_2(\omega;j;n) + \mathcal{I}_2(\omega;j;n)\right] \tag{5}$$

The instability occurs if, by the beam spectrum-impedance spectrum interaction, the imaginary part of the coherent frequency shift $\Delta\omega_n$ is positive:

$$\mathcal{I}m(\Delta\omega_n) > 0. \tag{6}$$

The bunch spectrum is given by $|h(\omega)|^2$. In Fig. 5 we show the bunch spectrum for the PSB parameters Tab. 1.

For a bunched beam the coherent tune shift for the mode $n$, involves a sum over the bunch spectrum

$$\Delta\omega_n = -\frac{i}{n+1}\frac{e^2 N_b}{4\pi Q_x bl\gamma m_0 c}\frac{\sum_p Z_\perp(\omega_p)h_n(\omega_p - \omega_\xi)}{\sum_p h_n(\omega_p - \omega_\xi)}, \tag{7}$$

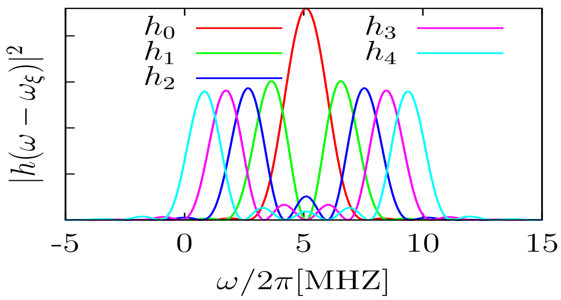



**Figure 5:** Bunch profile for the PSB bunch with a parabolic distribution. We used the data Tab. 1 and the beam parameters at ctime 378 ms.

where $Z(\omega) = (\text{sgn}\omega - i)\dfrac{R}{b^3}\sqrt{\dfrac{2\rho}{\epsilon_0|\omega|}}$, $\omega_p = (p + Q_x)\omega_0 + n\omega_s$, $\omega_s$ is the synchrotron frequency and we assumed $\rho = 10^{-6}\Omega$m and $b = 3.5$cm. For the instability growth rates $\tau_n = \mathcal{I}m(\Delta\omega_n)^{-1}$ we obtain the results in Fig. 6, while keeping the bunch intensity at $N_b = 5 \cdot 10^{12}$ ppb.

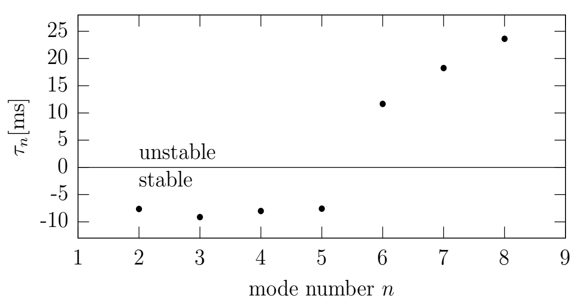



**Figure 6:** Growth rate of the PSB instability as a function of the mode number $n$.

As expected from the Fig. 5 the first unstable mode is the $n = 6$ one using a resistive wall impedance. Using a Broad Band impedance will give and higher mode in a even bigger contrast with the experimental observations.

## MEASUREMENTS OF GROWTH RATES VS. BUNCH CURRENT

In this section we show the measurement of the growth rates as a function of the bunch intensity.

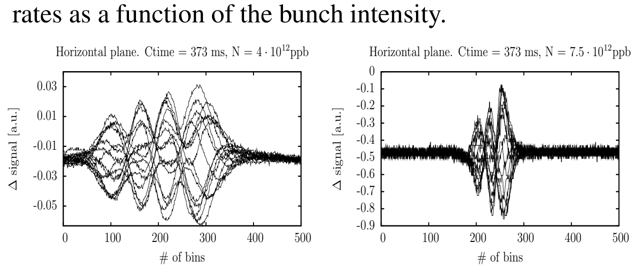



**Figure 7:** Pick-up signal at 378 ms. Left: each bin is $\Delta t = 1$ns. Right: each bin is $\Delta t = 2$ns.

From Fig. 7 we might observe a clear a mode "3" (left) and a mode "2" (right) oscillation while increasing the bunch intensity. In fact from Fig. 8 we see that the higher the intensity, the smaller is the rise time, but that also the mode number $n$ is going from $n = 3$ to $n = 2$. For the first instability at 378 ms we took a set of 5 pick-up data acquisitions for each current and we analyzed the results fitting the beam pick-up envelope with an exponential curve.

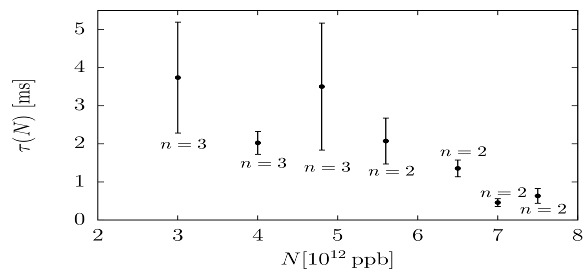



**Figure 8:** Growth rate of the first (100 ms after the injection) PSB instability as a function of the mode number $n$.

The observations and the theory together with HEADTAIL code [3] simulations show discrepancies concerning the numbers of nodes: while from one side numerical simulations agree with the theory, the data exhibit a smaller number of nodes. Using a resistive wall impedance (which drives the smaller $n$ mode unstable) the theory and the simulations show that the fist unstable mode is $n = 6$.

## CONCLUSIONS

We observed clear signs of head tail instability in the PSB and experimentally obtained the growth rates as a function of the bunch intensity. We calculated the modes for a parabolic bunch and found a discrepancy between theory/numerical simulations and experimental observations. This might be explained taking into account space-charge effects: as reported in recent literature [4] space charge forces might play a role in head tail instabilities when the ratio between space charge incoherent tune shift and the synchrotron tune is big $\Delta Q_{\text{s.c.}}/Q_s \gg 1$: in the PSB case under discussion we have $\Delta Q_{\text{s.c.}}/Q_s \approx 50$. Futher numerical and experimental studies are ongoing.

## REFERENCES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[2] F. J. Sacherer,*Transverse bunched-beam instabilities*, CERN/PS/BR 76-21 (1976).

[3] G. Rumolo, F. Zimmermann, *Practical user guide for HEADTAIL*, SL-Note-2002-036-AP (2002).

[4] A. Burov, *Head-tail modes for strong space charge* Phys. Rev. ST Accel. Beams **12**, 044202 (2009).