CRYSTAL AS A LINAC STRUCTURE FOR X-RAYS

T. Tajima
University of Texas, Texas, USA

M. Cavenago
Scuola Normale Superiore, Pisa, Italy*

ABSTRACT
A self-contained description of the crystal X-ray accelerator concept is given here. This concept features the acceleration of channelled muons in a crystal, which is an ultimate focusing system. High energy X rays (~ 40 KeV) are injected into the crystal at the Bragg angle to cause the Bormann anomalous transmission, yielding the proper accelerating field: the crystal superlattice acts as a "soft" irised waveguide for X-rays. In contrast to ordinary linac structures, the accelerating field is much smaller than the transverse X-ray field (by a factor 10^{-4}) and the group velocity of the fields is only a few percent less than the speed of light. The muons lose energy by bremsstrahlung, channelling radiation emission, ionization and phonon lattice interaction.

1. SCALING LINACS AND BEAM PIPES AT THE ATOMIC SIZE

In the well-known linac principle an irised waveguide supports electromagnetic waves which have longitudinal components suitable for particle acceleration. The wavelength λ of the fields, which determines the scale of the linac cells, is of the order of decimeters in the present machines. Considerable speculation [1] is now devoted to the most economically convenient wavelength, by assuming estimates about the feasibility of new microwave sources. There are two considerations which suggest smaller wavelengths:

i) the available field energy density $E^2/8\pi$ scales as λ^{-2}.
ii) we require that the luminosity

$$L = \frac{fN^2}{\pi a_0^2} \tag{1}$$

where N is the number of particles in the bunch, f is the repetition rate and a_0 is the beam radius, should scale as the particle energy ε to the inverse square power. Evidently $fN\varepsilon$ is bounded by available power, so that we need smaller beams and stronger focusing fields for higher energy colliders, which result in smaller pipes.

The optimum wavelength depends strongly on technological considerations of all the components of the collider: with conventional equipment λ should be of the order of centimeters according to Palmer [1].

New linac concepts may change this optimum and here we are concerned with the most fundamental limitations. The smallest scale we can think of is the atomic one, which can now be manipulated by man in a certain degree. Indeed alternate strata of similar crystals may be stacked to obtain a superlattice [2]. In comparison it is more difficult to machine gratings or droplets at the micron scale.

There is a fundamental change when we scale the linac structure at the atomic size. In an ordinary linac the microwaves cannot penetrate the wall: we call it a hard wall regime (see Fig. 1). When the photon energy becomes of the order $\hbar\omega \sim mc^2\alpha^2 \sim 30$ eV, corresponding to a scale length $\lambda \sim 500$ Å, the metallic wall begins to strongly absorb the photon; note that $mc^2\alpha^2$ is also of the order of the plasma frequency ω_{pe} corresponding to the crystal electron density.

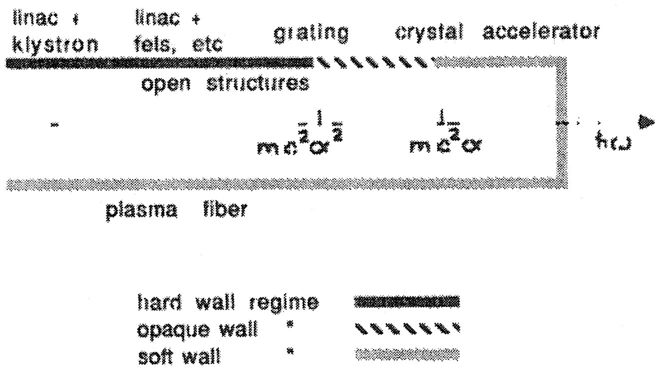

Fig. 1 Accelerator types against frequency

When $\hbar\omega \sim mc^2\alpha$ the photon wavelength is of the order of the lattice periodicity, so that the photon and the crystal have the same relative proportions of a microwave and a linac structure. Moreover, as the photon energy greatly exceeds $mc^2\alpha^2$ and becomes $> mc^2\alpha$ the crystal structure ceases to be opaque. Indeed the mean free path of the photon is given by Bethe Bloch as $\ell_i = (3/2^8\pi)\ a_B^{-2}\ \alpha^{-1} n^{-1}\ (2\hbar\omega/Z^2_{eff}mc^2\alpha^2)^{7/2}$ where a_B is the Bohr radius, n is the electron density and Z_{eff} the effective charge of the lattice ion

Note that the photons go straight into the electron cloud of the lattice ions, thus developing a small longitudinal component: we say that the crystal is a soft linac structure

The idea of using a crystal as a linac structure may also be considered to be an evolution of the plasma fiber accelerator [3] (see Fig. 1): indeed, increasing the plasma density to increase the longitudinal field, we reach the solid state electron density.

2. CRYSTAL X-RAY ACCELERATOR CONCEPT

In Fig. 2 we draw a schematic diagram of the X-ray crystal accelerator concept [4]. The muons travel almost parallel to the axis z of the rows of atoms, so that only the scattering from a row as a whole is important, but the collisions with each single atom of the row are not: this is the channelling effect [5]. The X-rays are injected at the Bragg angle θ_B with the axis z given by:

$$\frac{\lambda}{2b} = \sin\theta_B \tag{2}$$

where 2b is the distance between two rows, but they travel along z inside the crystal: indeed the coherent reflections between the crystal planes generate a standing wave pattern along x, with modes at the crystal planes resulting in reduced absorbtion (Bormann effect [6]). Note that the X-rays split into two beams at the exit from the crystal. The group velocity of the X-rays is closer to the speed of the light, and satellite longitudinal waves with phase velocity equal to c are generated by beating with the periodicities of the crystal electron density as discussed in the appendix. From Eq. (A.3) the amplitude of this wave is 10^{-4} that of the X-rays. Equation (A.2) shows that it is convenient to use the periodicity of a superlattice instead of the shorter crystal periodicity. Each stratum of the superlattice may contain even only one layer.

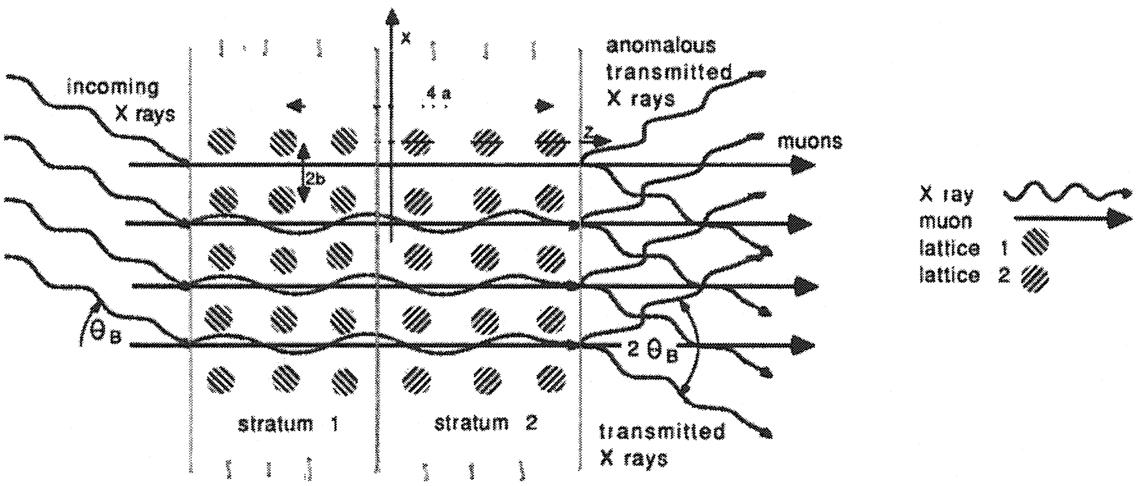

Fig. 2 The Bormann anomalous transmission of X-rays and the channelled positive muon beam. Only five crystal planes instead of some hundreds are shown, and only one superlattice period.

We prove elsewhere [7] with the adiabatic invariants [8] that the effective potential ϕ determines the average trajectory of the particle:

$$\phi(x,y) = \frac{1}{a}\int_0^a dz\ \phi(x,y,z) \tag{3}$$

In Fig. 3 we show the profile of ϕ, stressing the fact that corrections must be added to Eq. (3) to be valid near the rows of atoms; eventually the adiabatic approximation breaks down. The positive muons are confined away from the rows of atoms. The negative muon is attracted to a row of atoms, and must have an angular momentum l_z if it is not to fall on this row [7].

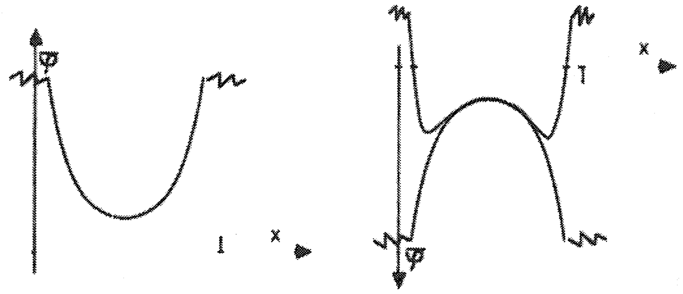

Fig. 3 Chanelling potential versus distance from a row of atoms for negative (left) and positive (right) particles. Note the contribution of the angular momentum in the right hand graph. The curves are rough where the adiabatic approximation breaks down.

There are studies, both theoretical and experimental, of the channelling phenomena [5]. In high energy regimes they were finalized for bending proton beams with crystals [9-11] or to obtain hard photons from electron beams [12,13].

We recall now the main features of the single-particle muon motion in the crystal. The ionization losses are about 5 MeV/cm for a muon, and we take this datum in our estimate.

The oscillation of the particle in the average channelling potential ϕ acts as an antenna, emitting the so-called channelling radiation [14,15]. (The channelling radiation is also called synchrotron radiation due to the fact that a sinusoidal path is approximated with arcs of circles, and vice versa.) But the particle also makes fast small oscillations with frequency $2\pi c/a$ and harmonics, in response to the remaining part of the electrostatic potential $\check{\phi} = \phi - \bar{\phi}$. This is the classical origin of the bremsstrahlung whose features are easily computed with the virtual photon approach, which is directly applicable to a classical particle moving with $r = \sqrt{x^2 + y^2}$ being constant or slowly changing.

The energy losses due to the bremsstrahlung are roughly proportional to the energy as usual,

$$\frac{d\varepsilon}{dz} = -\lambda_R^{-1}(r)\varepsilon$$

but the radiation length λ_R increases [7] with the distance r between the particle and the nearest row of atoms in a dramatic way:

$$\lambda_R^{-1}(r) = 4\alpha Z_{eff}^2 r_c^2 \frac{1}{(2b)^2 a} K_0'^2\left(\frac{2\pi}{a} r\right) \ln\left(\frac{2\gamma\alpha m_e}{m_c}\right) \tag{4}$$

where 2b is the distance between rows, a is the distance between two adjacent atoms of the same row (a = 2b), $r_c = e^2/m_c c^2$ with m_c the mass of the channelled particle and K_0' is the derivative of the modified Bessel function.

For a negative muon, the typical distance from the nearest row of atoms may be $r \approx {}^1/_2 b \approx {}^1/_4 a$; therefore taking $Z_{eff} \sim 20$ we obtain a radiation length of 10 km. Therefore bremsstrahlung losses are dominant above 5 TeV for a negative muon; for a positive muon $r = {}^1/_2 a$ so that bremsstrahlung losses are irrelevant.

Channelling radiation depends on the amplitude of channelling oscillations and it has a complicated behaviour for a strong oscillation, i.e. when $\gamma v_\perp$ is not neglible with respect to the speed of light [5]; however this radiation seems to be reduced by an effect of relativistic detuning.

Therefore the first test for our concept is to compare the energy losses, 5 MeV/cm, to the acceleration at the maximum transverse field sustainable by the crystal bonds; this field is very high, 10^{13} V/cm, for the very reason that X-rays interact weakly with the crystal. In order to obtain a net muon acceleration "only" a transverse field of about 10^{11} V/cm is required, which may be feasible by focusing the X-rays to a tiny spot and/or by guiding them with an optical fiber. There are other proposed schemes of acceleration in solid matter [16,17], which differ from our rippled-waveguide concept in the acceerator mechanism.

3. REMARKS ABOUT BEAM EMITTANCE: CRYSTAL AS THERMOSTAT

A second feasibility test must consider the dechannelling length of high-energy muons. The dechannelling happens when a particle receives a hard transverse kick, where hard means the kick is greater than the maximum acceptance of the channelling potential and soft means the opposite. The origins of the kicks are the emission of a bremsstrahlung photon, the emission of a channelling radiation photon, the ionization of a crystal electron and the scattering with a lattice phonon.

Soft kicks are also important: they excite the channelling oscillation. However the average effect of the energy losses is the damping of channelling oscillations. For example the bremsstrahlung radiation is emitted in the direction of the channelling oscillation motion and thus exerts a friction on this motion. Therefore the crystal acts also as a thermostat, because it excites betatron oscillations in the channelled beam at a given rate while it damps the existing betatron oscillations.

Note that the probability of hard kicks increases rapidly as the particle distance r from the nearest row of atoms decreases. For example in the case of bremsstrahlung emission the mean free path between hard kicks is similar to the radiation length (4), which depends on r in a dramatic way.

Therefore progressive heating/cooling of the beam and fatal scattering probability are strongly related. This point is important for all solid state accelerators |16,17|.

We are working on the transverse master equation which governs the evolution of the muon beam considering kicks, and the details of heating/cooling due to soft kicks from the four processes stated above.

ACKNOWLEDGEMENTS

We thank Prof. N. Rostoker for his encouragement and interest in the project. We thank Profs. L. Foa, K. Zioutas and M Spighel for introducing us to the relevant literature on channelling. The work was partly supported by an NSF grant, the Air Force, the U.S. Department of Energy Contract DE-FG05-80ET53088 and the Istituto Nazionale Fisica Nucleare, sezione di Pisa.

* * *

REFERENCES

|1| R. Palmer, "Parameter interdependence", this Workshop.

|2| L Eski and R. Tsu, IBM J. Res. Develop., 14, 61 (1970); also L. Esaki and L.L. Chang, Phys. Rev. Lett., 33, 495 (1974).

|3| K. Mima, T. Ohsuga, H. Takabe, K. Nishihara, T. Tajima, E. Zaidman and W. Horton, Phys. Rev. Lett., 57, 1421 (1986).

|4| T. Tajima and M. Cavenago, "Crystal X-ray accelerator", submitted to Phys. Rev. Lett.

|5| J. Linhard, Kong. Danske Vid. Selsk. mat.-fys. Medd., 34, No. 14 (1965) (in English).

|6| B.W Battermann and H. Cole, Rev. Mod. Phys., 36, 681 (1964); also P.O. Ewald, 50 Years of X-ray Diffraction (International Union of Crystallography. Utrecht, 1962), p. 242.

|7| M Cavenago and T. Tajima, "Bremsstrahlung from channelled charged particles: application to a crystal X-ray accelerator" (available on request).

|8| M. Kruskal, Journal Math. Phys. 3, 806 (1962).

|9| E. Uggerhoj, Physica Scripta, 28, 331 (1983).

|10| R.A. Carrigan, Jr., The application of channelling in bent crystal to charged particles beams, in Relativistic Channelling, NATO ASI Series, Plenum.

|11| R.A. Carrigan, Jr., and A. Tsyganov, Nucl. Instr. Meth. 194, 205 (1982).

|12| M. Atkinson, J.F. Bak, P.J. Bussey, P. Christensen, J.A. Ellison, R.J. Ellison, K.R. Eriksen, D. Giddings, R.E. Hughes-Jones, B.B Marsh, D. Mercer, F.E. Meyer, S.P. Moller, D. Newton, P. Pavlpoulos, P.H Sharp, R. Stensgaard, M. Suffert and E. Uggerjoh, Phys. Lett., 110B, 162 (1982).

[13] A. Belkacem, G. Bologna, M Chevallier, N. Cue, M.J. Gaillard, R. Genre, J.C. Kimball, R. Kirsch, B. Marsh, J.P. Peigneux, J.C. Poizat, J. Remillieux, D. Sillou, M. Spighel and C.R. Sun, Phys Lett , 177B, 211 (1986).

[14] M.A. Kumanov, Phys. Stat. Sol. (b), 84B, 581 (1977).

[15] M.A. Kumakhov and Ch. G. Trikalinos, Phys. Stat. Sol. (b), 99B, 449 (1980).

[16] R. Hofstadter, "The atomic accelerator", Stanford university HEPL Report 560 (1968).

[17] P. Chen and R.J. Noble, "Channelled particle acceleration by plasma wave in metals", SLAC-PUB-4187, SLAC/AP-55 (1987)

APPENDIX

CRYSTAL SUPERLATTICES AS RIPPLED WAVEGUIDES

In this Appendix we compute analytically the longitudinal field generation in a crystal, obtaining the results (A.13) and (A.11), which show the importance of the superlattice.

As the photon frequency ω is much greater than the atomic frequencies, the current is given by $\vec{j} = (\omega^2{}_{pe}/i\omega)\vec{E}$, where the "local plasma frequency" $\omega^2{}_{pe}(\vec{x}) = 4\pi e^2 n_e(\vec{x})/m$ depends on the electron density $n_e(\vec{x})$.

Considering fields of the form $A = R(e^{-i\omega t}A(\vec{x}))$ and choosing the Lorentz gauge $\phi = \text{div}\,\vec{A}/i\omega$, we start from the Lagrangian

$$L = \int d^3x(\omega^2 - \omega^2_{pe})|A|^2 - c^2|\nabla\vec{A}|^2 - c^2(\omega^2_{pe}/\omega^2)|\text{div}\,\vec{A}|^2 , \tag{A.1}$$

whose last term will be subsequently negligible. We look for a solution of the vector $A_i = a_i(x,y)f_i(z)$, where the repeated index $i = 1,2,3$ does not mean summation. Variations with respect to a_i and f_i give rise to two coupled equations

$$[\partial^2_z + c^2k^2_z(z)]\, f_i(z) = 0 \tag{A.2}$$

$$[\nabla^2_\perp + c^2k^2_\perp(x,y)]\, a_i(x,y) = 0 \tag{A.3}$$

where the effective wave numbers are

$$c^2k^2_z(z) = \omega^2 - \langle\omega^2_{pe}\rangle_\perp - \langle(|\nabla_\perp a_i|^2/|a_i|^2)\rangle$$

and

$$c^2k^2_\perp(z) = \omega^2 - \langle\omega^2_{pe}\rangle_z - \langle(|\partial_z f_i|^2/|f_i|^2)\rangle_z \;;$$

the averages $\langle\rangle_\perp$ and $\langle\rangle_z$ are, respectively, the averages on the (x,y) plane with the weight $|a_i|^2$ and on the z axis with the weight $|f_i|^2$. Since $c^2k^2 \sim \omega^2 \gg \omega^2{}_{pe}$, Eq. (A.2) may be solved by the WBK method: $f_i = k^{-1/2}(z)\exp[iv(z)]$ with $\partial_z v = k_z$.

The general solution of Eq. (A.3) has the Bloch form:

$$a_i(x,y) = e^{ik_{Bx}x + ik_{By}y}\, u(x,y) \tag{A.4}$$

where u is periodic on the lattice. For the sake of simplicity we drop y-dependence henceforth.

We note $k_\perp^2(x)$ is a periodic function with period 2b; let $k_b = \pi/b$. Fourier components of a_i, i.e.

$$a_0 = u_0 e^{ik_{Bx}x} \quad \text{and} \quad a_N = u_N e^{i(k_{Bx}-Nk_b)x}$$

are then coupled.

When $k_{Bx} \simeq 1/2\ Nk_b$, these two components have approximately the same frequency, therefore they resonate. From (A.3) and (A.4) we obtain

$$\begin{aligned} &[V_0 - k_{Bx}^2]\, u_0 + V_{-N}u_N = 0 \\ &V_N u_0 + [V_0 - (k_{Bx} - Nk_b)^2]\, u_N = 0 \end{aligned} \qquad \text{(A.5)}$$

where the coupling coefficients are:

$$V_N = (2b)^{-1} \int_0^{2b} dx\, k_\perp^2(x) \exp[-iNk_b x]$$

Averaging (A.2) over x and y, we obtain the dispersion relation

$$\omega^2 = \langle\omega_{pe}^2\rangle|_z + c^2 \bar{k}_z^2 + c^2 V_0^2$$

where $\langle\rangle|_z$ is the average over both z and (x,y) and

$$\bar{k}_z = \lim_{z\to\infty} \phi(z)/z = \sqrt{\langle|\partial_z f_i|^2\rangle_z} \ .$$

Solving the determinant equation for V_0 in Eqs. (A.5) we obtain

$$\omega^2 - \langle\omega_{pe}^2\rangle|_z - c^2\bar{k}_z^2 = \frac{1}{4} N^2c^2k_b^2 + c^2(k_{Bx} - \frac{1}{2} Nk_b)^2 - sc^2 \sqrt{V_N V_{-N} + N^2k_b^2(k_{Bx} - \frac{1}{2} Nk_b)^2} \qquad \text{(A.6)}$$

where s = -1 corresponds to a standing wave with nodes near the atomic planes: $a_i(x) = a_i\sin(1/2Nk_b x)$. We choose $k_{BX} = 1/2\ Nk_b$ for our accelerator because then the transverse group velocity $\partial\omega/\partial k_{Bx} = 0$, that is the Bormann effect.

The slow wave is generated by sidebands: E_z (or A_z) has the main wave number k_z, but $\phi(z)$ has a slight modulation. When there is a component with wave number k in the plasma frequency, the phase is given by:

$$\phi(z) = \bar{k}_z + \frac{\omega_{pe}^2(k)}{2\bar{k}_z kc^2} \cos(kz) \qquad \text{(A.7)}$$

where

$$\omega_{pe}^2(k) = 2a^{-1} \int_0^a dz \langle\omega_{pe}^2\rangle_I \sin(kz) .$$

The amplitude of the ℓ-th satellite $E_z(\ell)$ with $k_z = \bar{k}_z + \ell k$ is

$$E_z(\ell) \approx \frac{Nk_b\bar{k}_z^{-1/2}}{2\omega} a_x i^\ell J_\ell \left(\frac{\omega_{pe}^2(k)}{2\bar{k}_z kc^2}\right) \cos\left(\frac{1}{2} Nk_b x\right) \tag{A.8}$$

Note that even N modes are required to accelerate positive muons which are channelled around x = b.

The amplitude for the carrier wave is

$$E_x = i\omega a_x \bar{k}_z^{-1/2} \sin\left(\frac{1}{2} Nk_b x\right) .$$

Thus the coupling ratio R for $\ell = 1$ which is the largest satellite, is:

$$R = \frac{1}{4} N \frac{k_b\omega_{pe}^2(k)}{k\omega^2} . \tag{A 9}$$

We require that the phase velocity of this sideband:

$$v_{ph} = \frac{\omega}{\bar{k}_z + k} = c \frac{\sqrt{\bar{k}_z^2 + \frac{1}{4} N^2k_b^2 + \omega_{pe}^2(0)}}{\bar{k}_z + k} \tag{A.10}$$

be equal to c for the high energy acceleration.

Introducing the parameter $\xi \equiv k_b/2k$, in the high energy regime we have:

$$\bar{k}_z = \frac{1}{4} \xi(N^2 - \xi^{-2})\, k_b \tag{A.11}$$

$$v_{gz} = c(N^2 - \xi^{-2})(N^2 + \xi^{-2})^{-1} \tag{A.12}$$

$$R = 8N\xi^3(N^2\xi^4 + 2N^2\xi^2 + 1)^{-1} \omega_{pe}^2(k)c^{-2}k_b^{-2} \tag{A 13}$$

It is quite straightforward to check that the sidebands of the lattice periodicity with $k = 2\pi/a \sim k_b$ require $N > 2$ in order to propagate forward into the crystal.

In the case of a superlattice with periodicity $k = k_s$ the parameter ξ is big, so that v_{gz} is near the speed of light also for $N = 1,2$. For example $k_s = 0.05\ k_b$; then for $N = 1$ we get $k_z = 2.5\ k_b$, $v_{gz} = 0.98c$ and $R = 0.8\ \omega^2_{pe}(k_s)c^{-2}k_b^{-2}$. Similarly for $N = 2$ we get $\bar{k}_z = 10\ k_b$, $v_{gz} = 0.996\ c$ and $R = 0.4\ \omega^2_{pe}(k_s)c^{-2}k_b^{-2}$.